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**HIGH PERFORMANCE
SPINDLE DESIGN METHODOLOGIES
FOR HIGH SPEED MACHINING**

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1 Introduction and motivation

In many components especially for aerospace industry up to 90% of the material is removed from the workpiece during cutting therefore it is necessary to focus the research efforts on the Material Removal Rate maximization in order to decrease the manufacturing time and the corresponding costs. The importance of the Material Removal Rate (MRR) improvement can be well understood looking at Fig. 1.1 that depicts a cutting phase of an aeronautical component.

Basically the MRR is limited by the spindle power availability and/or by the occurrence of the instability of the cutting process otherwise called chatter phenomenon.

Generally the first kind of limitation occurs during heavy roughing machining at low cutting speeds while the second limiting aspect becomes very crucial especially in high speed machining where the spindle often plays an important role.



Fig. 1.1: Relevance of the MRR maximization

Chatter is a problem of instability in the metal cutting process. The phenomenon is characterized by violent vibrations, loud sound and poor quality of surface finishing. Chatter causes a reduction of the life of the tool and affects the productivity by interfering with the normal functioning of the machining process. These vibrations can be very dangerous also for some important machine tool components such as the spindle bearing systems.

The cutting process instability has affected the manufacturing community for years and has been a popular topic for academic and industrial research. The regeneration phenomenon proposed first by Tobias [1] and Tlustý [3] has been referred to by any researcher investigating chatter instability. A feedback model

explaining chatter as a closed loop interaction between the structural dynamics and the cutting process was presented by Merrit [4].

The stability lobes diagram is used for any study on chatter, since it gives a quantitative idea of the limits of stable machining in terms of two cutting parameters that the machine tool user has to define for any machining operation: the width of contact between tool and the workpiece otherwise called the axial depth of cut and the spindle speed.

In Fig. 1.2 a typical stability lobe diagram has been reported.

The lobes split the plane into two regions: in the cutting operations that can be located in the upper region the chatter vibration occurs while operations related to the lower zone are stable ones.

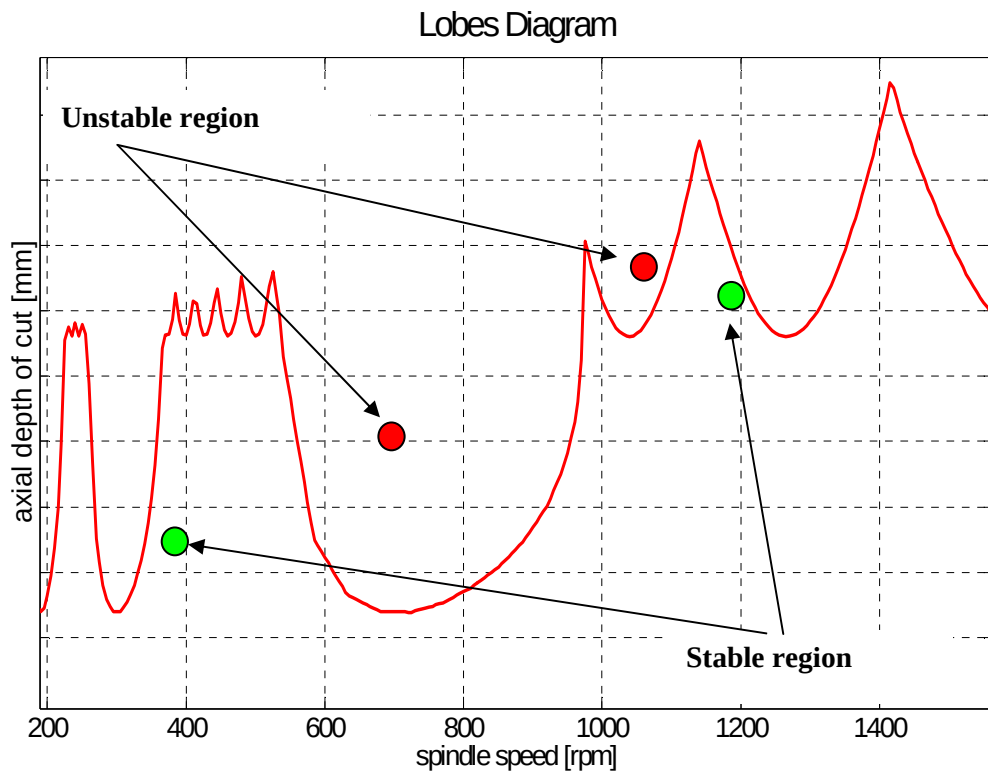


Fig. 1.2: Lobes diagram – regenerative chatter analysis

In Fig. 1.3 it is possible to observe the chatter vibration effects on the quality of the machined surface; a significant comparison between stable and unstable is reported.

The milling operations were performed by means of *Jotech* (built by Jobs, Italy). The stability limit is strongly affected by the complex dynamic behaviour of the machine tool and in some cases also by the workpiece-table system. We can roughly assert that each sequence of lobes of the stability chart is generally linked to a machine tool eigenmode therefore in order to increase the MRR it is useful to analyze and comprehend the complex machine dynamics and moreover

how each component affects the performance of the overall system. In Fig. 1.4 a schematic representation of a machine tool system and its main components is presented.

In this thesis the attention has been focused on the High Speed Machining (HSM) thus, the chatter instability and the spindle performances are the most critical aspects.

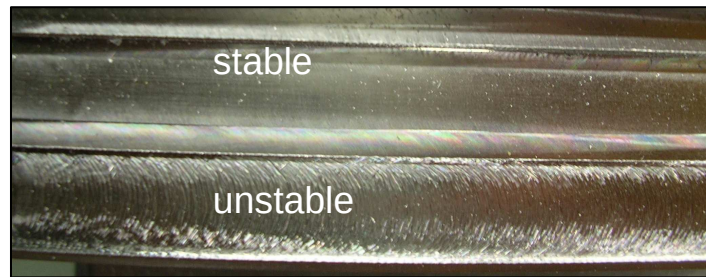


Fig. 1.3: Machined surfaces – stable cutting vs unstable cutting

Fig. 1.5 explains the reason of the importance of the spindle dynamic behaviour in High Speed Machining: even if the machine structure could be the limiting aspect for low cutting speeds its contribute, for the typical structure of the lobes diagram, vanishes at higher cutting speeds and consequently the spindle performance becomes crucial.

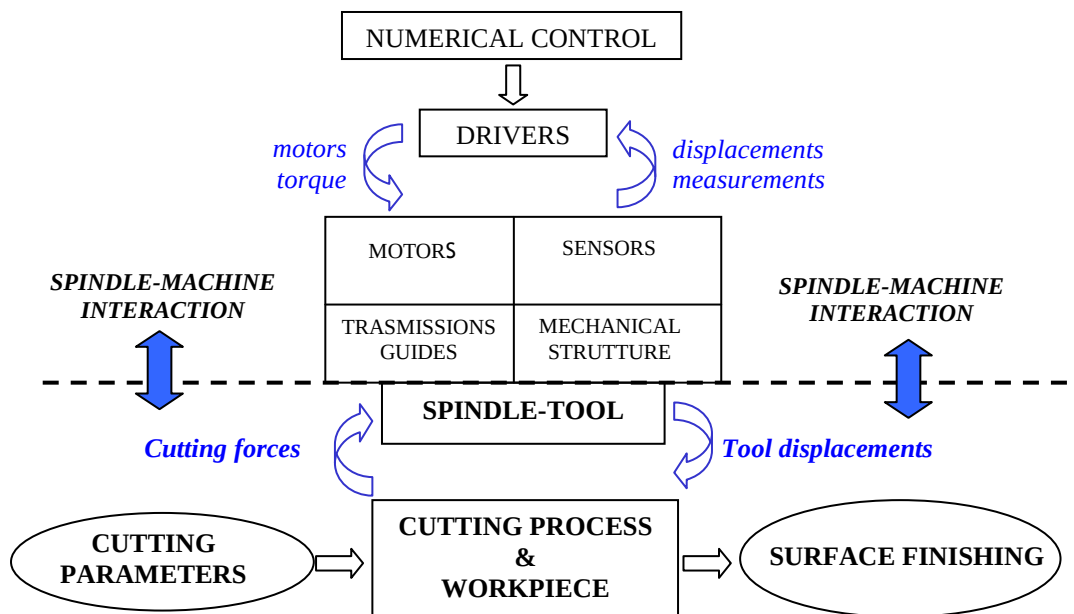


Fig. 1.4: Machine Tool system – main components

It is therefore very important to comprehend the complex behaviour of the spindle systems in order to test different design solutions to get higher cutting performances; this can be more easily done modelling the spindle systems.

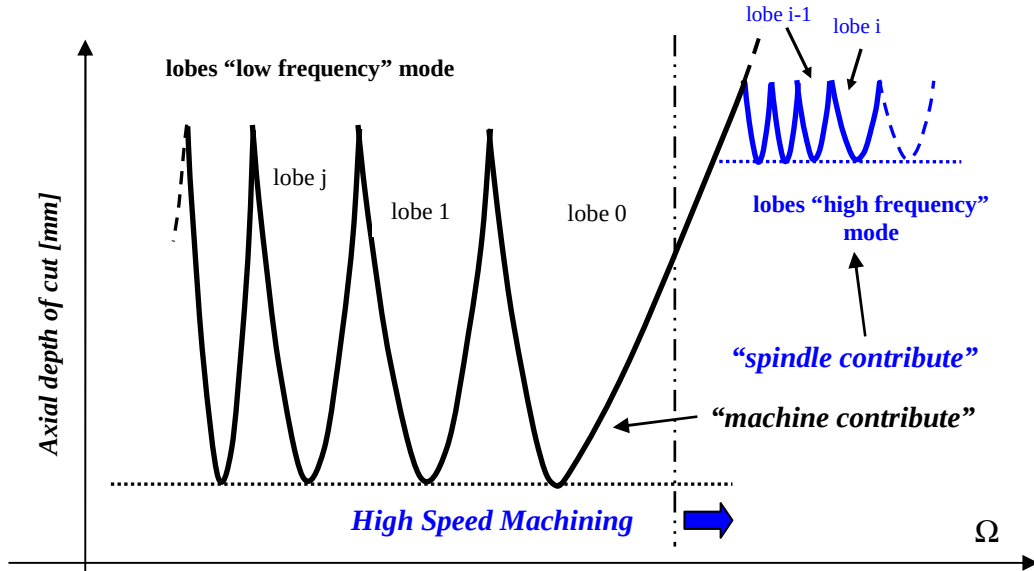


Fig. 1.5: Spindle relevance in HSM

Having described the general relevance of the topics a more accurate description of the regenerative chatter instability, the chatter suppression techniques and the methods to analyze and to develop spindle systems is necessary in order to obtain the desired enhancements. The main literature works linked to these topics have been presented in this chapter.

Chapter 2 presents the results of an experimental approach that was used to define some guidelines to create accurate spindle models. An alternative spindle design solution that uses a spindle shaft made by composite material is presented in order to show the importance and the utility of the modelling efforts.

Chapter 3 describes the role of the dynamic interaction between the spindle system and the machine tool on the overall performances and it suggests how to consider these aspects.

Chapter 4 deals with a chatter suppression technique called Spindle Speed Variation (SSV) that consists into continuously varying the spindle speed in order to break the regenerative phenomenon that brings to instability. Some guidelines to select the more appropriate SSV parameters will be defined.

Moreover, in Chapter 5 an active vibration control technique has been applied to a spindle system in order to improve its performance during cutting.

1.1 Chatter instability: state of the art

The machine tool chatter vibrations occur basically due to a self-excited mechanism in the generation of chip thickness. Obviously this phenomenon is caused by the machine/workpiece flexibility. During the cutting the dynamic of the system is forced by the cutting forces, the consequent tool vibrations leave an oscillatory surface finishing on the machined surface. In this contest the instantaneous chip thickness doesn't depend only on the actual position of the relative cutting edge but obviously on the track left on the workpiece by the last cutter that formerly removed material as presented in Fig. 1.6.

Fig. 1.6 shows a dynamic milling model in which is underlined the regenerative phenomenon. Obviously the instantaneous chip thickness modulation depends on the spindle speed because it affects the time delay between two consequent tooth passings. In the regenerative theory the time delay plays the main role.

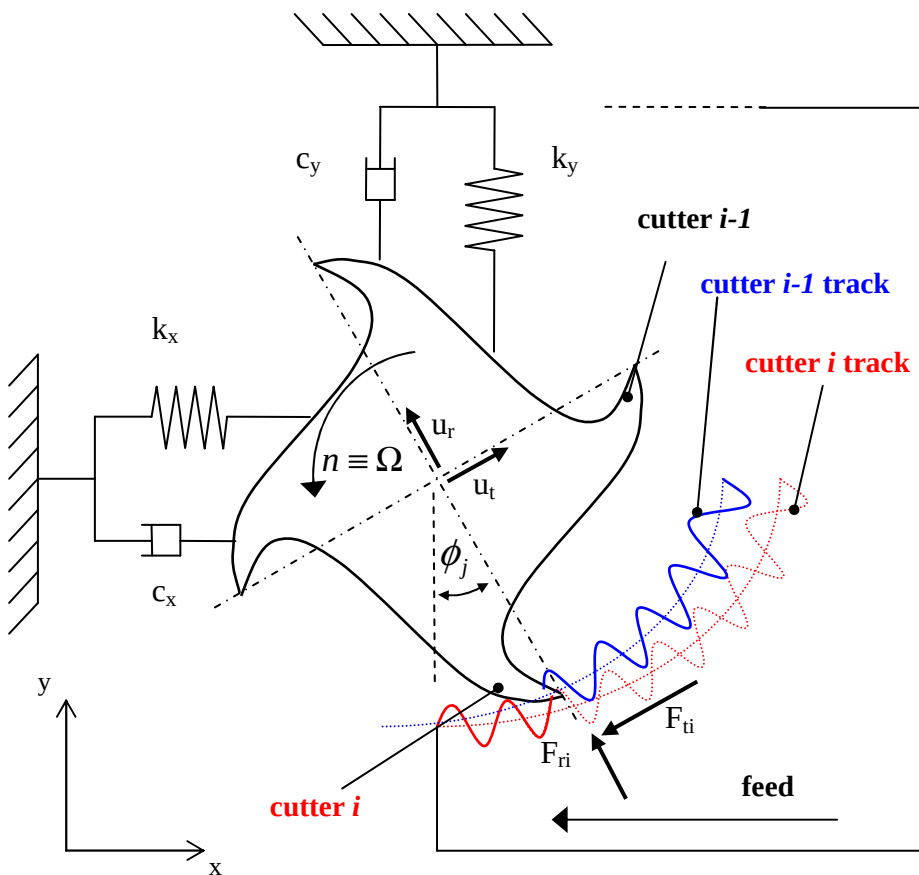


Fig. 1.6: Dynamic model of milling – regenerative effect

The regenerative phenomenon was studied first analyzing the easiest orthogonal cutting case: only some particular turning operations can be approximated as orthogonal cutting case. A stability analysis can be performed in order to seek

how the free depth of cut is affected by both the machine parameters and the cutting process ones.

The stability analysis is performed on the closed loop system depicted in Fig. 1.7 increasing the depth of cut, Merit [4].

Thusty [3], [30] and Koenigsberger [18] derived the fundamental stability theory and they found that the free axial depth of cut is inversely proportional to the real part of the transfer function between the tool and the workpiece.

Tobias [1] presented a similar stability analysis but he added the process stiffness effect and moreover he studied the phase shift between the inner and outer tracks. He presented a method to generate the stability lobes diagram.

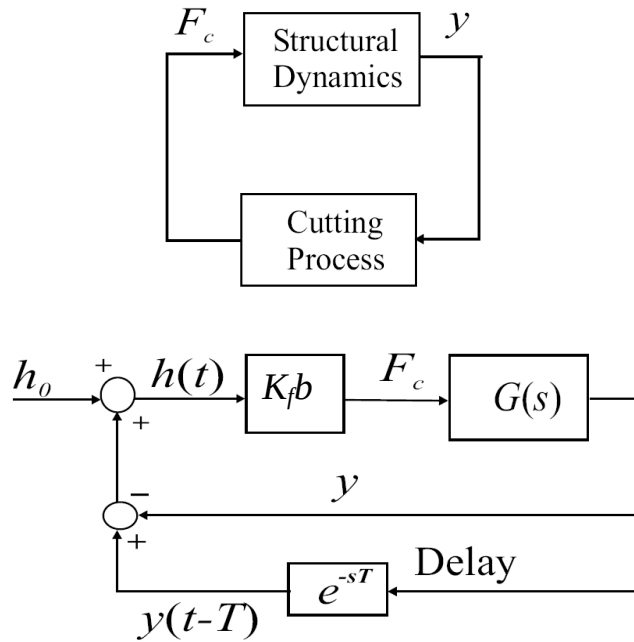


Fig. 1.7: Closed loop representation of chatter for turning

The work done by Tobias [1], Thusty [3] and Merit [4] can be used to draw stability lobes diagram only for turning operations that can be considered similar to the orthogonal cutting condition where cutting forces and chip thickness generation mechanism are time invariant.

In the case of milling the chip thickness, the cutting forces vary and are intermittent so that the governing equation for regenerative chatter in milling is a periodic delay differential equation, which cannot be analyzed directly by frequency domain techniques. Therefore, time domain simulations are extensively used to demonstrate various aspects of chatter instability in milling, see Ganguli [100].

Thusty et. al. [3], [5] presented a method of generating stability lobes using time domain simulations of milling operations.

Moreover, they considered the velocity dependent process damping and the basic nonlinearity in the chatter which arises when the tool jumps out of the cut due to excessive vibration.

The velocity dependant damping effects are well summarized in Fig. 1.8.

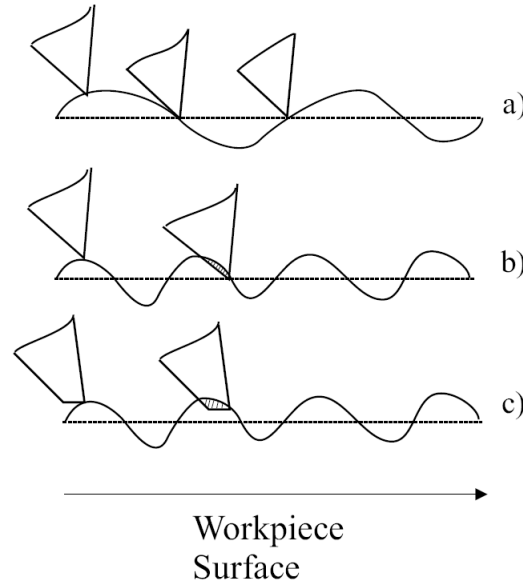


Fig. 1.8: Rubbing of the tool – speed (a, b) and wear effects(c)

Altintas [6] incorporated the process damping in the time domain model and moreover he predicted the surface finish obtained with the occurrence of chatter vibration.

This phenomenon was also considered by Lee in [28].

The influence of various physical parameters, such as the type of milling operation, the feed direction and changes in the structural flexibility on the stability of milling can be investigated by time domain simulations.

Haisel in [29] presented a method to identify chatter vibration characteristics from surface measurements.

Although time domain simulations methods (Smith [9]) are quite powerful since they allow the nonlinearities and various tool geometries a pure analytical model is desirable in order to obtain a rapid and easy estimation of the chatter free regions.

Unlike turning, where a single tool is in contact with the workpiece surface, milling involves a rotating cutter with multiple teeth cutting the workpiece. Consequently the governing differential equation is a Differential Delayed Equation (DDE) with periodic coefficients. Frequency domain techniques cannot be directly applied unless some approximations are made in the system formulations.

The milling chatter problem can be studied using the cutting process model depicted in Fig. 1.6. The first author that proposed a solution was Altintas et. al. in [10] and [11] but the issue was formerly faced by Shridar [7] that proposed

the first detailed mathematic model of the dynamic milling process and Minis et. al. [8].

A brief explanation of the approach will be further presented.

Where τ is the time delay, n is the angular velocity and N is the number of teeth.

$$\text{Eq. 1.1:} \quad \tau = \frac{60}{n \cdot N}$$

It is possible to express the instantaneous chip thickness:

$$\text{Eq. 1.2:} \quad h(\theta) = f_t \sin \phi_i + \begin{bmatrix} \sin \phi_j & \cos \phi_j \end{bmatrix} \cdot \begin{Bmatrix} x(t) - x(t - \tau) \\ y(t) - y(t - \tau) \end{Bmatrix}$$

And using the Koenisberger et. al. relationships [2] and [18]:

$$\text{Eq. 1.3:} \quad \begin{Bmatrix} F_{xi} \\ F_{yi} \end{Bmatrix} = \begin{bmatrix} -\cos \phi_j & -\sin \phi_j \\ \sin \phi_j & -\cos \phi_j \end{bmatrix} \cdot \begin{Bmatrix} K_t \cdot h(\phi_j) \cdot b \\ K_r \cdot K_t \cdot h(\phi_j) \cdot b \end{Bmatrix}$$

Where f_t is the feed rate, K_t and K_r are the cutting coefficients, F_{xi} and F_{yi} are the projections of the radial and tangential components on the x and y directions.

Considering the contributes of all the cutting edges N and:

$$\text{Eq. 1.4:} \quad x(t) - x(t - \tau) = \Delta x, \quad y(t) - y(t - \tau) = \Delta y$$

$$\text{Eq. 1.5:} \quad \begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = K_t \cdot b \cdot \sum_{i=1}^N \left(\begin{Bmatrix} \alpha_{11_j} \\ \alpha_{21_j} \end{Bmatrix} \cdot f_t + \begin{bmatrix} \alpha_{12_j} & \alpha_{22_j} \end{bmatrix} \cdot \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} \right) \cdot g(\phi_j)$$

And when $g(\phi_j) = 1$ it means that the cutting edge is in the workpiece.

And:

$$\text{Eq. 1.6:} \quad \begin{Bmatrix} \alpha_{11_j} \\ \alpha_{12_j} \\ \alpha_{21_j} \\ \alpha_{22_j} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2} \cdot [-\sin 2\phi_j - K_r \cdot (1 - \cos 2\phi_j)] \\ \frac{1}{2} \cdot [-(1 + \cos 2\phi_j) - K_r \cdot \sin 2\phi_j] \\ \frac{1}{2} \cdot [(1 - \cos 2\phi_j) - K_r \cdot \sin 2\phi_j] \\ \frac{1}{2} \cdot [\sin 2\phi_j - K_r \cdot (1 + \cos 2\phi_j)] \end{Bmatrix}$$

That are, like reported in [10]:

$$\text{Eq. 1.7:} \quad \begin{Bmatrix} \alpha_{xx} \\ \alpha_{xy} \\ \alpha_{yx} \\ \alpha_{yy} \end{Bmatrix} = \begin{Bmatrix} g(\phi_j) \sum_{j=1}^N \alpha_{11_j} \\ g(\phi_j) \sum_{j=1}^N \alpha_{12_j} \\ g(\phi_j) \sum_{j=1}^N \alpha_{21_j} \\ g(\phi_j) \sum_{j=1}^N \alpha_{22_j} \end{Bmatrix}$$

And consequently:

$$\text{Eq. 1.8:} \quad \overline{F} = \overline{\Gamma(t)} \cdot \overline{f_t} \cdot K_t \cdot b + [\Psi(t)] \cdot K_t \cdot b \cdot (\overline{x} - \overline{x}(t - \tau))$$

Where $\overline{\Gamma(t)}$ and $[\Psi(t)]$ are periodic functions with period τ .

The forcing function consists of two components: a periodic component arising out of the feed force and a regenerative component, coming from a periodic modulation of the chip thickness. The stability of the system is determined by the stability of the motions due to the regenerative component of the forcing function.

It is possible to system dynamic equation $\underline{x} = [x \quad y]^T$

$$\text{Eq. 1.9:} \quad [M] \cdot \ddot{\underline{x}} + [C] \cdot \dot{\underline{x}} + [K] \cdot \underline{x} = [b] \cdot \overline{F}$$

Where $[M]$, $[C]$, $[K]$ are respectively the generalized mass, damping and stiffness matrixes and $[b]$ is the influence matrix containing the information about the degrees of freedom where the force vector $\bar{F} = \begin{bmatrix} F_x & F_y \end{bmatrix}^T$ is acting.

Eq. 1.9 can be viewed in a state space representation:

Eq.

1.10:

$$\begin{aligned} \begin{Bmatrix} \dot{\underline{x}} \\ \ddot{\underline{x}} \end{Bmatrix} &= \begin{bmatrix} 0 & I \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \cdot \begin{Bmatrix} \underline{x} \\ \dot{\underline{x}} \end{Bmatrix} + \dots \\ &\dots + \begin{bmatrix} 0 & 0 \\ K_t \cdot b \cdot [M]^{-1} \cdot [b] \cdot [\Psi(t)] \cdot b^T & 0 \end{bmatrix} \cdot \begin{Bmatrix} \underline{x} - \underline{x}(t-\tau) \\ \dot{\underline{x}} - \dot{\underline{x}}(t-\tau) \end{Bmatrix} + \begin{bmatrix} 0 \\ K_t \cdot b \cdot [M]^{-1} \cdot [b] \cdot \overline{\Gamma(t)} \end{bmatrix} \cdot f_t \end{aligned}$$

Eq. 1.10 represents a system of linear delay differential equation (DDE) with periodic coefficients. The solution for $\underline{x}$ will consist of two parts: a particular solution, due to the forcing function with period τ , arising from the feed and the solution for the regenerative forcing function, which decides the stability of the whole solution. The first part of the solution has the same periodicity as the tooth pass and therefore the Fourier spectrum of the displacement signal will always contain multiples of the tooth passing frequency. The second part of the solution will decide the frequency of chatter at instability, Ganguli [100].

A brief explanation of the approaches that can be used to analyze the cutting process stability follows.

The governing differential equation for a linear time invariant system is:

Eq. 1.11:
$$\dot{\underline{q}} = [A] \cdot \underline{q}$$

where A is a constant matrix. The general solution is of the form $\underline{q} = e^{[A](t-t_0)} \underline{q}(t_0)$, where $\underline{q}(t_0)$ denotes the initial condition of the system. The exponential term is called the state transition matrix $[\Phi(t, t_0)]$ as it gives information of the state of the system. For asymptotic stability, the eigenvalues of $[A]$ should have negative real parts, which means that the eigenvalues of $[\Phi(t, t_0)]$ should have modulus, less than 1.

For time varying systems, there is no general solution of the transition matrix. However, periodic systems can be analyzed by the Floquet theorem [19].

In this case the state transition matrix is $[\Phi(t, t_0)] = [P(t, t_0)] \cdot e^{[R](t-t_0)}$ where $[R]$ is a constant matrix and $[P(t + \tau, t_0)] = [P(t, t_0)]$. The system is asymptotically stable if the eigenvalues of $[R]$, also known as the characteristic exponents have negative real parts.

If the eigenvalues of $[R]$ have negative real parts, the eigenvalues of $[\Phi(t, t_0)]$ would have magnitude less than 1. Therefore, if the eigenvalues of the Floquet Transition matrix are located inside the unit circle, the system is asymptotically stable.

This theory can be extended to DDE with periodic coefficients like in Eq. 1.10. Neglecting the contribute due to the feed we can obtain:

$$\text{Eq. 1.12:} \quad \dot{\underline{q}}(t) = [A_1(t)] \cdot \underline{q}(t) + [A_2(t)] \cdot \underline{q}(t - \tau)$$

Where $[A_1(t)]$ and $[A_2(t)]$ are periodic matrixes.

In this case the dimensions of the state transition matrix for one period $[\Phi(\tau, t_0)]$ is infinite therefore an approximation is necessary to obtained a finite sized matrix.

Minis [8] and Altintas [10], [11] proposed the expansion of the periodic function $[\Psi(t)]$ with a zero-th order coefficient $[\Psi_0]$. This assumption makes the DDE time invariant and it is possible to study the stability of the characteristic equation assuming the chatter frequency ω_c .

Eq. 1.13:

$$\det \left(sI - \begin{bmatrix} [0] & [I] \\ -[M]^{-1} \left([K] - K_t \cdot b \cdot (1 - e^{-s\tau}) \cdot [b] \cdot [\psi_0] \cdot [b]^T \right) & -[M]^{-1} \cdot [C] \end{bmatrix} \right) = 0$$

It is therefore possible to compute the chatter free axial depth of cut.

This introduced approximation is satisfying for high radial immersion milling operations in fact in these cases the coefficients reported in Eq. 1.6 can be substituted by their mean values.

For low immersion milling operations this approximation is not suitable, this can be appreciated in Fig. 1.9 as described in [100].

The Floquet theorem has been applied by various authors after obtaining a finite sized state transition matrix for one period “T”. Sridhar et al [7] presented a graphical method for stability analysis of milling. Recently two methods, have been proposed to obtain a finite sized state transition matrix for one tooth passing period: the Temporal Finite Element Analysis method (TFEA) and the Semi-Discretization Method.

The TFEA method is proposed by Bayly et al [20] for low immersion milling for a single toothed cutter in a SDOF milling operation. The formulations are extended to a 2 DOF milling operation by Mann et. al. [21], Gabor et. al. propose the Semi-Discretization method in [22] and present experimental verification in [23] for a SDOF milling operation. The method is extended to the general 2 DOF milling operation by Gradisek et al in [24].

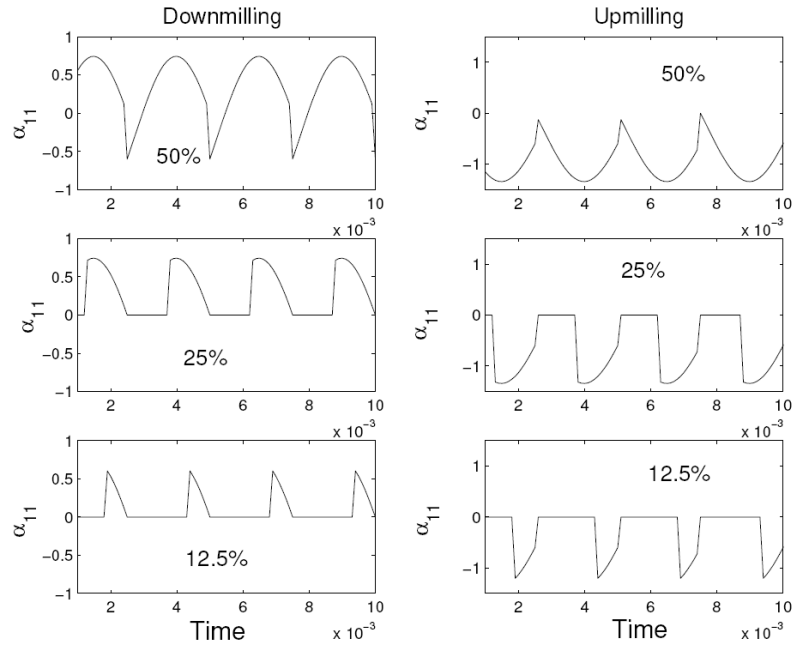


Fig. 1.9: Milling coefficients

These approaches allow to study the cutting process stability problem in a more detailed way than the zero-th order method.

The stability limits correspond to eigenvalues ($\mu = e^{j\omega_c \tau}$) of the Floquet transition matrix that lies on the unit circle.

It is possible to distinguish different situations:

- When μ is complex and $|\mu|=1 \rightarrow$ the chatter frequencies are $\omega_c \pm \frac{2k\pi}{\tau}$ and this instability is called Hopf Bifurcation and the most frequent stability phenomenon in milling.
- When $\mu = -1$ the chatter frequencies are $\frac{\pi}{\tau} \pm \frac{2k\pi}{\tau}$ and is called Flip bifurcation.
- When $\mu = 1$ the Bifurcation is called Saddle Node Bifurcation and it can not arise in case of milling.

Gabor et al [12] discuss about the existence of multiple chatter frequencies in the milling process and the authors identified three kinds of frequencies : tooth passing frequencies, Hopf Bifurcation ones that are close to the main resonances and Flip Bifurcation frequencies that are odd multiples of half of the tooth passing frequency. The TFEA technique and the Semi-Discretization method demonstrate both kinds of bifurcations and characterize them by plots of the evolution of the eigenvalues in the complex plane. The Flip Bifurcation regions manifest as a group of high stability regions in the stability lobe diagram, as observed by Davies et. al. [25], Gabor et. al. [26].

The zero-th order single frequency solution proposed in [10] cannot predict the Flip Bifurcation regions accurately and thus the stability lobes differ from the ones obtained by the TFEA or the Semi Discretization Method especially for low immersion milling operations.

Ganguli in [100] and in [97] presented an interesting approach to study the problem of chatter instability: the authors gave an interpretation of the physical meaning using the root loci both for turning operations and milling ones. Referring to turning operations Ganguli asserted that not always the responsible of the chatter vibration is a structural pole but sometimes it is a poles linked to the “delay” effect.

The author critically analyzed the relationship between the inner-outer modulation phase shift and the different regions of the lobes diagram.

Furthermore, Ganguli presented the results from time domain simulations in order to verify the occurrence of different chatter instabilities that depend on the main cutting parameters such as radial immersion of the tool and the kind of milling (upmilling/downmilling).

Faasen [17] proposed a method to perform the lobes diagram for high speed machining in fact he considered the cutting coefficients and machine dynamics dependence on the cutting velocity.

Recently also Movahhedy [27] et. al. analyzed the gyroscopic effects due to the spindle behaviour on the lobes diagram.

Park in [16] provided a novel method, based on the robust stability theorem, to predict chatter-free regions for machining processes, by taking in account the unknown uncertainties and changing dynamics for machining.

1.2 Chatter suppression techniques: state of the art

Analyzing the specific literature concerning the chatter instability it is possible to detect different strategies that can be used to increase the Material Removal Rate and suppress the chatter vibrations. They are summarized in Fig. 1.11. Obviously the first effort have to be focused on the machine tool and spindle dynamic performance improvements. The goal is to increase the dynamic stiffness of the system.

A mechatronic approach can be used to make easier the design procedure and to let the designer to investigate multiple solutions. Various authors dealt with these topics: Albertelli et. al. [38], Bianchi [39], Brecher [40].

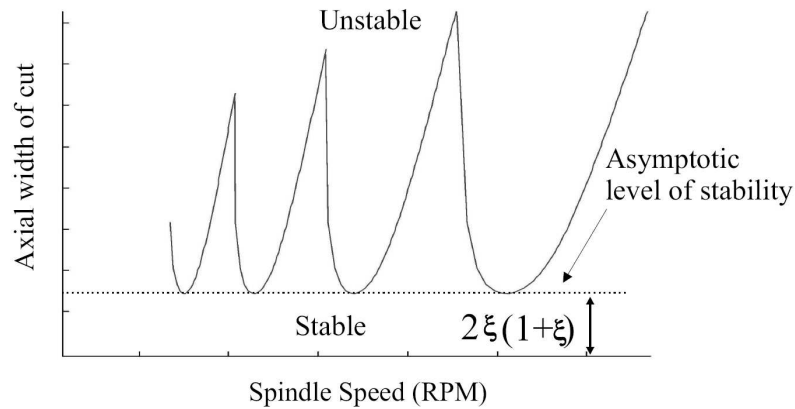


Fig. 1.10: Damping influence

In High Speed Machining (HSM) the spindle plays an important role so there is an increasing necessity to model and analyze the spindle bearing systems. A review of the previous works has been reported in section 1.3.

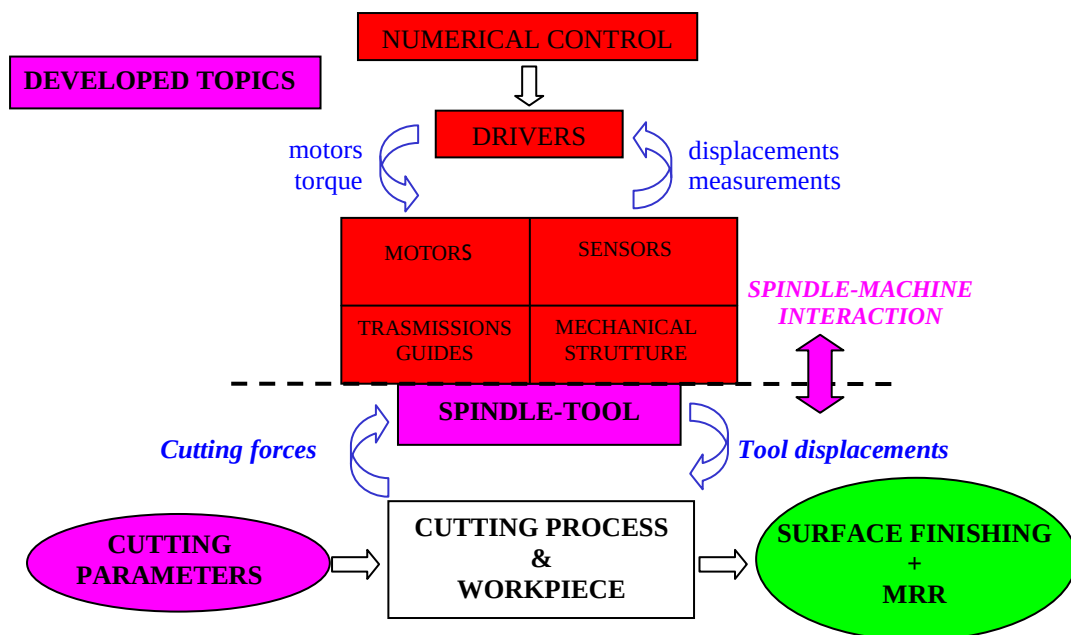
Fig. 1.10 shows the role of the damping ratio ξ on the increment of the free chatter depth of cut, consequently different strategies used to increase the damping reveal themselves to be effective as MRR improvement methods.

Many works concerning active and passive damping techniques can be found in literature. A comprehensive review of these works will be presented in section 5.1.

Another possibility to improve the performances of the spindle system consists in the use of special materials. In literature we can find some applications in which smart materials like electrorheological ones were used.

For example Wang in [53] proposed a novel design method for a tunable stiffness boring bar containing an electrorheological (ER) fluid to suppress chatter in boring. The ER fluid undergoes a phase change when subjected to an external electrical field and serves as a complex spring behaving nonlinearly in the structure. The deformation modes of the ER fluid are dependent on the

applied electric field and the strain amplitude. As a result, the global stiffness and energy dissipation properties of the boring bar under an electric field change drastically at different amplitudes of vibration. It is found that the amplitude of chatter can be prevented from increasing by using the nonlinear vibration characteristic of the ER fluid. It is shown experimentally that the chatter can be suppressed under a certain range of the electric field related to the cutting conditions. A further extension was proposed by the same author in [54].



OPTIMAL INTEGRATED DESIGN: **SPINDLE + MACHINE TOOL**
 INCREASE SYSTEM DAMPING
 SMART MATERIAL
COMPOSITE MATERIALS
ACTIVE DAMPING & PASSIVE DAMPING
 CUTTING PARAMETERS SELECTION
SSSV

Fig. 1.11: Machine Tool system

Wang J. in [55] used a magneto-rheological (MR) fluid in place of lubricating oil in a traditional squeeze film damper (SFD). A variable-damping SFD controlled by a magnetic field was designed and it can be used to control the vibration of rotor systems. The structure of an MR fluid SFD is introduced. The mechanical properties of the squeeze film and the unbalance response characteristics of the MR fluid damper rigid rotor system are analyzed theoretically.

Wei in [56] demonstrated that the electrorheological (ER) sandwich structure can be proposed for vibration control of the rotating flexible beams with variable speed/acceleration. A sandwich beam specimen, which is treated with ER fluid sandwiched between two aluminium surface layers, is constructed.

The experimental results demonstrate that the vibration of the beam caused by the rotating motion at different rotating speed and acceleration can be quickly suppressed by applying the electric field to the ER beam, and evaluate the feasibility of ER fluid in attenuating the vibration of rotating beams.

The use of composite materials for the spindle design will be presented in paragraph 2.3.

Composite materials generally have an higher specific stiffness compared to metal ones so their use in the machine tool industry is quite promising nevertheless their high costs.

Choi in [62] used a carbon fiber-epoxy composite material to design a shaft of a ball bearing spindle system. The goal of the work was to get enhancements in the spindle dynamic performance. A comparison between the new design solution and the conventional steel one was presented. Both simulated and experimental results are reported. Meaningful performance improvements were found that is an increment of the natural frequency values and of the corresponding damping ratios.

Chang [63] designed a composite air spindle system made by an high-modulus carbon fiber composite shaft, powder-containing epoxy composite squirrel cage rotor, and aluminium tool holder is introduced. For the optimal design of the composite air spindle system, the stacking sequence and thickness of the composite shaft were determined by considering the fundamental natural frequency and deformation of the system.

Kim [64] designed a composite boring bar in order to improve the stability limit during cutting. An analytical approach was proposed to evaluate the new solution performance.

Argento in [65] reported the experimental results from the test performed on the composite boring bar prototype designed in [64].

Lee [58] and [59] designed and manufactured a rotating boring bar with high stiffness pitch-based carbon fiber epoxy composite to meet the requirements of boring at high rotating speed because carbon fiber epoxy composite materials have a much higher specific stiffness and higher damping than conventional boring bar materials. The dynamic characteristics of the composite boring bar developed were measured by the boring operation of aluminium engine blocks. From experiments, it was found that the dynamic stiffness of the composite boring bar was about 30% higher than that of the tungsten carbide boring bar. Also a 30% stability limit improvement compared to the tungsten boring bar was observed.

Another chance to obtain enhancements in the chatter free axial depth is done taking advance of the particular structure of the stability charts: a proper selection of the spindle speeds allows higher stability limit. Obviously in order

to select the more appropriate spindle speeds it is necessary to accurately know the machine tool and workpiece dynamics.

Some works proposed automatic cutting parameters adjusting.

In Lian [50] an on-line fuzzy controller that automatically select the proper cutting parameters (spindle speed and feed) in order to suppress the chatter was developed. Vibration energy and the peak value of a vibration frequency spectrum are jointly used as chatter indicators and inputs to the proposed fuzzy controller

A similar fuzzy controller was designed in order to limit the cutting forces to a threshold value adjusting the spindle speed and the feed, Harber [52].

In order to suppress the chatter vibration the spindle speed modulation technique can be effectively used. A complete analysis of the state of the art has been proposed in paragraph 4.1.

1.3 Spindle bearing system modelling: state of the art

As formerly stated in many High Speed Machining (HSM) operations the Material Removal Rate (MRR) is a key performance factor, limited by instability of the milling process: cutting force variation produce a poor machined surface quality, can accelerate cutting edge wear and overload spindle ball bearings.

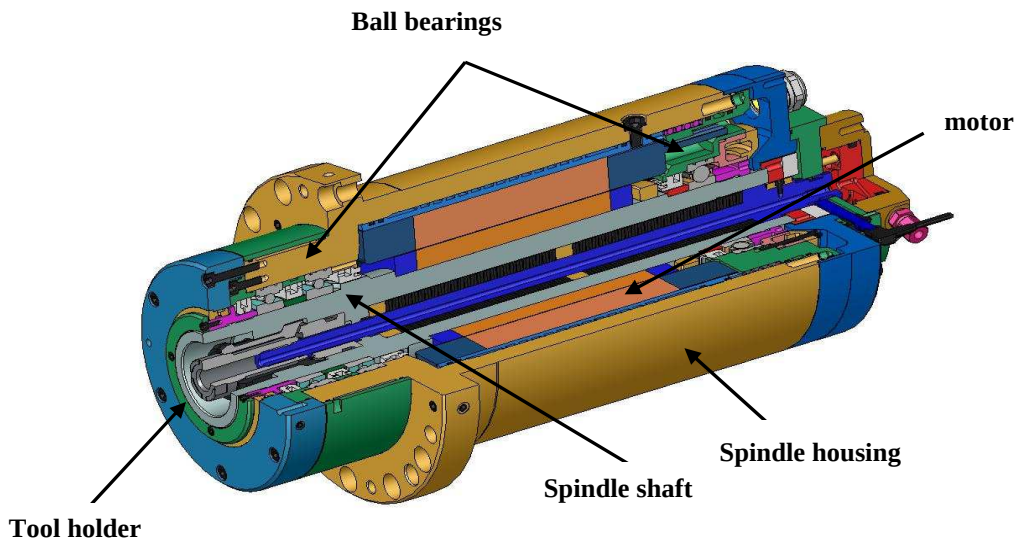


Fig. 1.12: Spindle bearing system with a built in electrical motor

Process instability has been associated to the regenerative chatter phenomena as described in chapter 1.1.

The stability lobes diagram can be preformed knowing the dynamic compliance between tool and workpiece and estimating the cutting forces considering machining process data, like workpiece material and tool geometry.

Chatter vibrations principally occur near the frequency of the most flexible eigenmodes of the machine tool-spindle system: in the range of frequency involved in high speed machining, this dynamic compliance heavily depends on the spindle itself. In order to forecast the material removal capacity and optimize the design of a new spindle it is therefore necessary to correctly predict its dynamic behaviour.

A spindle model allows the designer to investigate by means of simulations different design solutions and to understand in which way the performances can be improved.

In this section a brief overview of the most meaningful works from the specific literature have been reported in fact many authors faced with the prediction of dynamic behaviour of machine tool spindle bearing systems, both analytically and experimentally.

Generally in order to be able to accurately predict the spindle bearing system dynamic and static behaviour it is necessary to model different aspects such as the flexibility of the spindle shaft and the spindle housing, centrifugal and gyroscopic effects of the shaft and of the disks, the structural contributes due to additional components (i.e. rotor of electrical motor), the flexibility of the tool, the tool holder characteristics, the ball bearings and the spindle speed bearing stiffness dependence.

In (Cao et. al.) [31] a finite element model based on the Timoshenko's beam, look at Nelson [41] and Cowper [34], considering both gyroscopic and centrifugal effects of the spindle bearing system was proposed.

Basics of rotor dynamics can be found in Genta [42].

The test case spindle is depicted in Fig. 1.13. This spindle is different from the previous one shown in Fig. 1.12 in fact the last one has a built in electrical motor and consequently doesn't have the pulley system that is used to transfer the motion from the external motor to the spindle shaft. Generally the electro-spindle dynamic performances are higher than those of the traditional spindles.

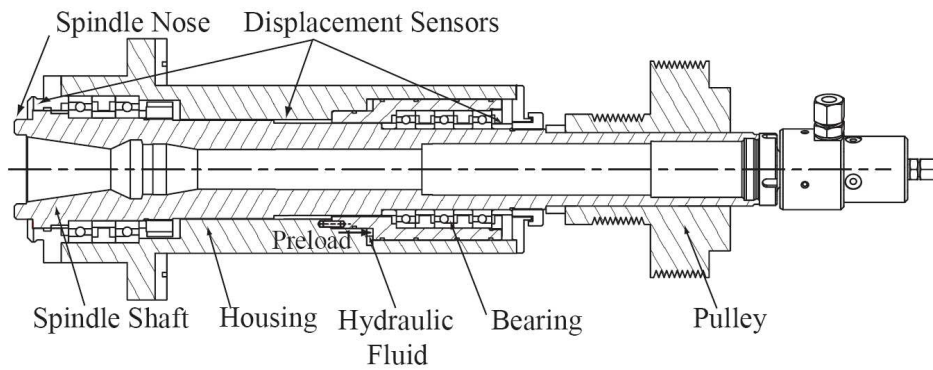


Fig. 1.13: Spindle bearing system

The corresponding Finite Element model is reported in Fig. 1.14. A beam modelling approach was used to model the spindle housing.

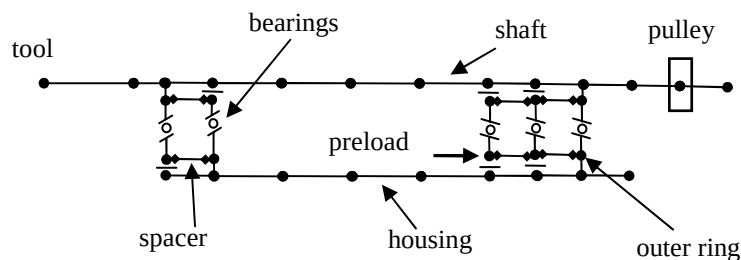


Fig. 1.14: Spindle system beam model

The following equations describe the dynamic behaviour of the shaft.

$$\text{Eq. 1.14:} \quad [M^b] \cdot \{\ddot{q}\} - \Omega [G^b] \cdot \{\dot{q}\} + ([K^b] + [K^b]_p - \Omega^2 \cdot [M^b]_c) \cdot \{q\} = \{F^b\}$$

Where:

$[M^b]$: is the mass matrix

$[M^b]_c$: mass matrix in order to consider the centrifugal effects

$[G^b]$: is the gyroscopic matrix

$[K^b]$: is the stiffness matrix

$[K^b]_p$: is the stiffness matrix due to the axial load

$[F^b]$: this matrix represents the external load

$\{q\}$: is the vector containing the 5 degrees of freedom of the Timoshenko formulation and Ω is the spindle speed.

Moreover, the equations that describe the rigid disk behaviour are reported:

$$\text{Eq. 1.15:} \quad [M^d] \cdot \{\ddot{q}\} - \Omega [G^d] \cdot \{\dot{q}\} = \{F^d\}$$

$[G^d]$: is the gyroscopic matrix for the disk

$[F^d]$: and it is the unbalance force vector.

In Eq. 1.15 and Eq. 1.14 the damping contributes are not considered: generally the damping can be introduced into the system by the components (i.e. the ball bearings introduce damping in the system) or using a modal approach.

The ball bearing was modelled taking into account rotational speed effects on the rolling elements and the furthermore the bearing stiffness depends on the actual load that is the resultant effect due to the preload and the operative load.

The used bearing model is a non linear one.

The implemented ball bearing model considers the spindle speed effects and how the contact geometry changes at different operative spindle speeds, like depicted in Fig. 1.3 and described by Harris in [43].

Generally the ball bearings, in spindle assembly, are axially preloaded. The choice of the proper preload value is an important issue because there is a trade off between the bearing stiffness and the bearing life.

Different systems can be used to preload the bearings: a constant preload spring system or a hydraulic one.

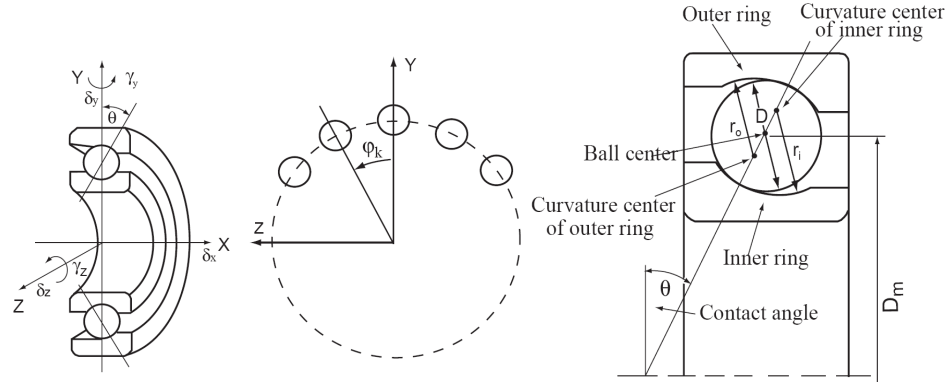


Fig. 1.15: Ball bearing contact geometry

In Fig. 1.16 it is possible to note the centrifugal force F_{ck} and the gyroscopic moment M_{gk} acting on a singular ball bearing. Both F_{ck} and M_{gk} depends on the rotational spindle speed, the contact geometry and the ratio between Ω and the orbital ball speed.

Q_{ok}, Q_{ik} are the Hertzian contact forces respectively between outer ring, inner ring and the balls.

θ_{ok}, θ_{ik} are the contact angles referred to the outer and inner rings. These angles vary with the spindle speed.

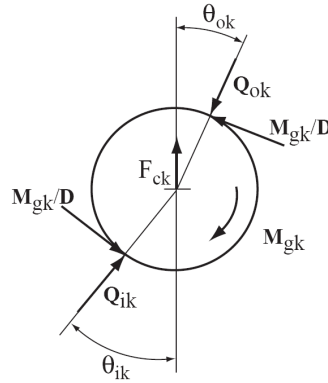


Fig. 1.16: Forces and moments on the ball bearing

It is possible to write the equilibrium equations for the balls considering the relationships between outer and inner rings in order to compute the contact Q_{ok}, Q_{ik} during the operative conditions.

It is therefore possible to compute the bearing stiffness matrix.

Assembling the descriptive equations associated to the different components it is possible to obtain the following equations:

$$\text{Eq. 1.16:} \quad [M] \left\{ \ddot{x} \right\} + [C] \left\{ \dot{x} \right\} + \{R(x)\} = \{F(t)\}$$

Where:

$$\text{Eq. 1.17:} \quad [M] = [M^b] + [M^d]$$

$$\text{Eq. 1.18:} \quad [C] = [G^b] + [G^d] + [C^s]$$

$$\text{Eq. 1.19:} \quad \{F(t)\} = \{F^b\} + \{F^d\}$$

And $[C^s]$ is the modal damping matrix that can be evaluate by means of experimental tests on the spindle bearing system.

Furthermore $\{R(x)\}$ is the equivalent stiffness that depends on the nodal coordinates, this is caused by the non linearity of the ball bearings.

This modelling approach used by Cao et. al. [31], [32] and [44] can be exploited to analyze the influence of the axial preload and the spindle speed on the bearing stiffness and on the spindle natural frequencies, Fig. 1.17 and Fig. 1.18.

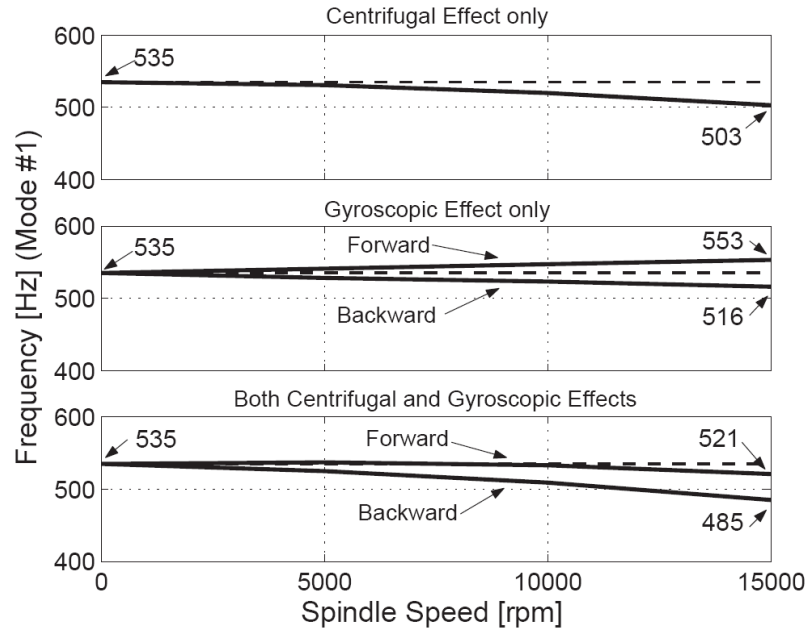


Fig. 1.17: Shaft speed effects

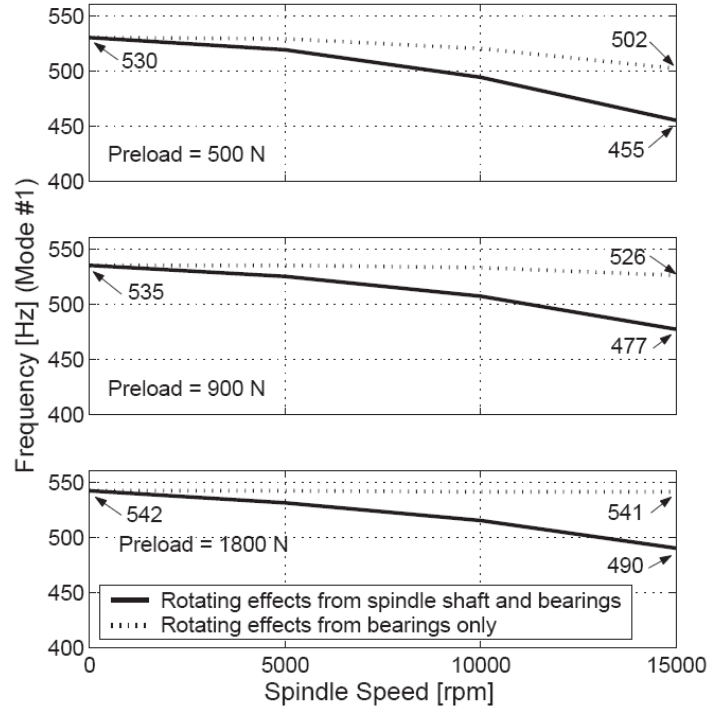


Fig. 1.18: Speed effects and preload effects

Gyroscopic and centrifugal effects influence the dynamic behaviour of the spindle bearing system, Genta [42]. In Fig. 1.19 it is reported how the Frequency Response Function (FRF) evaluated at the tool tip changes due to the rotating effects, Gangol [33]. The author moreover performed some experimental tests in order to check the adequateness of the proposed model.

Basically we can observe a reduction of the values of the spindle bearing resonances. This can affect the spindle performances during cutting operations. In [32] and [44] the authors introduced also a simple beam model of the cutter head.

In this work the thermal effects were not considered.

In the specific literature there are some works, Lin et. al. [45] and [46] that have explained the thermal influence on the spindle bearing behaviour.

In this kind of work the most critical issue is the chance to accurately predict the thermal field in the spindle, especially when the electro spindles are considered. Hypnotizing a thermal field it is possible to define the following quantities:

T_o^0, T_i^0, T_b^0 are the initial temperatures respectively of the outer ring, the inner ring and the balls.

Conversely T_o^1, T_i^1, T_b^1 are the final temperatures. T_{ir}^1, T_{or}^1 are the temperatures of the inner ring and the outer ring at generic time " t ". $P_{a,t}$ is the induced thermal preload.

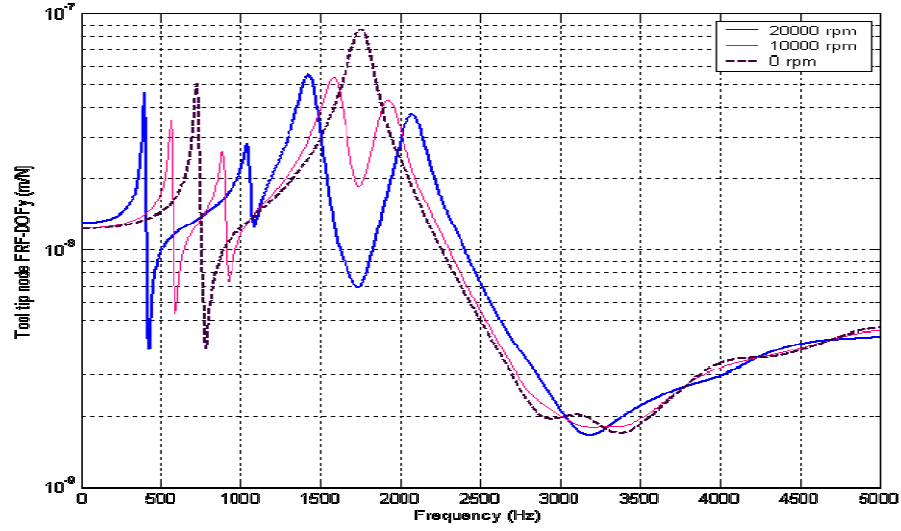


Fig. 1.19: Rotating effects on tool tip FRF

From Fig. 1.20 it is observed that the axial thermal expansion difference is mostly caused by the shaft and the spacer. Their different expansion can be expressed as:

$$\text{Eq. 1.20:} \quad \Delta_1 = \alpha \cdot x_i \cdot (T_i^1 - T_i^0) - \alpha \cdot x_o \cdot (T_o^1 - T_o^0)$$

Eq. 1.20 considers the axial direction, Eq. 1.21 the radial direction and the Eq. 1.22 takes into account the expansion of the bearing balls.

$$\text{Eq. 1.21:} \quad \Delta_2 = \alpha \cdot [D_{io} \cdot (T_{ir}^1 - T_{ir}^0) - D_{oi} \cdot (T_{or}^1 - T_{or}^0)]$$

$$\text{Eq. 1.22:} \quad \Delta_3 = \frac{1}{2} \alpha \cdot D_b \cdot (T_b^1 - T_b^0)$$

And therefore the total deformation in the direction of line contact.

$$\text{Eq. 1.23:} \quad \Delta = \Delta_1 + \Delta_2 \cos \theta - \Delta_1 \sin \theta$$

And consequently the induced preload

$$\text{Eq. 1.24:} \quad P_{a,t} = k_t \Delta^{1.5}$$

Where k_t is a constant that can be obtained from experimental tests.

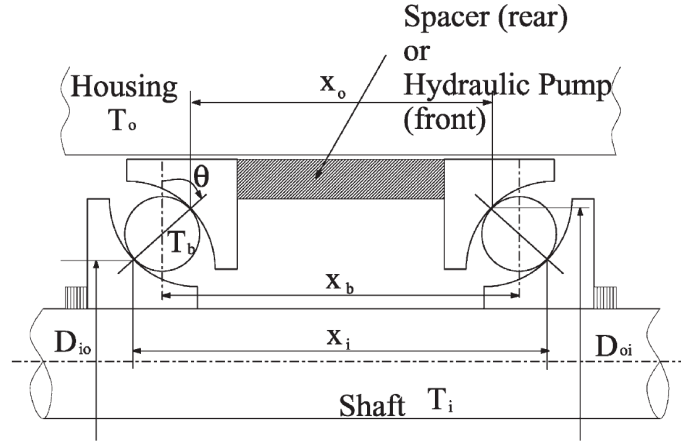


Fig. 1.20: Thermal preload

The thermal preload can be used to compute the bearing stiffness like described in the following empirical relationships, Fig. 1.21.

Eq. 1.25:

$$k_{as} = c_a P_a^{1/3} N_b^{2/3} \sin^{5/3} \theta \cdot D_b^{1/3}$$

$$k_{rs} = 0.64 c_a P_a^{1/3} N_b^{2/3} \sin^{5/3} \theta \cdot \cos \theta \cdot D_b^{1/3}$$

$$P_a = P_{a,i} + P_{a,t}$$

Where c_a is an empirical constant, $k_{a,s}$, $k_{r,s}$ are respectively the axial and radial stiffness, D_b the balls diameter, N_b the number of balls and $P_{a,t}$ the global preload.

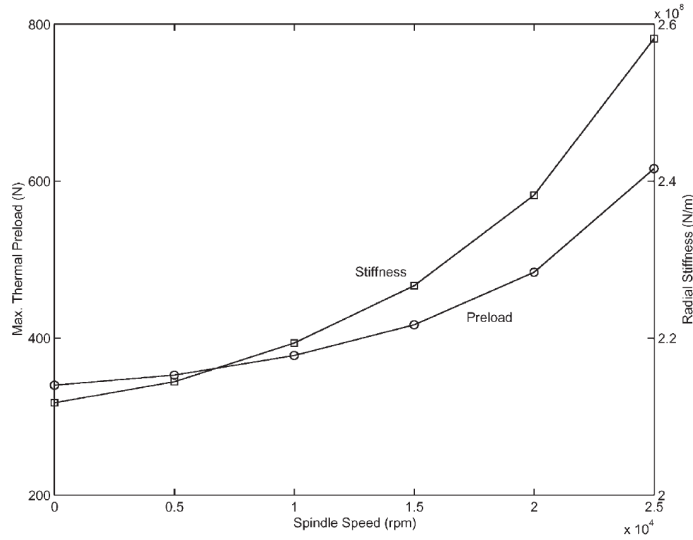


Fig. 1.21: Preload and radial stiffness

2 Spindle modelling and experimental characterization

Chatter vibrations principally occur near the frequency of the most flexible eigenmodes of the machine tool-spindle system: in the range of frequency involved in high speed machining, this dynamic compliance heavily depends on the spindle itself.

This work presents a study performed to build a reliable motor spindle dynamic model that can be used to forecast the machine performance by the corresponding cutting stability diagram.

In order to obtain reliable spindle models an important model updating procedure is recommended. The created model describes the following elements: tool cutter, tool holder, spindle shaft, spindle housing, bearings, tool clamping system and bearing preload system.

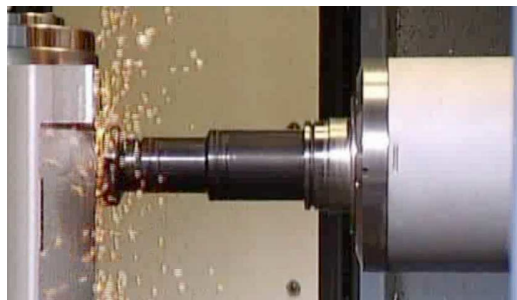


Fig. 2.1: Spindle system during a milling operation

The analysis is based on a FE model and on an experimental modal campaign on several motor spindle setups in order to characterize some spindle components and tune the model.

The influence of aspects such as bearing preload, tool clamping force, cutting forces on the spindle dynamic behaviour and on the cutting stability lobes have been also experimentally investigated.

The analyzed spindle, shown in Fig. 2.2, has a built-in electrical motor and five ball bearings in back-to-back configuration.

Bearing preload is applied to the rear bearing pack by a hydraulic system that acts on the outer rings of the bearings, the preload force is transmitted to the front bearings through the inner rings and the shaft.

Tool holder is HSK 100; the tool is locked and unlocked by a draw bar, pulled by a set of springs.

Timoshenko beams are used to model the spindle shaft and the spindle housing but also for the motor, the clamping system, the tool and the tool holder.

The Timoshenko beam formulation can take into account centrifugal forces and gyroscopic moments acting on all rotating elements, like presented by Cao et. al. [31]. The tool holder is modeled as a lumped compliance. The spindle shaft and housing are therefore described by the matrix equations Eq. 1.14.

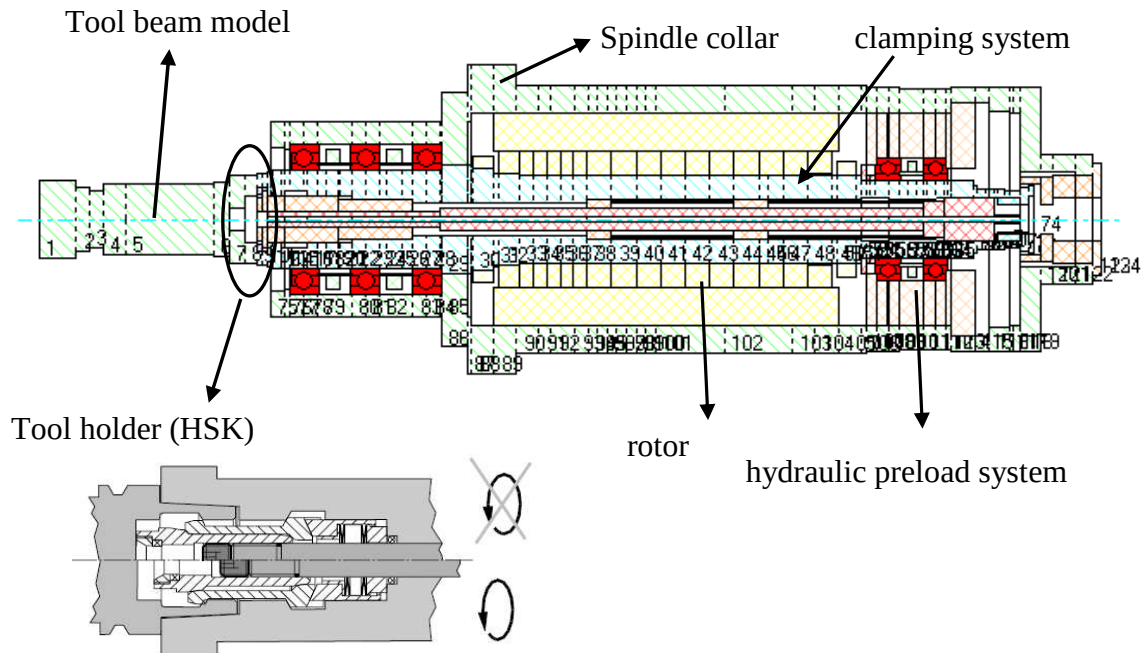


Fig. 2.2: FEM spindle model - components

In this work we generally have introduced damping in the system using a modal approach.

As described by Cao [31] and by Harris [43], a Jones' bearing model is used. This model considers elasticity of balls and rings. Forces and displacements are computed using the Hertzian theory.

The model also takes into account speed effects on the rolling elements, such as centrifugal forces and gyroscopic moments as illustrated in Fig. 2.3.

The bearing stiffness depends on the bearing geometry, the ball and ring materials, the load condition, including the bearing preload, and on the rotational speed.

In this paper we have used the complete non-linear model to produce a linearized representation at a defined spindle speed and a specific preload value. Experiments have been planned in order to provide guidelines useful to build reliable virtual spindle prototypes, to be used, while designing future generations of spindles, to forecast the material removal capability, after having taken into account its interaction with the whole machine.

The model has been updated using the information gathered by an experimental modal analysis [47] campaign, conceived in order to characterize the

contribution of the spindle motor rotor, the tool holder, the clamping system and the bearing preload hydraulic piston.

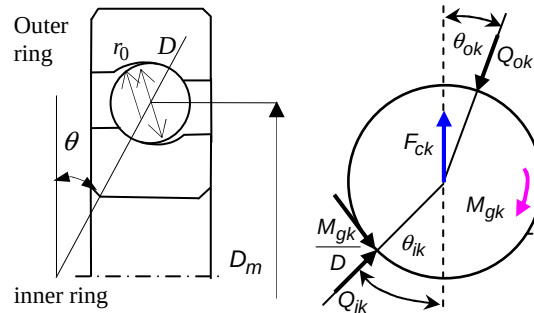


Fig. 2.3: Speed effects on rolling elements.

Experimental tests are also used to study the influence of some important operative parameters such as bearing preload force, tool clamping force and cutting forces on the spindle dynamic behaviour.

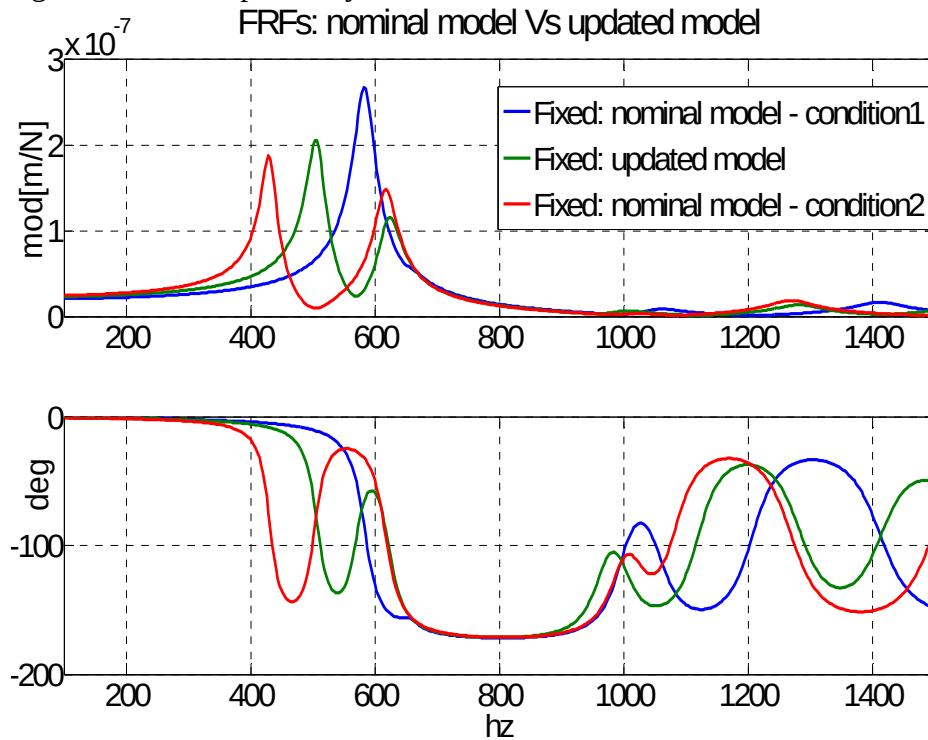


Fig. 2.4: Effect of rotor modeling on Tool Tip FRFs

As an example, Fig. 2.4 shows the effect on the dynamic compliance at the tool tip of different modeling approaches adopted to represent the motor rotor: (1) as a steel element completely connected to the shaft, (2) as a “mass only” element, with null elastic module: the influence on the dominant mode, in terms of both

frequency and peak height (i.e. modal mass) is evident. The third intermediate line has been obtained with the model updated on the experimental data.

This underlines the importance of the model characterization procedure in order to get reliable information about the dynamic behaviour from the spindle model and consequently to predict the cutting process stability limit.

Fig. 2.5 describes the model updating procedure, based on Experimental Modal Analysis on the whole spindle and partial spindle setups, also exploiting the spindle assembly stages. The following functional has been minimized:

$$\text{Eq. 2.1:} \quad F = \sum \frac{(f_i - f_{i_{\text{exp}}})^2}{f_{i_{\text{exp}}}}$$

Where f_i is the frequency of the i^{th} simulated mode and $f_{i_{\text{exp}}}$ the corresponding experimental one.

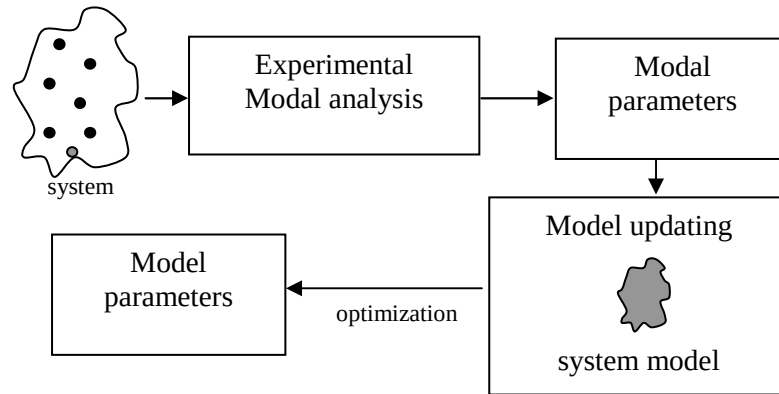


Fig. 2.5: Model characterization procedure

In this work we have used a simplified updating strategy that considers only simulated and experimental frequencies, rather than using all modal parameters.

We have tested four shaft-only setups and different spindle assembly setups.

The experimental tests performed on the shaft-only have been used to obtain data about the rotor behaviour, the clamping system and on the tool holder.

Tests performed on the spindle assembly have allowed us to verify the overall spindle model and to analyze the influence of the bearing preload and the clamping force on the spindle performances.

The setups were defined in such a way following the spindle assembling procedure, this approach made the characterization of the components we hadn't known their behaviour easier.

For each setup, impact tests and acceleration measurements were used to identify system modal parameters and modal shapes.

All spindle setups were in free-free configuration.

2.1 Rotating components characterization

2.1.1 Experimental shaft-only setup 1

In this section the results from experimental modal tests on different shaft-only set ups have been presented.

The first experimental shaft setup, as depicted in Fig. 2.6, consists of the spindle shaft and the balancing ring only. The figure shows the acceleration measurement points.

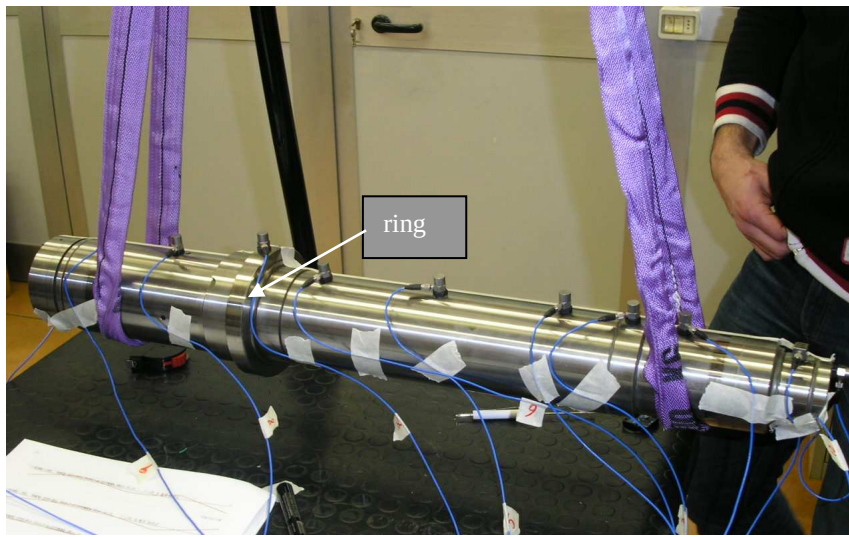


Fig. 2.6: Experimental set up 1 (shaft)

In Fig. 2.7 the corresponding FEM model is presented. This setup is used, properly acting on model parameters (Young's Module and shear coefficient [34]), to reduce the small inevitable geometrical approximation due to the beam model approach and to verify the shaft model.

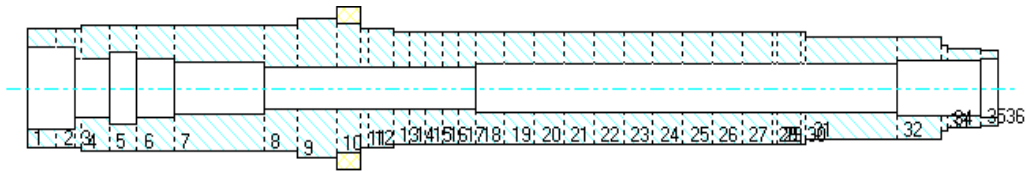


Fig. 2.7: Model setup 1 (shaft).

The equivalent density has been computed from the spindle shaft weight. In Fig. 2.8 the trend of the shear coefficient for hollow circle section is presented.

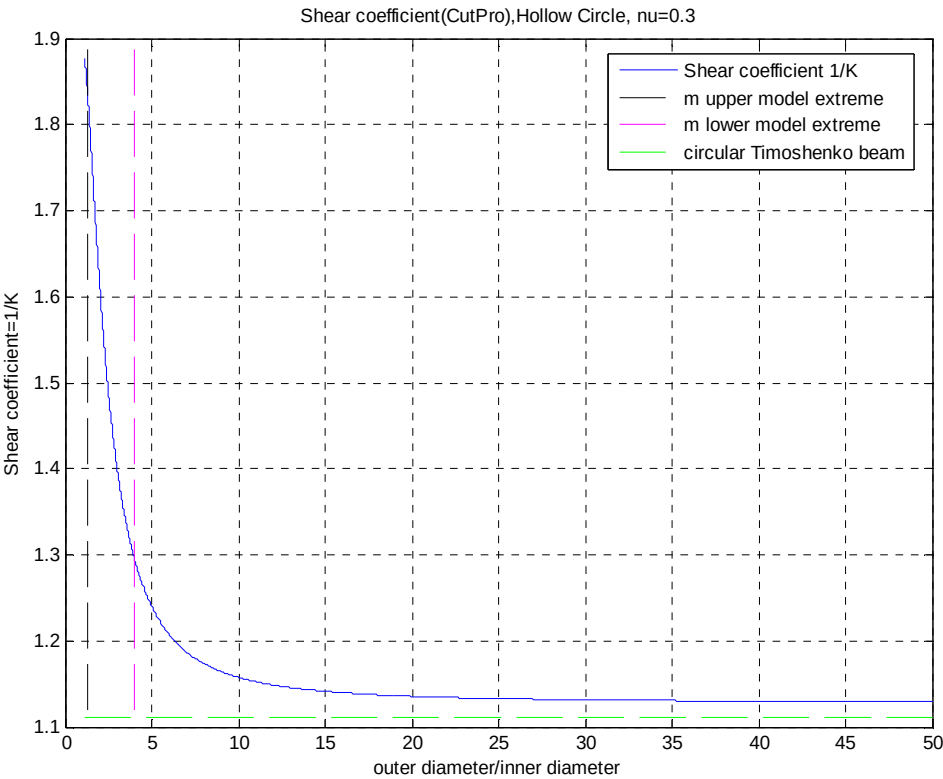


Fig. 2.8: Shear coefficient – hollow circle section

In Tab. 2.1 the equivalent parameters obtained from the optimization procedure are reported.

#	Material description	$E[N/m^2]$	Poisson	$D[kg/m^3]$	Shear cf.
1	Steel	1.984E11	0.3	7718	1.5

Tab. 2.1: Parameters setup 1

The following pictures explain how the frequencies of the model change in accordance with different parameters values.
These surfaces were used to minimize the functional F , presented in Eq. 2.1.
In Fig. 2.11, Fig. 2.12 and Fig. 2.13 the modal shapes linked respectively to the first, second and fifth modes are depicted. All these modal shapes refer to bending modes.

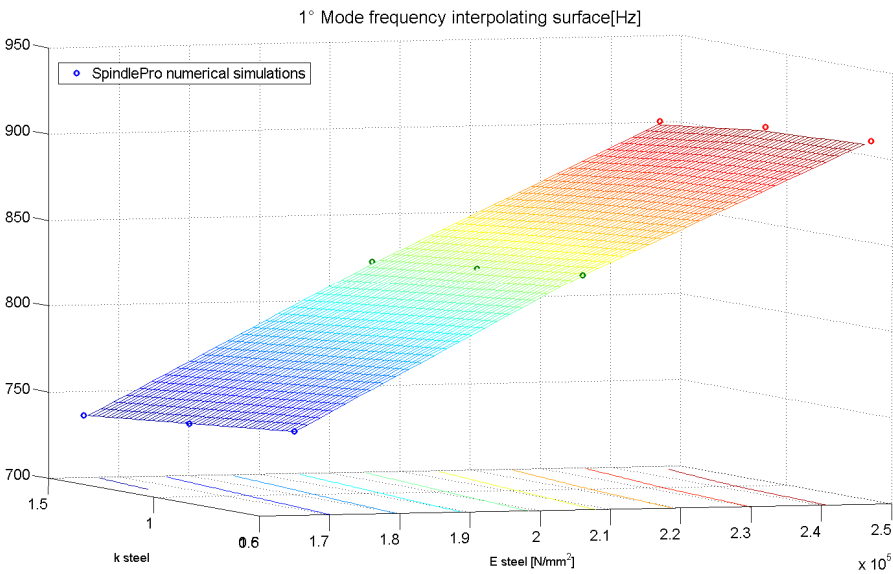


Fig. 2.9: Response surface – first mode

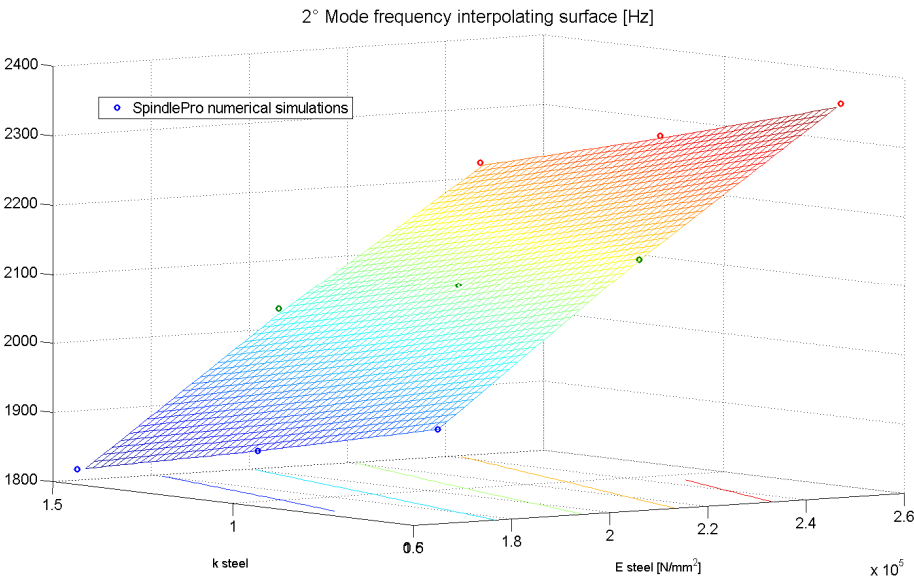


Fig. 2.10: Response surface – second mode

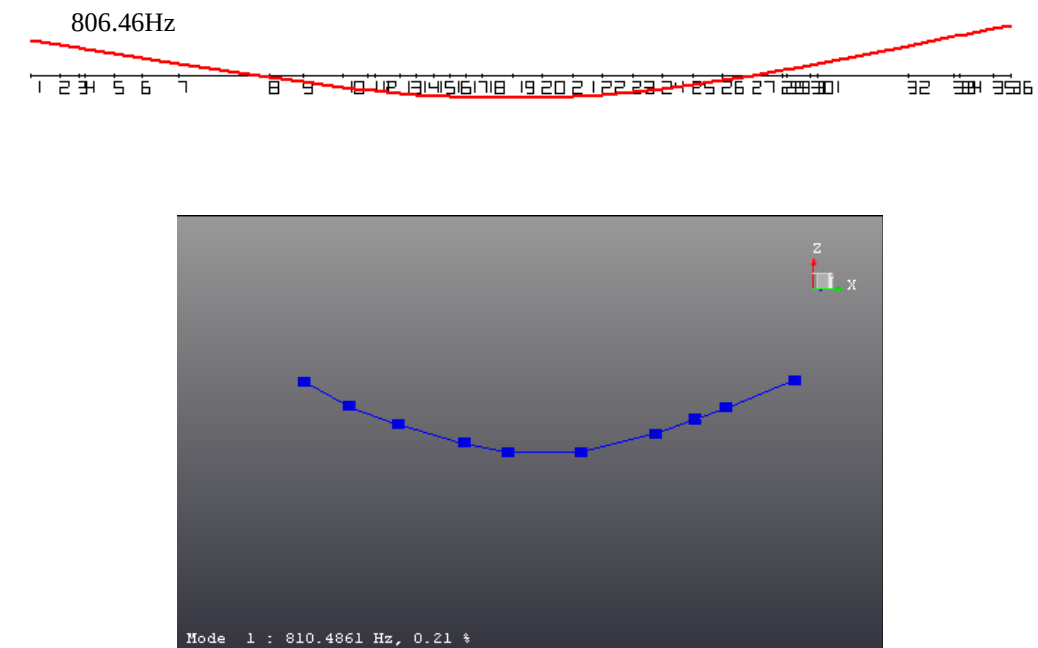


Fig. 2.11: Modal shape – experimental vs simulated – mode 1

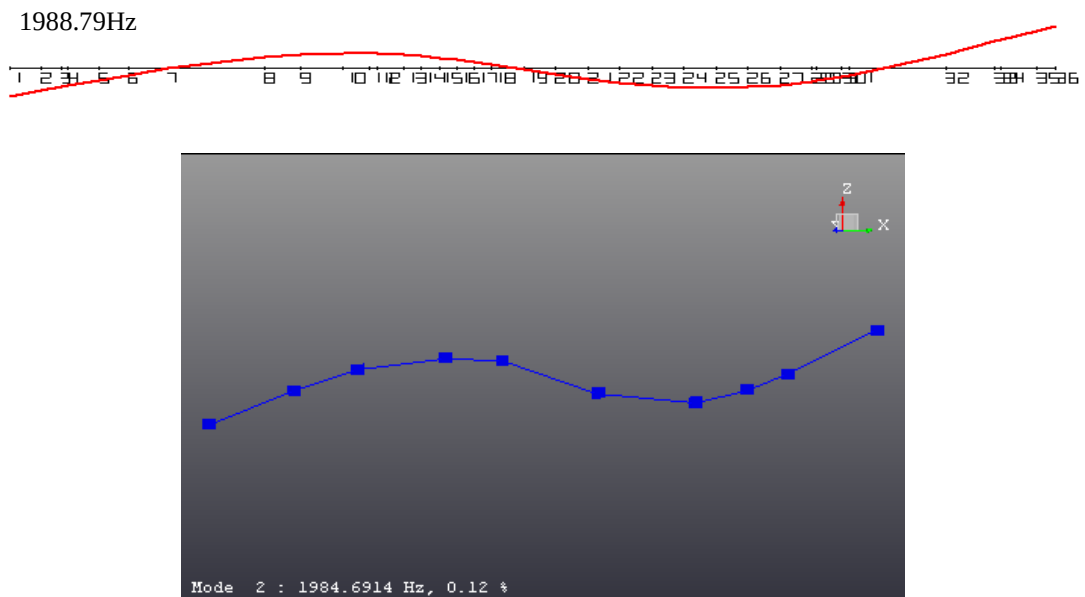


Fig. 2.12: Modal shape – experimental vs simulated – mode 2

After the optimization stage a good agreement with the experimental results was found.

Differences on frequency values are limited up to 1%.

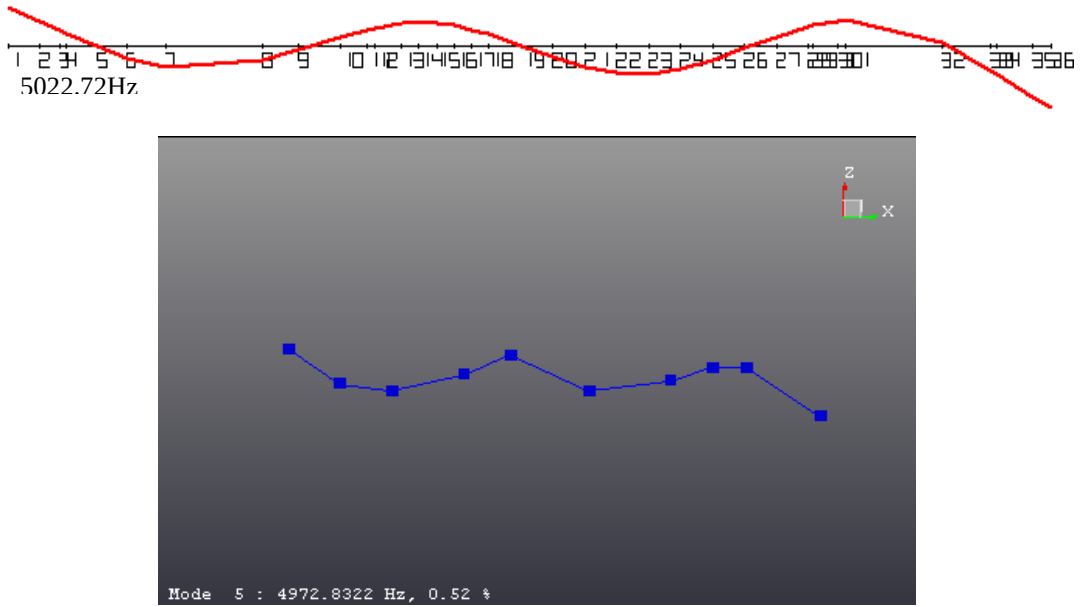


Fig. 2.13: Modal shape – experimental vs simulated – mode 5

2.1.2 Experimental shaft-only setup 2

The second experimental shaft setup is obtained by adding the motor rotor to the first setup

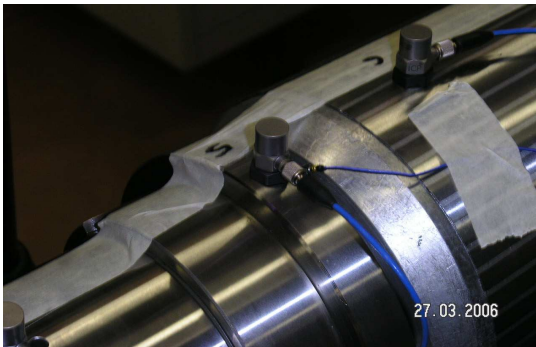


Fig. 2.14 : Motor Rotor - zoom

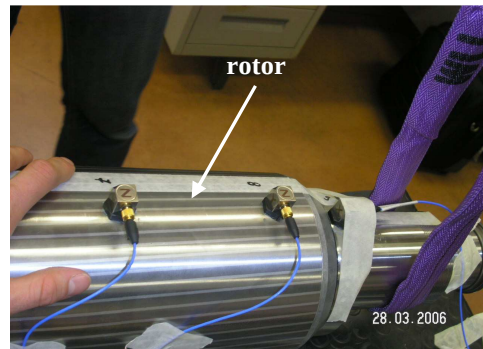


Fig. 2.15: Setup 2

The setup is used to evaluate the rotor structural contribution. In this work an asynchronous motor rotor has been used. Fig. 2.16 illustrates the corresponding model.

A rotor beam model is proposed. The goal is to define a homogeneous equivalent material that approximates the rotor behaviour.

The equivalent ρ_{ROT} was computed from the rotor weight.

In Tab. 2.2 are reported the optimized modal parameters.

#	Material description	$E[N/m^2]$	Poisson	$D[kg/m^3]$	Shear cf.
1	Steel	1.984E11	0.3	7718	1.5
2	Disk_equil	1.984E11	0.3	7718	1.5
3	Rotor	5.98E10	0.3	6787	1.86

Tab. 2.2: Caratteristiche materiali SETUP2

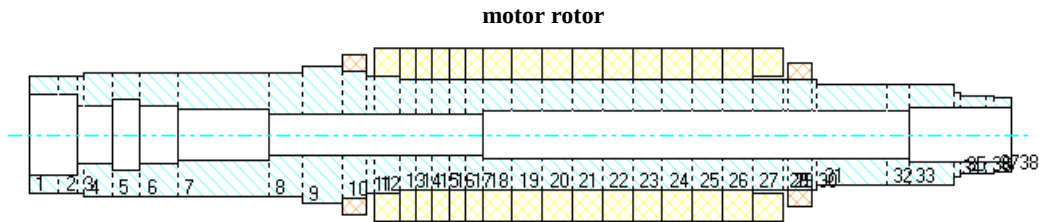


Fig. 2.16: Model setup2 (shaft).

Fig. 2.17 compares experimental and simulated FRFs at the spindle nose after model setup updating. The agreement is good, especially for the first two modes.

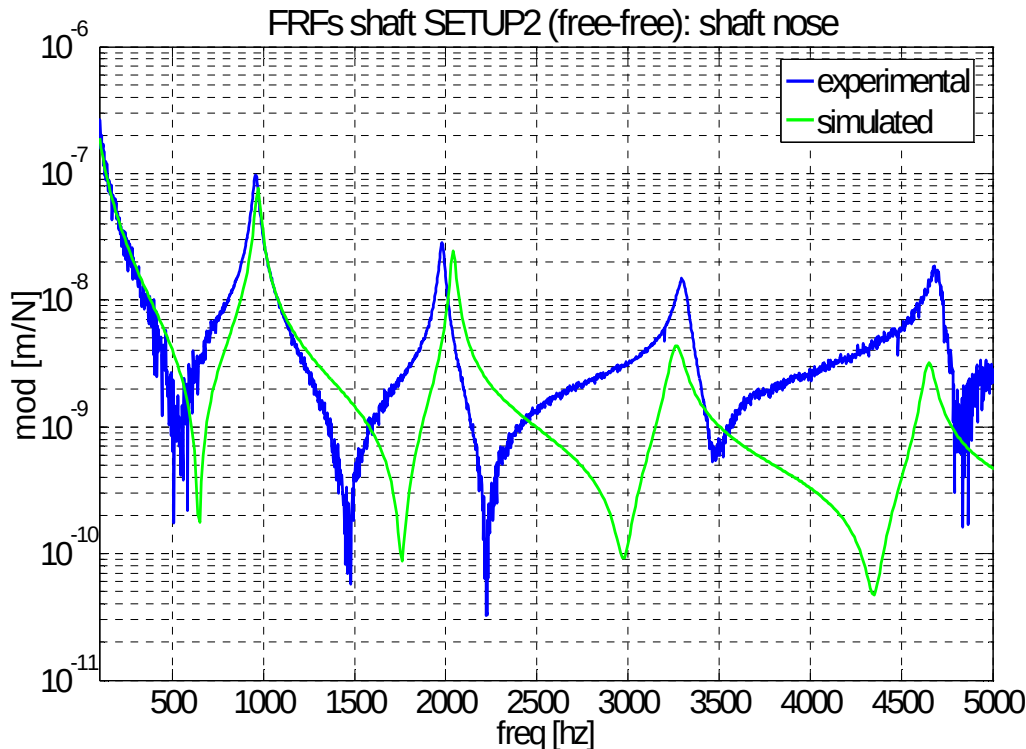


Fig. 2.17: Shaft nose FRFs (setup 2).

In Fig. 2.18 and Fig. 2.19 have been reported the response surfaces that show the model parameters influence on the modal frequencies.

Obviously the most influent parameter is the Young's module compared with the shear coefficient that takes into account the effect of the shear on the system flexibility.

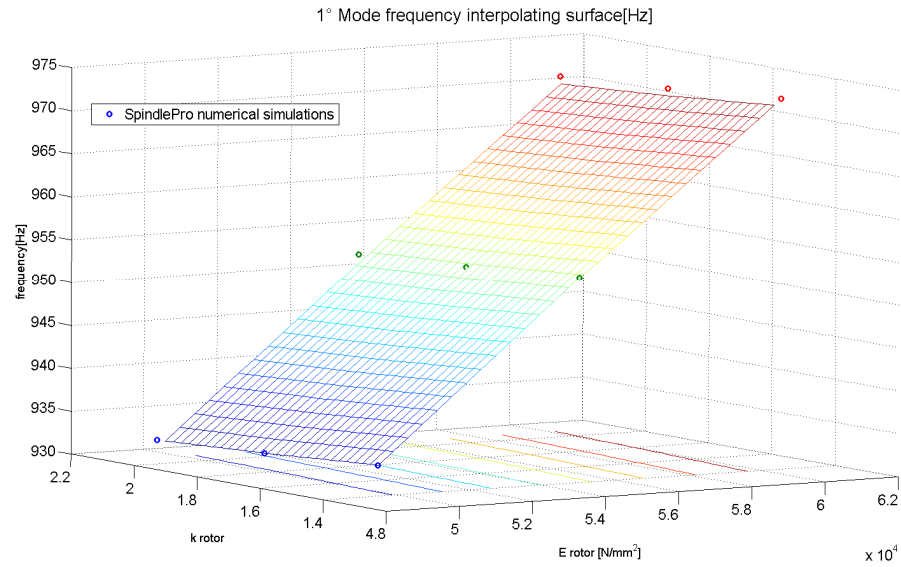


Fig. 2.18: Response surface – first mode (setup 2)

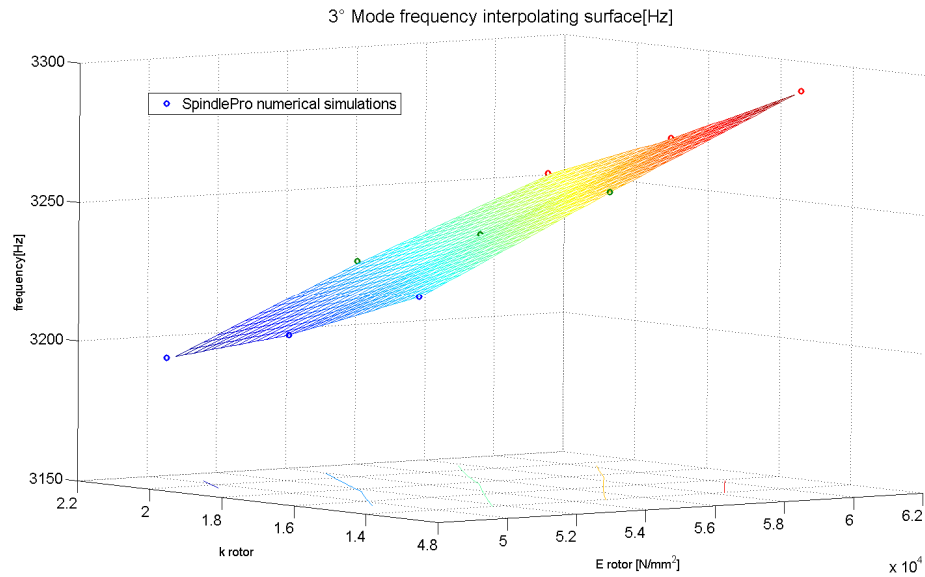


Fig. 2.19: Response surface – third mode (setup2)

A good matching also on the modal shapes was found, for example look at Fig. 2.20 and Fig. 2.21. Only two modal shapes are reported.

3261.67Hz

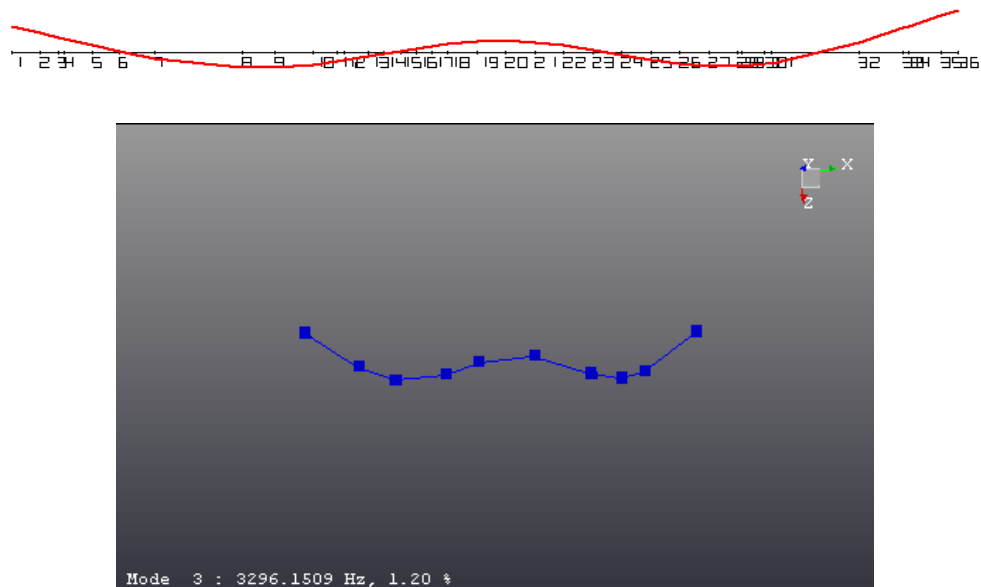


Fig. 2.20: Modal shape – experimental vs simulated – mode 3

4647.91Hz

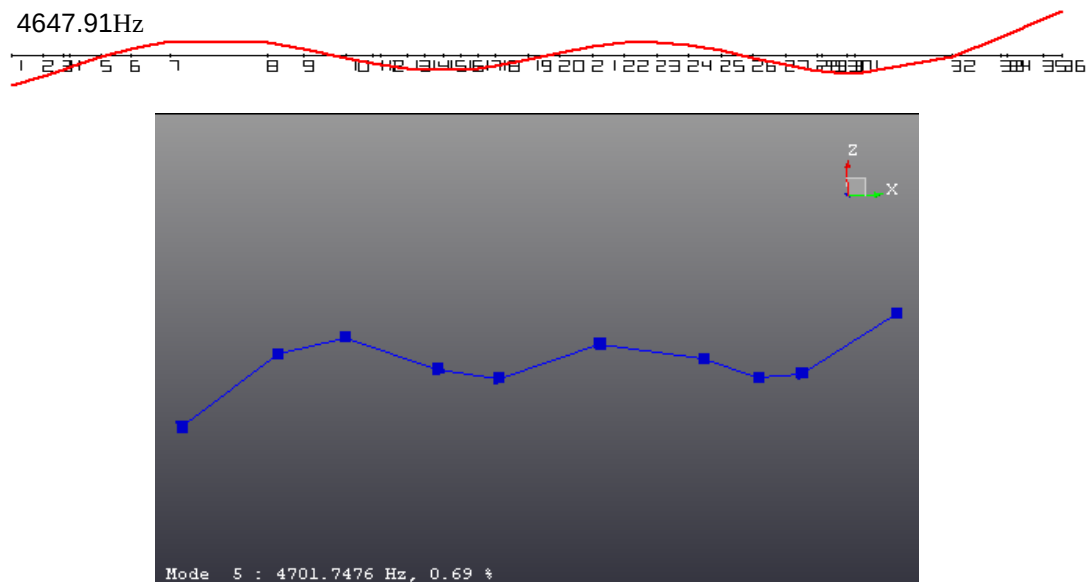


Fig. 2.21: Modal shape – experimental vs simulated – mode 5

A small deviation (less than 4%) between the simulated and experimental frequencies was noticed.

Referring to shaft setup 2, depicted in Fig. 2.16, a 95% Confidence Interval CI was computed for the rotor model parameters, propagating the uncertainty of the experimental frequency values using the Monte Carlo algorithm.

95% CI	$E_{\text{rotor(opt)}}[\text{MPa}]$	Shear(opt)
upper limit	60100	1.867
lower limit	59500	1.854

Tab. 2.3: Equivalent rotor material properties.

Moreover in order to evaluate the repeatability of the assembly operation performed by rotor shrinking on the shaft, experimental modal analyses on three nominally identical components has been made: Fig. 2.22 illustrates the values of the first natural frequency in the three different “setups 2”.

We can observe a slight influence of the rotor shrinkage on the first bending modes.

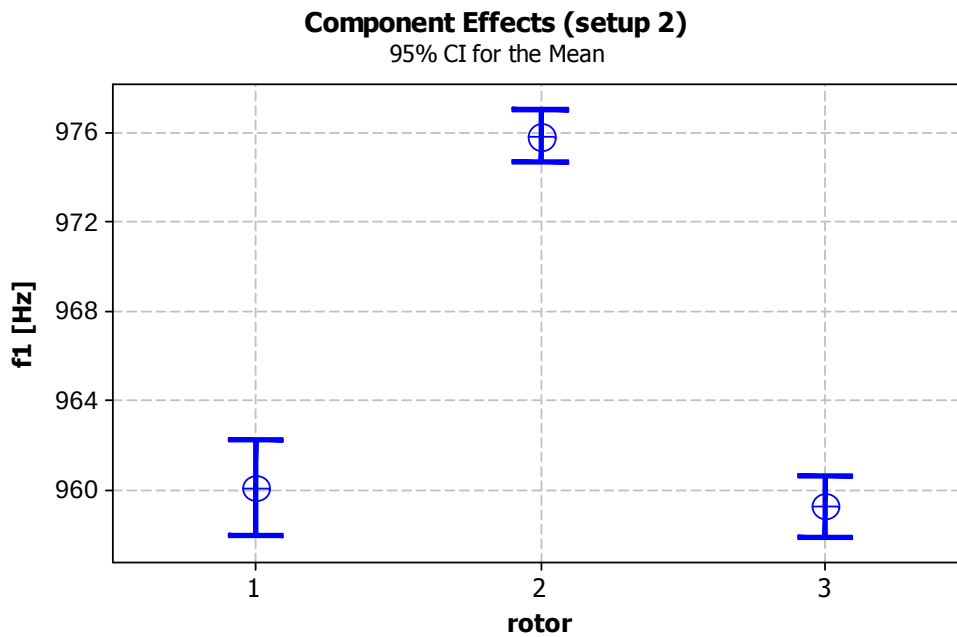


Fig. 2.22: First mode setup2 - shrinking effects.

Another important aspect has been evaluated: Monte Carlo method is also used to generalize the rotor characterization.

The equivalent material properties Tab. 2.4 can be used to build a new spindle model, obviously with asynchronous motor.

It is important to observe that the rotor motor introduces a significant damping in the system in fact the relative modal damping passed from 0.1% (setup1) to 1.7% (setup 2).

Quantity	$E_{\text{rotor(opt)}}[\text{MPa}]$	Shear(opt)
Mean	60241	1.74
Standard deviation	741.2	0.15

Tab. 2.4: Equivalent rotor material – shrinking effects.

2.1.3 Experimental shaft-only setup 3

The third shaft setup, as illustrated in Fig. 2.23 and Fig. 2., includes the clamping system, with the tool collect.

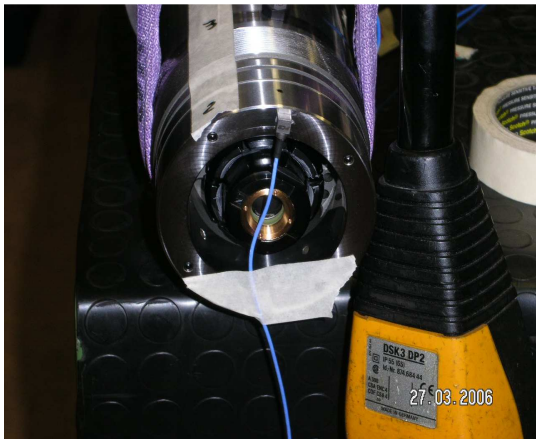


Fig. 2.23: Clamping system - zoom

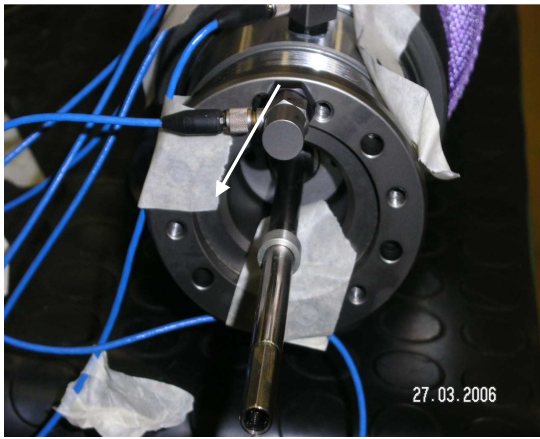


Fig. 2.: Setup 3

This setup constitutes an intermediate step between setup two and four, focused on the tool clamping stiffness. Fig. 2.24 shows the corresponding model. The clamping system, composed by a tie rod, a spring and a tool collect, is represented by corresponding beams.

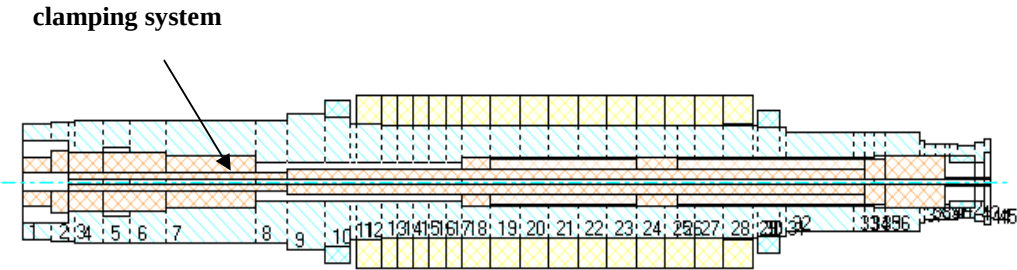


Fig. 2.24: Model setup 3 (shaft).

A sensitivity analysis on the model shows that the clamping system equivalent material properties lightly influences the global dynamic behaviour. Not deserving an updating procedure, nominal values have been used to describe it. This can be appreciated in Fig. 2.25 and Fig. 2.26.

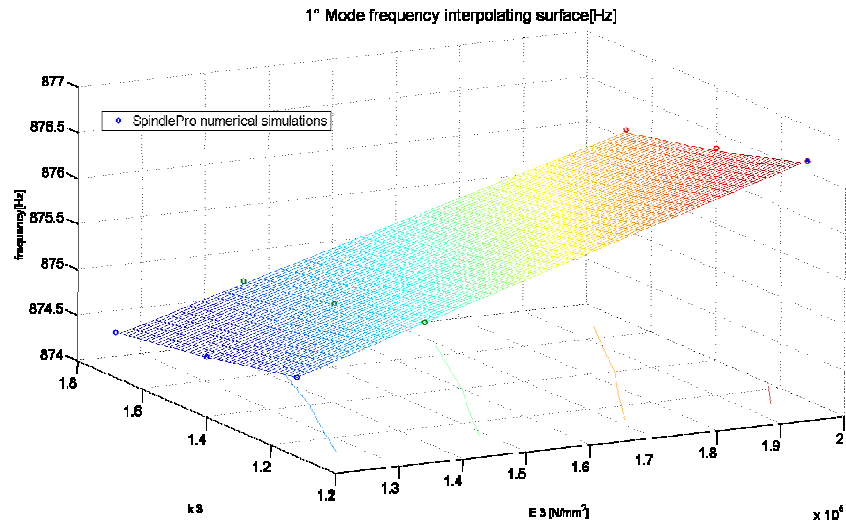


Fig. 2.25: Response surface – first mode (setup3)

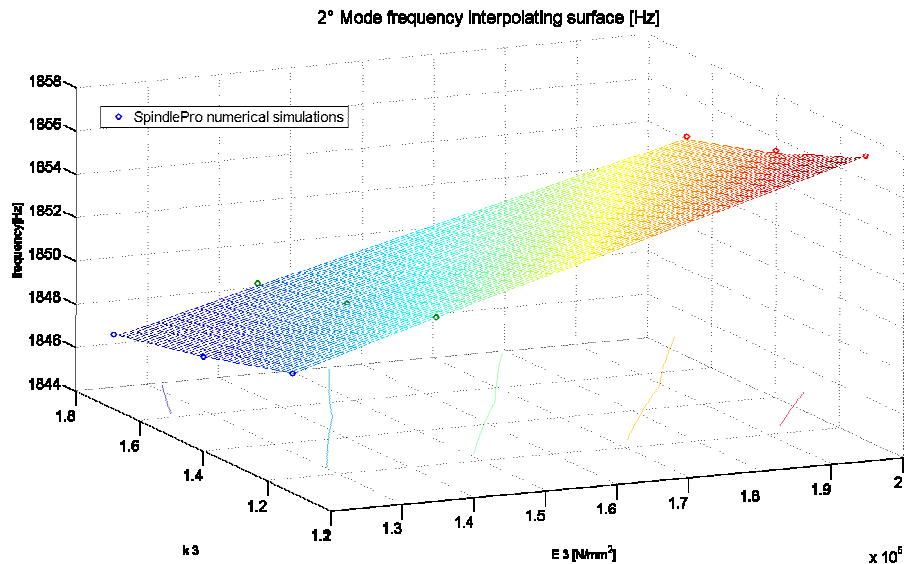


Fig. 2.26: Response surface – second mode (setup3)

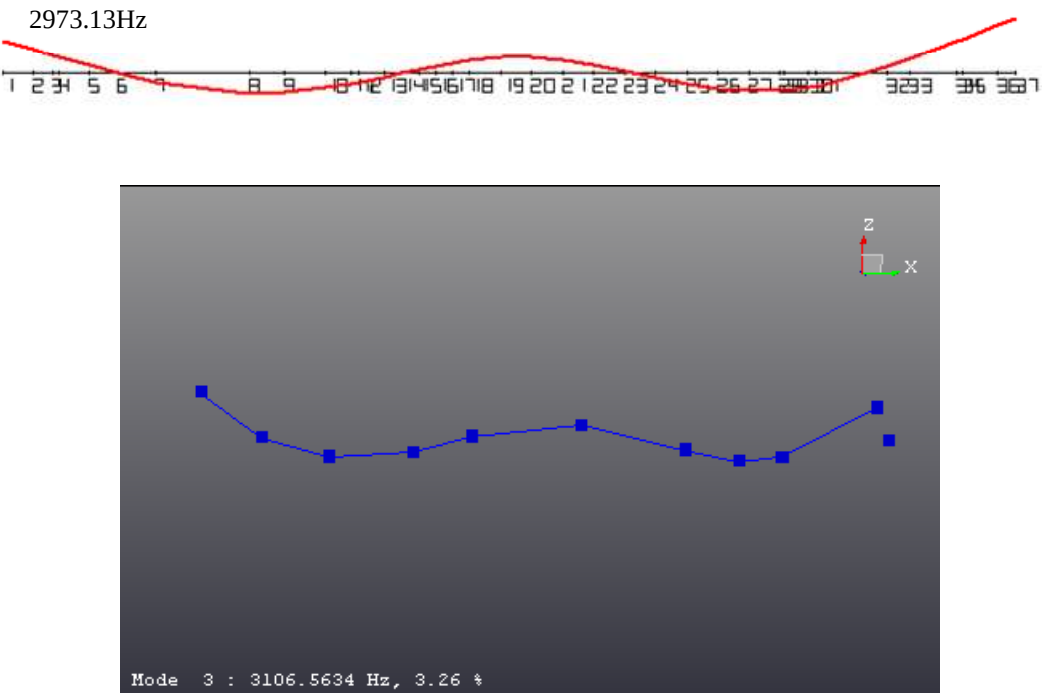


Fig. 2.27: Modal shape – experimental vs simulated – mode 3

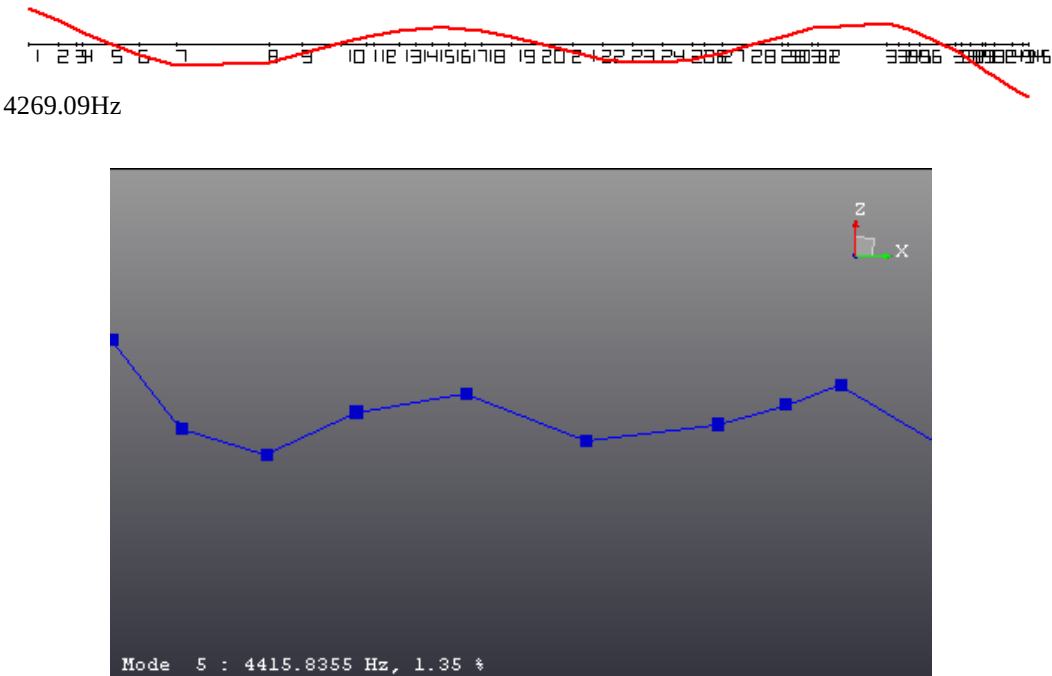


Fig. 2.28: Modal shape – experimental vs simulated – mode 5

From shaft setup 3, model errors on frequency values are less than 5%.

2.1.4 Experimental shaft-only setup 4

Adding a tool (an hollow boring bar) to the previous setup we obtained the fourth shaft setup, Fig. 2.29.

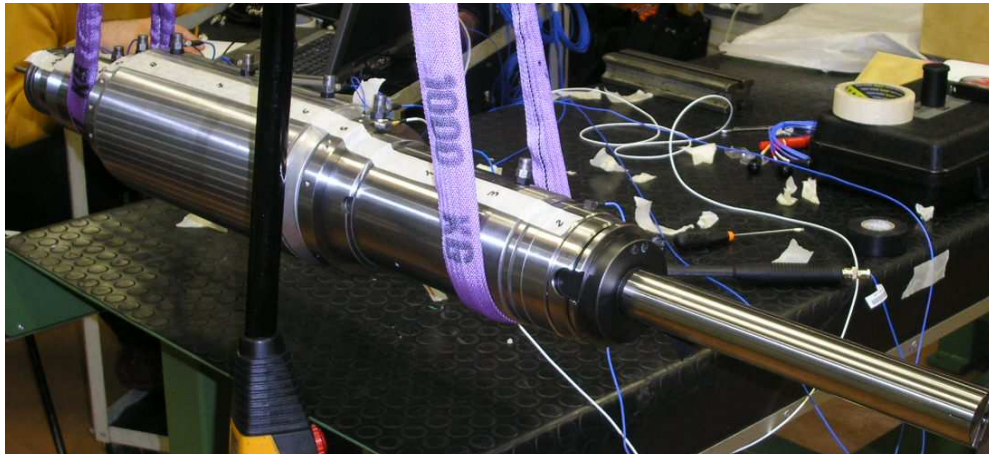


Fig. 2.29: Experimental set up 4 (shaft).

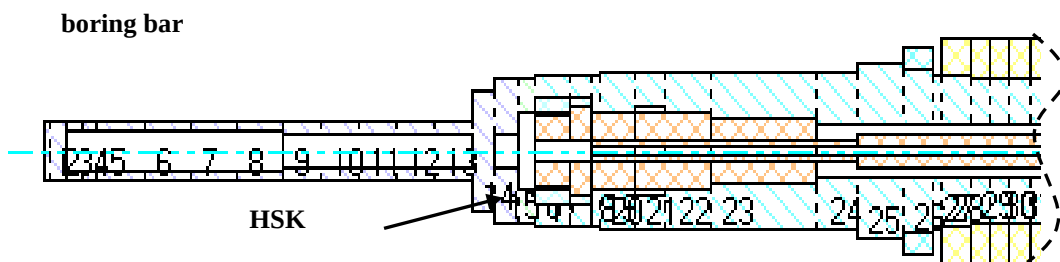


Fig. 2.30: Model setup 4 (shaft).

Fig. 2.30 shows the associated model.

The tool holder is modeled using a beam element as a lumped compliance. From a sensitivity analysis on the free-free numerical model this local compliance doesn't significantly alter the setup dynamic properties so nominal values for an HSK 100 tool holder, reported in Fig. 2.31 and [35], are used in the model.

Fig. 2.32, Fig. 2.33, Fig. 2.34 compare mode shapes for setup four.

The matching between experimental and simulated eigenvectors seems very good and as in other setups errors are limited up to 4-5%.

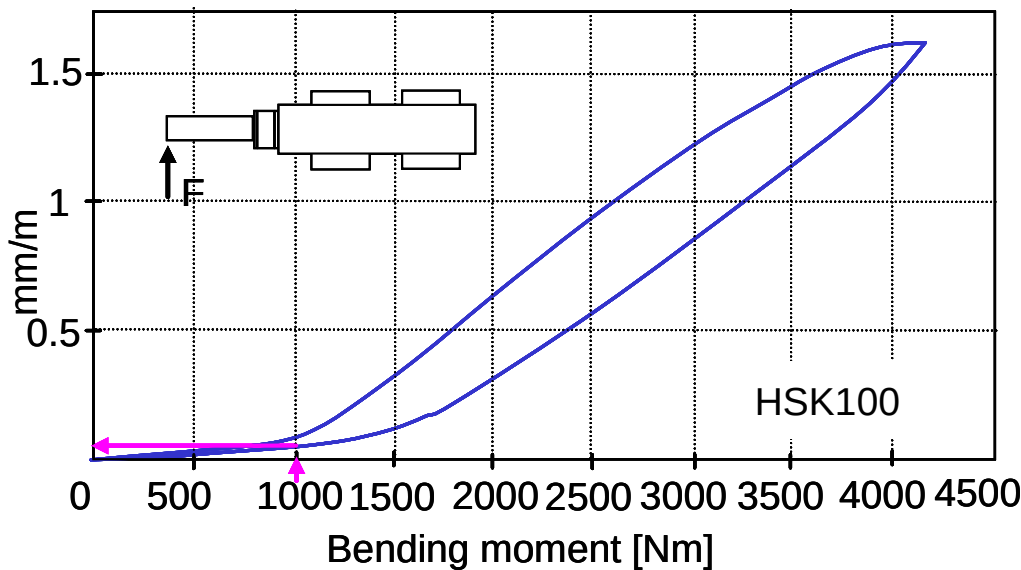


Fig. 2.31: HSK 100 tool holder stiffness

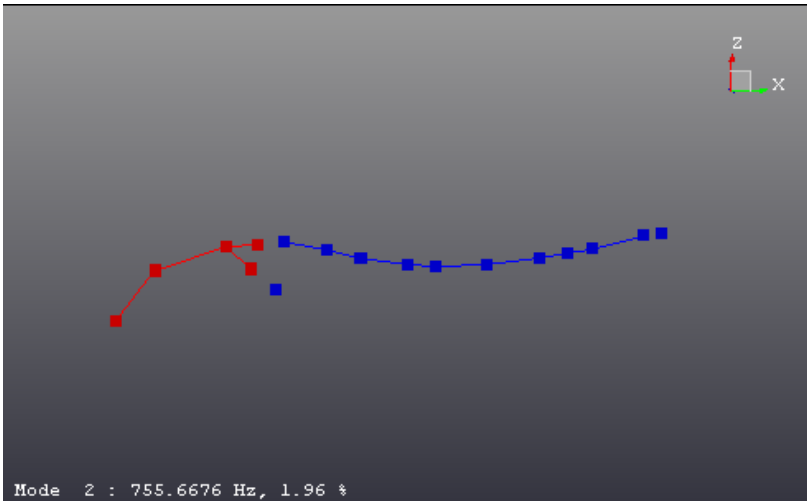
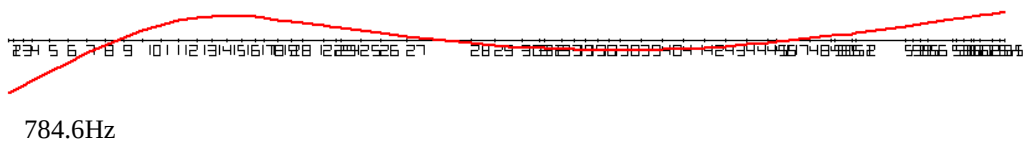


Fig. 2.32: Modal shape – experimental vs simulated – (mode 2)

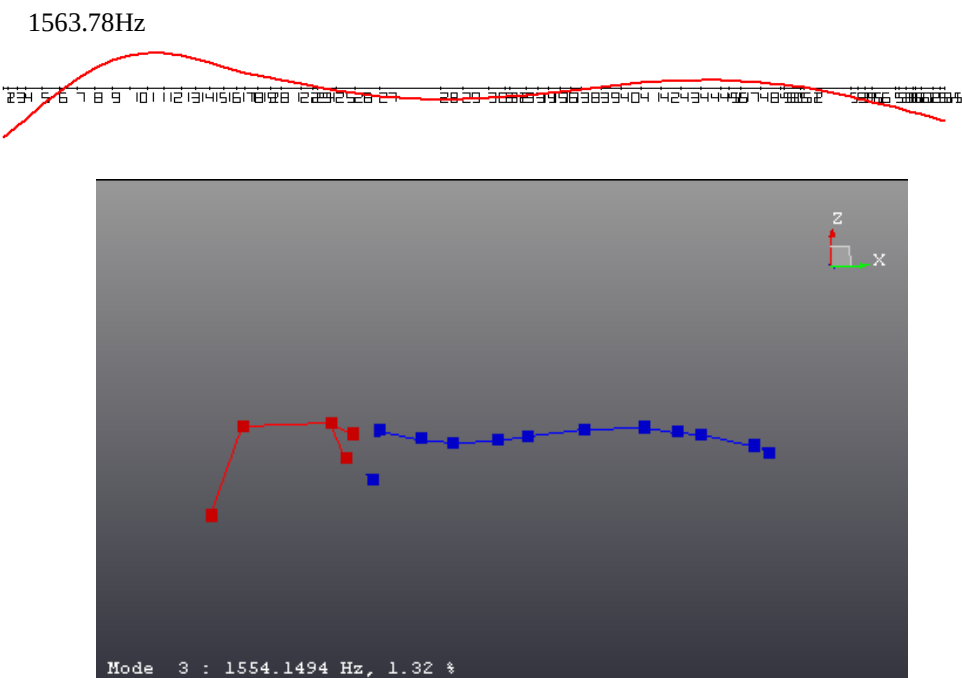


Fig. 2.33: Modal shape – experimental vs simulated – (mode 2)

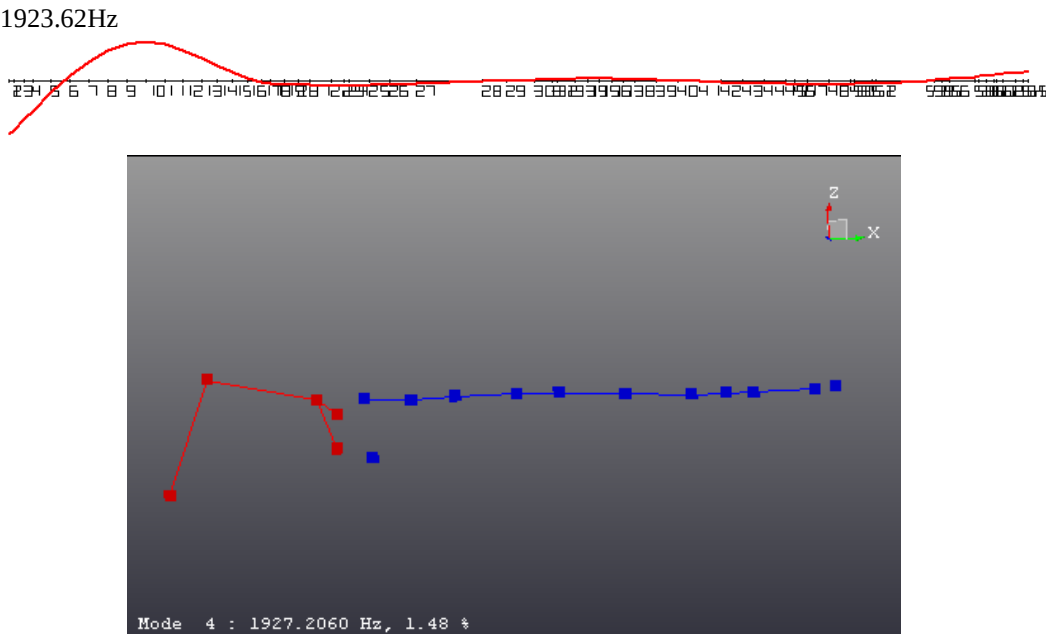


Fig. 2.34: Modal shape – experimental vs simulated – (mode 4)

2.2 Spindle assembly experimental characterization

Having characterized and verified all rotating elements, we passed to the assembled spindle.

Some phases of the experimental modal analysis are reported in Fig. 2.35.

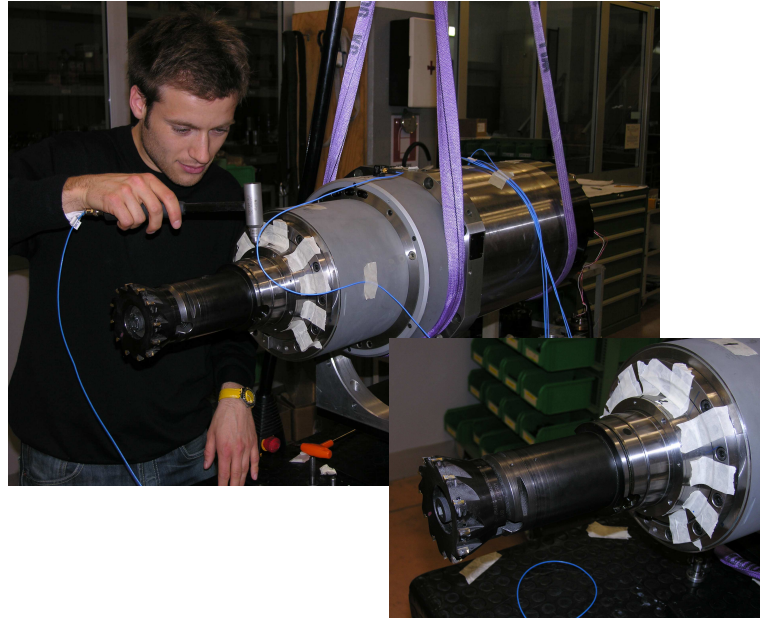


Fig. 2.35: Experimental Tests - spindle assembly

Fig. 2.36 illustrates the measurement points on the spindle, where the colour corresponds to various spindle components.

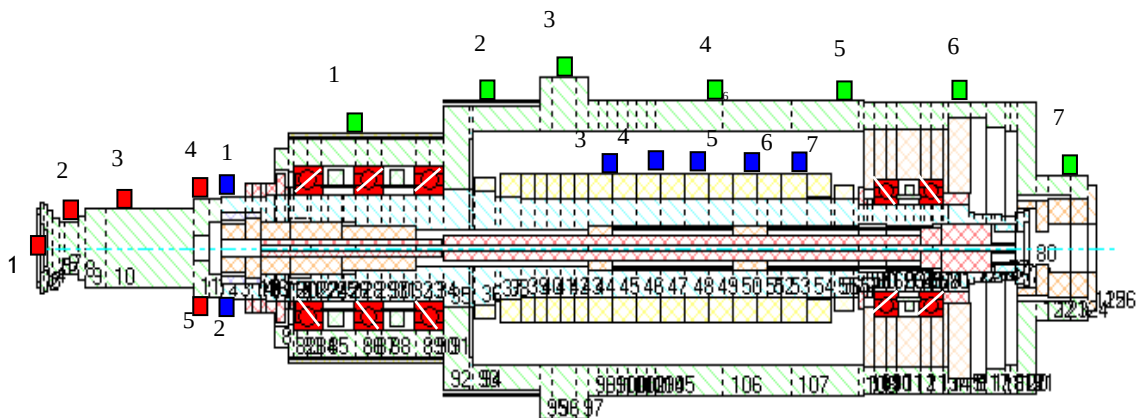


Fig. 2.36: Measurement points on the spindle.

This setup is used especially to verify the bearing stiffness estimation.

Experimental modal analysis on spindle assembly are also useful to study the bearing preload effects, the clamping force effects and the cutting force effects on the system modal dynamic properties.

Fig. 2.37 points out a comparison between simulated and experimental results: the updated model well reproduces basically the first and the second mode (the more flexible eigenmodes). A significant deviation from the experimental FRF is notable closeness to the 3rd mode and mostly to the 5th mode. The comprehension of these differences will be the goal of future research. A different and more complex updating strategy based on FRFs fitting will be investigated.

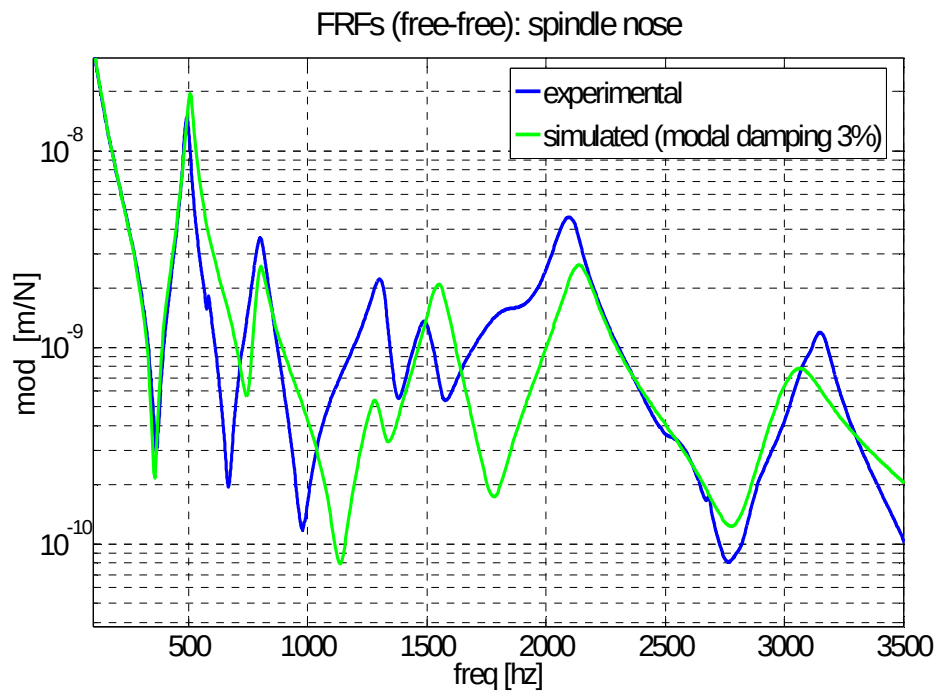


Fig. 2.37: Spindle nose FRFs.

The optimization procedure on the spindle assembly model has shown that the rear bearings, mounted on a ball bushing used to modulate the preload by an hydraulic piston, present a reduced equivalent radial bearing stiffness (-28%).

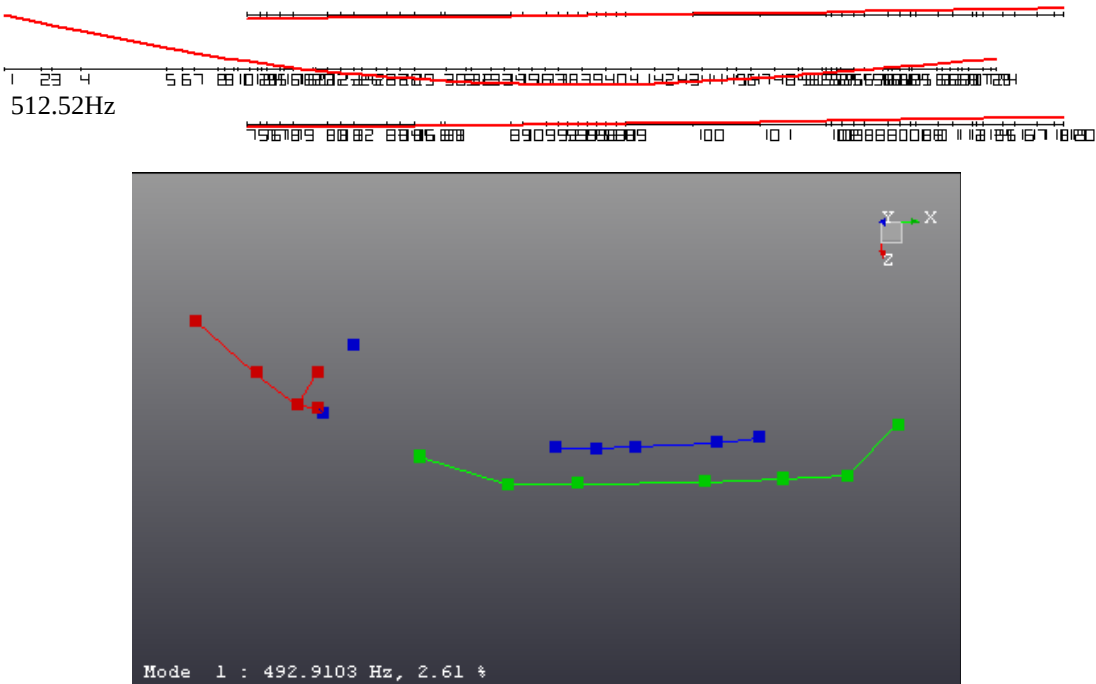


Fig. 2.38: Modal shape – experimental vs simulated – (mode 1)

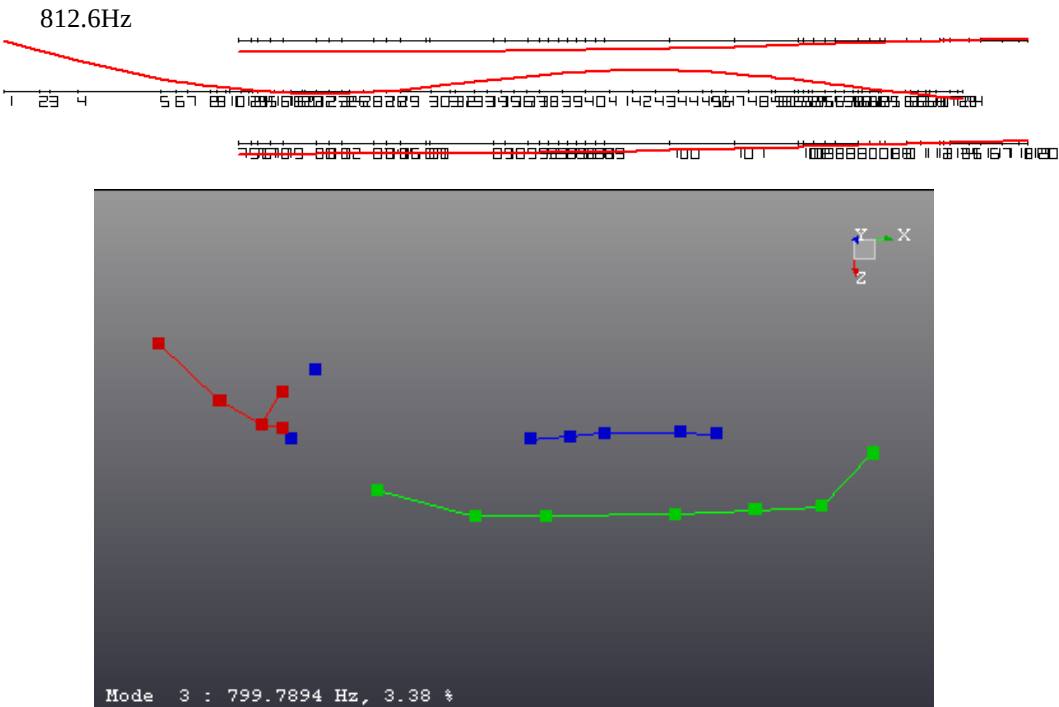


Fig. 2.39: Modal shape – experimental vs simulated – (mode 3)

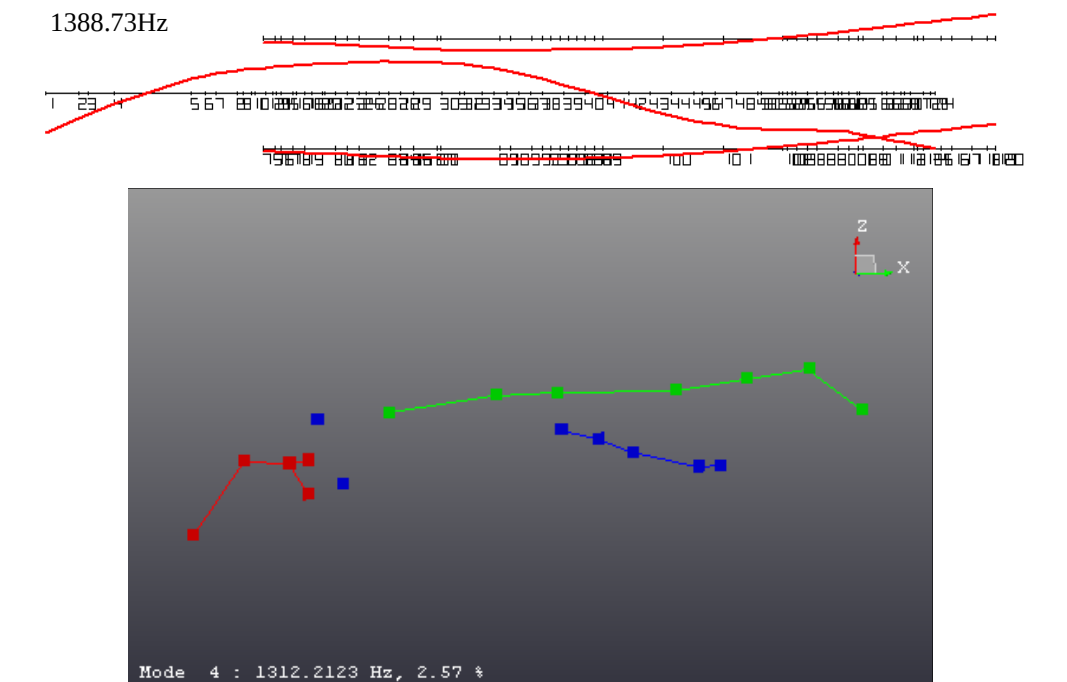


Fig. 2.40: Modal shape – experimental vs simulated – (mode 4)

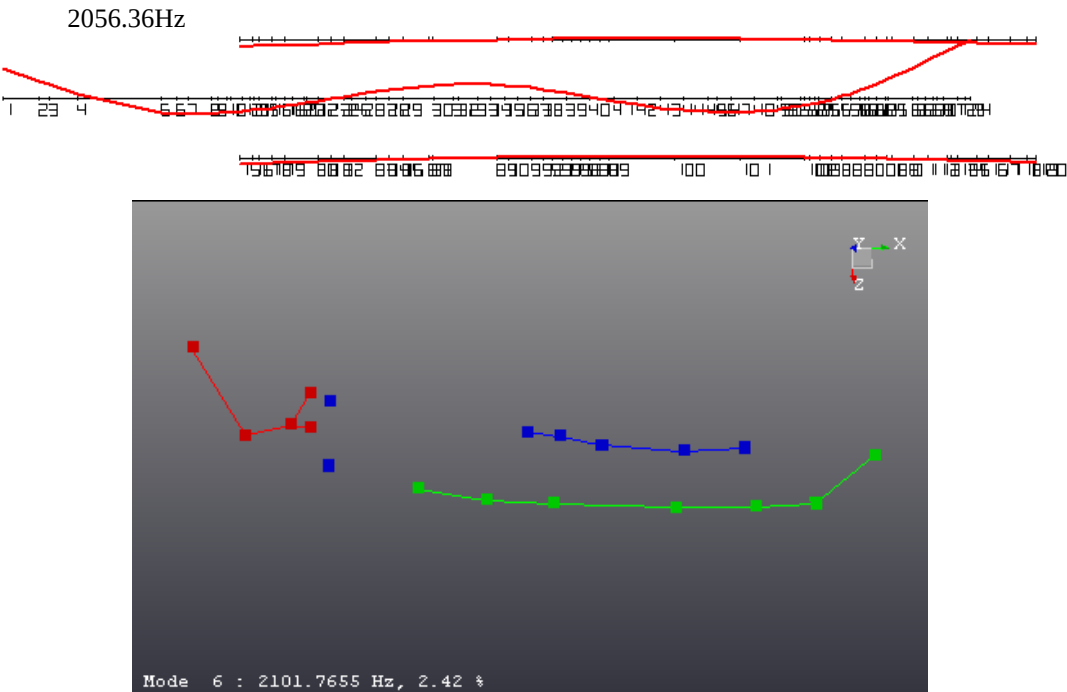


Fig. 2.41: Modal shape – experimental vs simulated – mode 6

2.2.1 Bearing preload, clamping force and cutting forces effects

Clamping force effects and the efficiency of the spring based clamping system have been studied. Three force levels for the tool clamping were investigated. In Fig. 2.42 experimental results are reported: the effect (at 290 Hz) of a local compliance at tool holder already appears with the medium force level (37,6kN). Only small flexibility increments have occurred near the other modes .

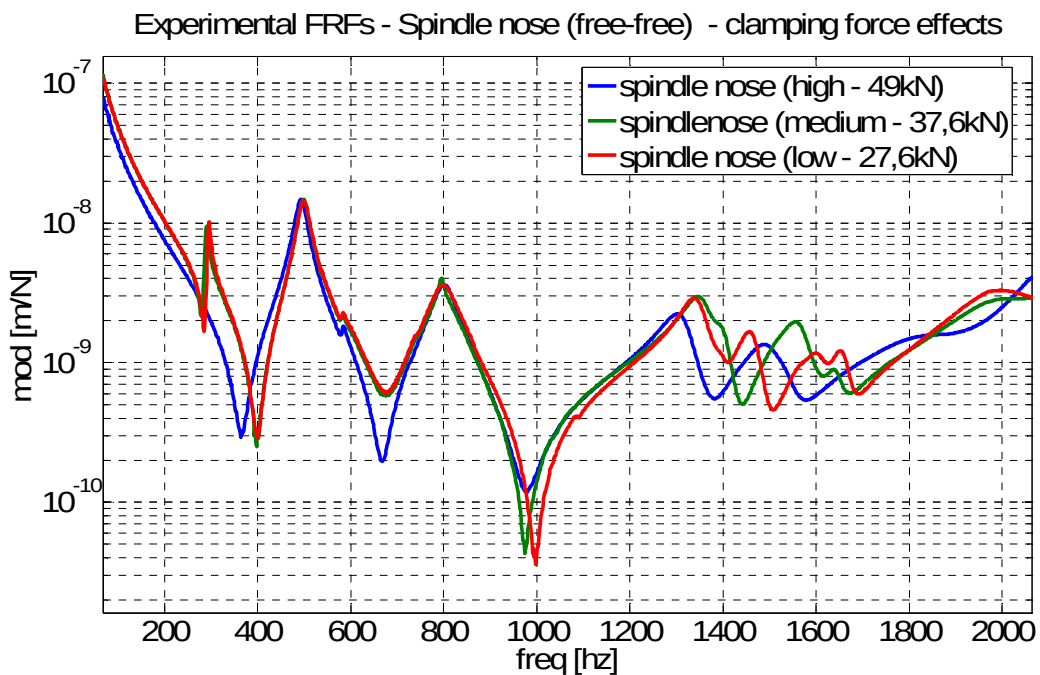


Fig. 2.42: Experimental FRFs - clamping force effects.

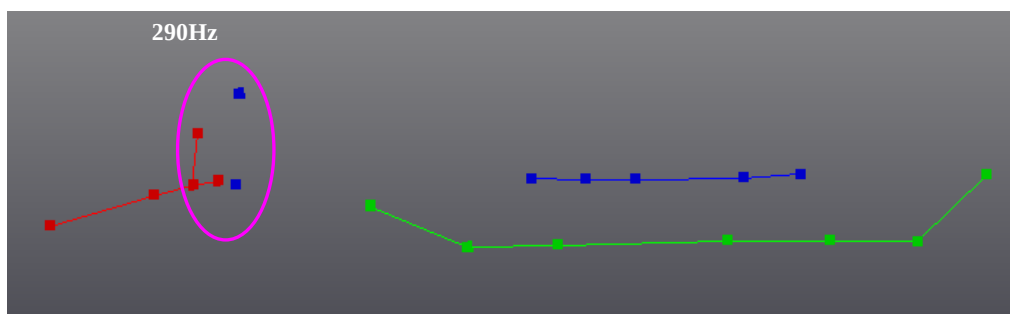


Fig. 2.43: HSK local dynamic compliance.

To get a first rough estimation of how cutting forces can alter the dynamic behaviour of the spindle during machining, a simple test was performed. Fig. 2.44 describes the experimental test: a mass (400N) was hung by a soft suspension to the tool tip, in order to simulate the cutting force mean value.

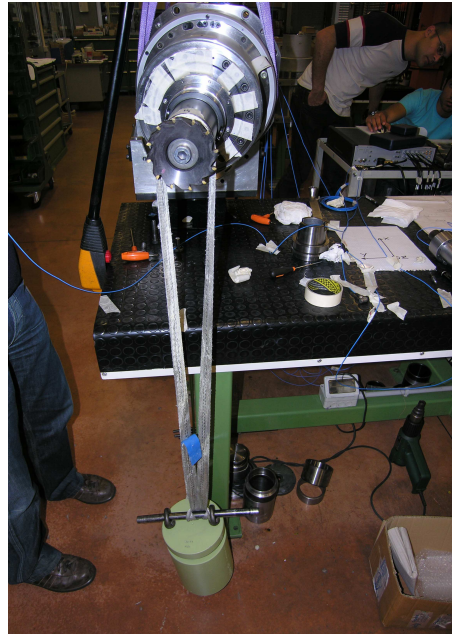


Fig. 2.44: Load effects – experimental tests

Measured FRFs are shown in Fig. 2.42.

A slight dynamic flexibility reduction can be noticed.

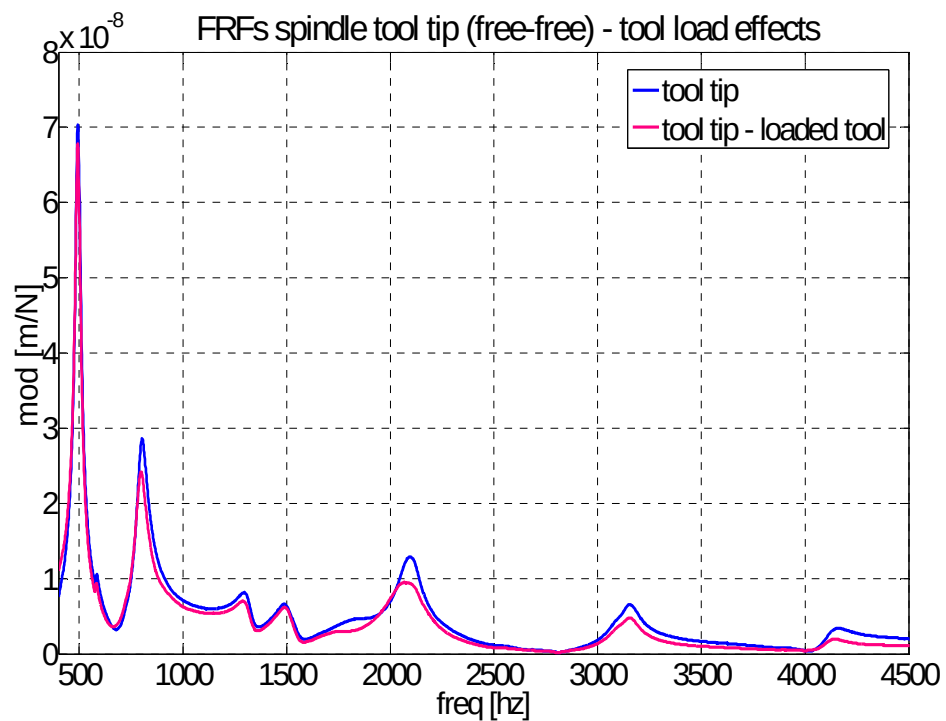


Fig. 2.45: Tool load effects

The effect of bearing preload has been investigated measuring the dynamic compliance at the tool tip with spindle directly mounted on a machine, as shown in the Fig. 2.46.

Two bearing preload levels were used: 9kN and 12kN.

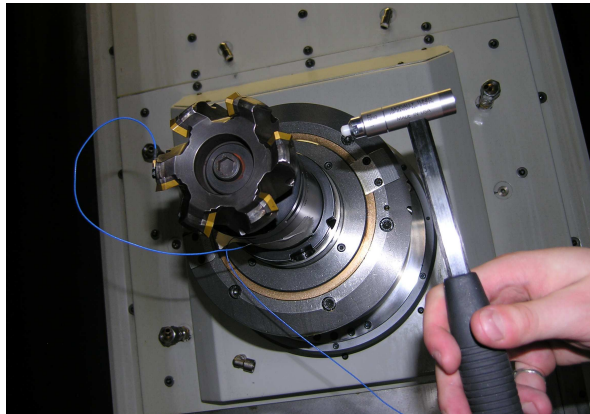


Fig. 2.46: Test case motor spindle . compliance measurements

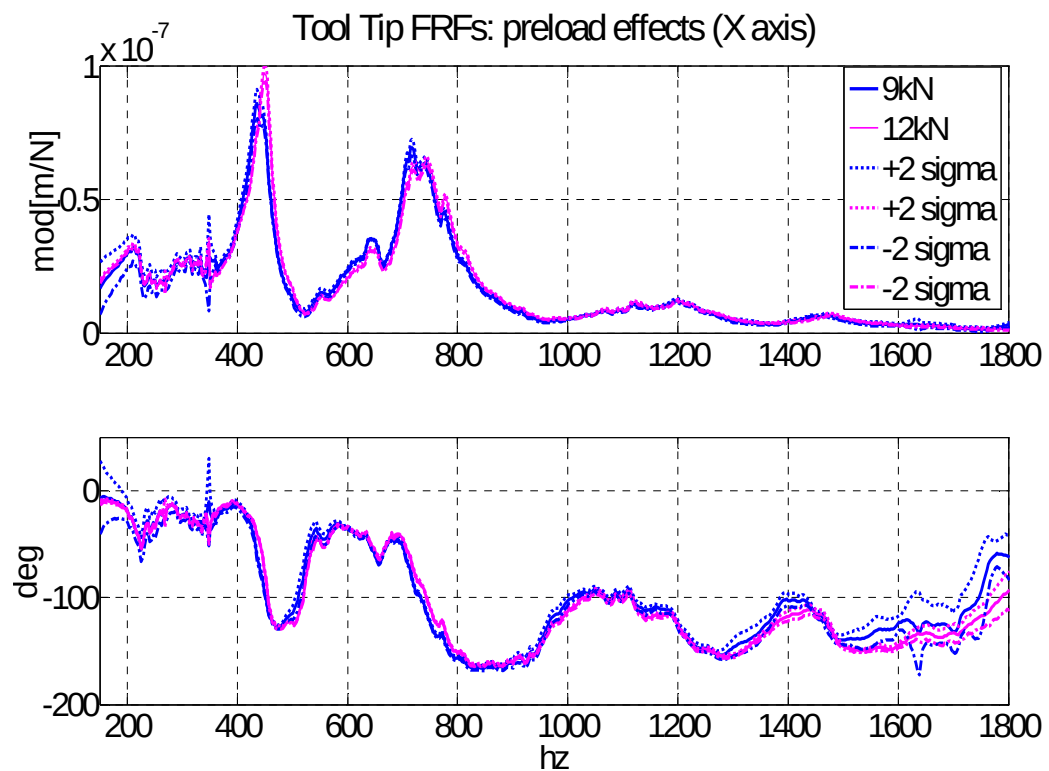


Fig. 2.47: Experimental tool FRFs - bearing preload effects.

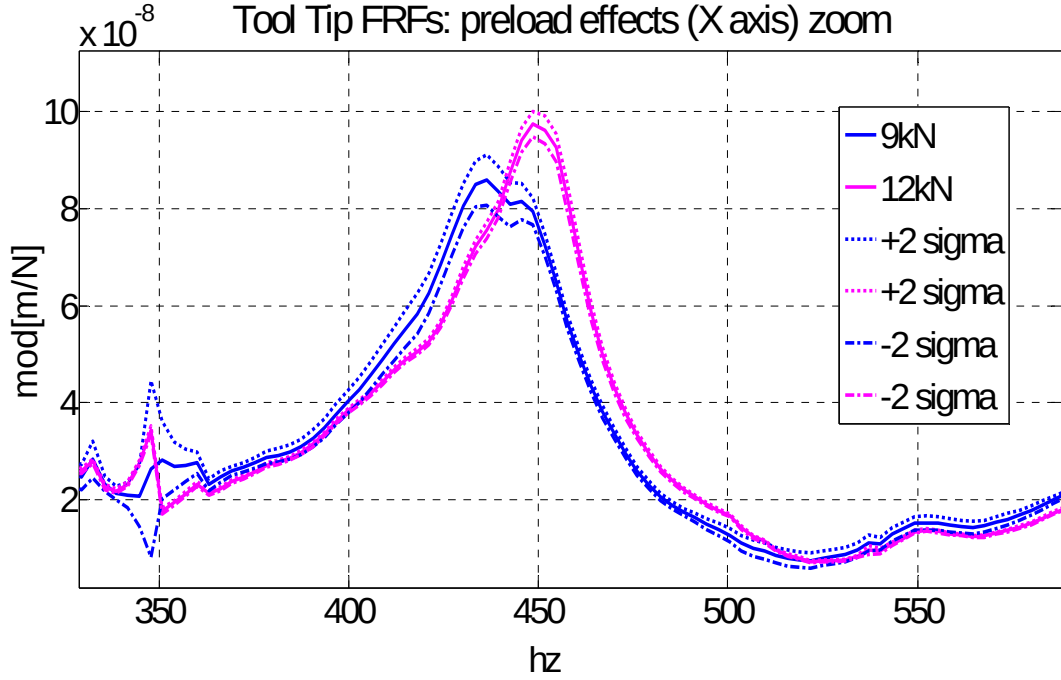


Fig. 2.48: Bearing preload effects – zoom.

Fig. 2.48 shows the 95% confidence interval of each Frequency Response Function.

The Confidence Interval has been computed from using the Coherence function as described in Ferrar [48], Bendat [49] and reported in Eq. 2.2:

$$\sigma\left(\left|\hat{H}_{xy}(\omega)\right|\right)=\frac{\sqrt{\left(1-\gamma_{xy}^2(\omega)\right)}}{\left|\gamma_{xy}(\omega)\right| \cdot \sqrt{2}}\left|\hat{H}_{xy}(\omega)\right|$$

Eq. 2.2:

$$\sigma\left(\operatorname{phase}\left(\hat{H}_{xy}(\omega)\right)\right)=\sin^{-1}\left(\frac{\sqrt{\left(1-\gamma_{xy}^2(\omega)\right)}}{\left|\gamma_{xy}(\omega)\right| \cdot \sqrt{2}}\right)$$

Where $\gamma_{xy}(\omega)$ is the coherence function and $H_{xy}(\omega)$ is the generic transfer function.

Bearing preload slightly influences the spindle dynamics : passing from 9kN to 12kN we have noticed a small frequency increase for example referring to the first spindle mode (from 436.4Hz to 448.6 Hz) and a modal damping reduction (from 3.9% to 3.3%). Similar results were found by Cao in [32] and [44].

Fig. 2.49 shows the bearing preload effects on cutting stability lobes. The preload increment reduces the chatter free stability limit, due to the associated modal damping reduction. So we can conclude that the bearing preload can't be used to improve the spindle performance as we initially had though.

In this paper we haven't experimentally investigated the spindle speed effects because simulation forecast a reduced influence, for the speed range of interest, on the spindle dynamic behaviour Fig. 2.50 and thus on cutting process stability. Movahhedy [27] dealt with the influence of the gyroscopic effects on the stability lobes diagram.

It has been considered only the speed effect linked to the ball bearing behaviour. It seems that the bearing stiffness weakening doesn't strongly affect the frequency response function and consequently the stability limit.

We can conclude that the accuracy in dynamic modeling is a real critical issue while developing innovative spindles so that in this work a spindle modeling methodology has been proposed.

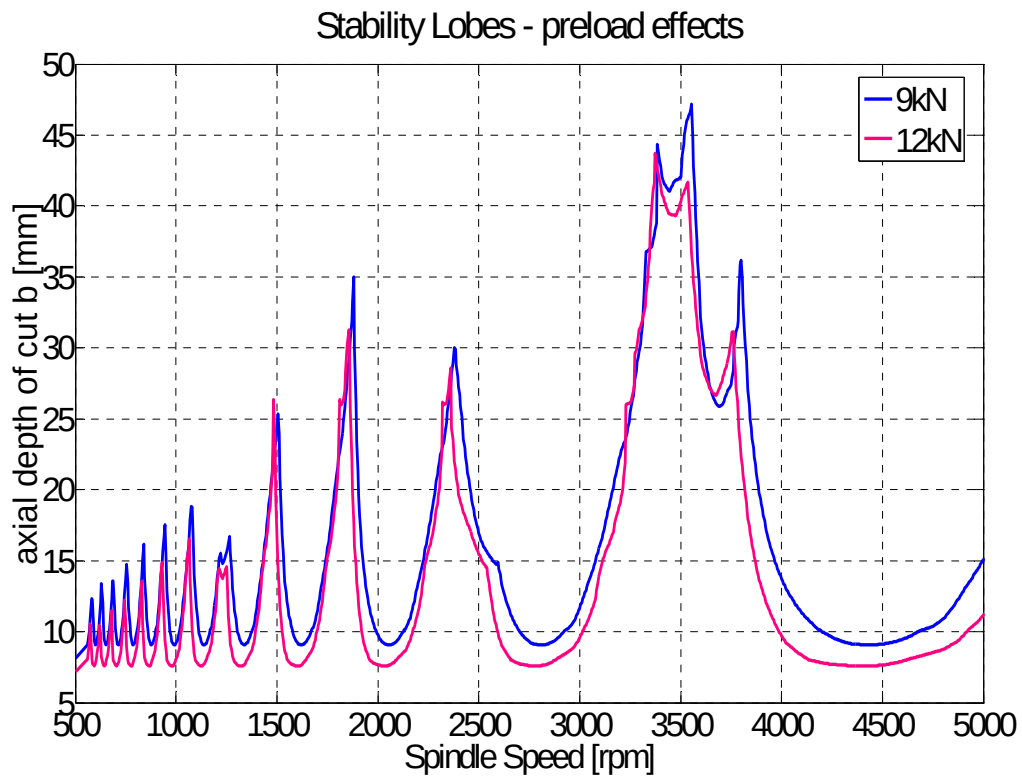


Fig. 2.49: Bearing preload effects – stability lobes.

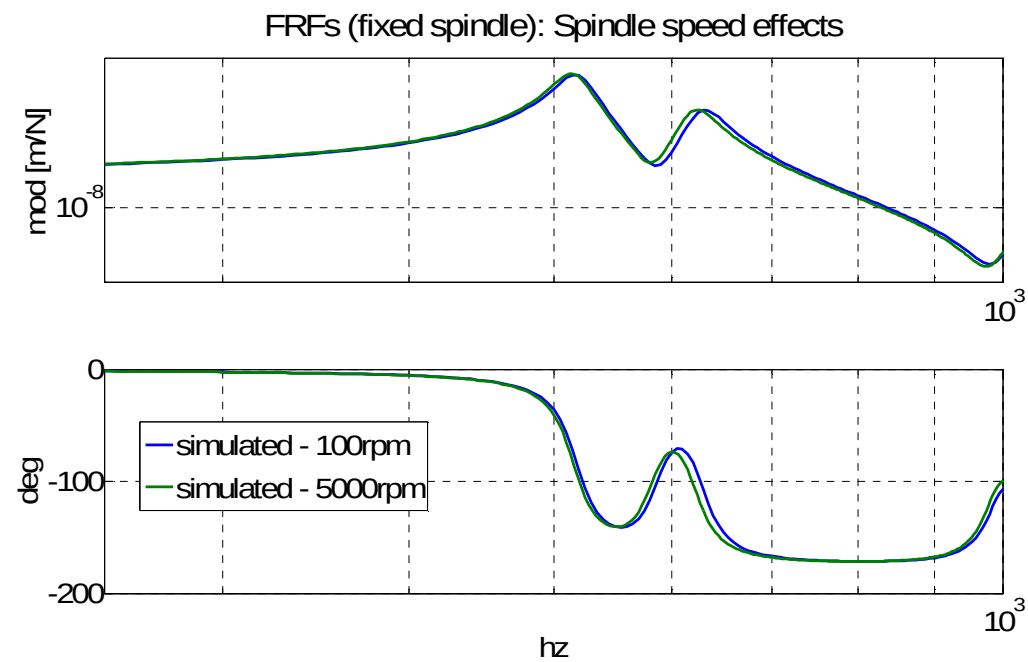


Fig. 2.50: Tool tip FRFs - spindle speed effects

2.3 Composite spindle shaft design

In this section a shaft made by carbon fiber-epoxy composite has been designed in order to exploit the high specific stiffness and the intrinsic damping properties of these materials as a mean to increase the spindle design performances and consequently the chatter free stability limit.

Various applications of the composite materials to rotating elements design can be found in the specific literature, see section 1.2.

A composite carbon fibre-epoxy composite material solution moreover allows getting enhancements in the unbalance issues.

The defined modelling guidelines will be used to test the new design solution.

It is necessary to state some preliminary considerations assumed in this work:

- The spindle dynamic behaviour was simulated by means of a beam model thus an equivalent isotropic material had used to describe the composite fiber material.
- In this first analysis the manufacturing aspects haven't been taken into account. The proposed solution was going to evaluate the extreme enhancements due to the use of composite materials and the spindle shaft wasn't redesigned.

We have decided to use laminated long carbon fiber –epoxy composite.

In order to study an orthotropic material [57], [60] and [61], the following relationships need to be used:

These are suitable for a plane model.

$$\text{Eq. 2.3:} \quad \{\sigma\} = [D]\{\varepsilon\}$$

$$[D] = \begin{bmatrix} \frac{E_{11}}{(1-\nu_{12}\nu_{21})} & \nu_{21}D_{22} & 0 \\ \nu_{12}D_{22} & \frac{E_{22}}{(1-\nu_{12}\nu_{21})} & 0 \\ 0 & 0 & G \end{bmatrix}$$

Eq. 2.4: D matrix

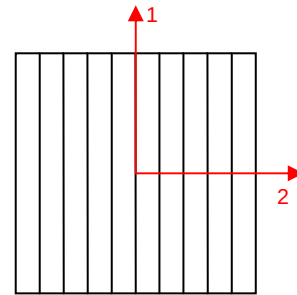


Fig. 2.51: Plane model

And it is possible to compute ν_{21} :

$$\text{Eq. 2.5:} \quad \nu_{21} \cdot E_1 = \nu_{12} \cdot E_2$$

Where E_{11} is the young's module, direction 1 and E_{22} is the young's module, direction 2.

If we rotate the coordinates system (i.e. $\bar{1}-\bar{2}$ as reported in) it is possible to define the T matrix as reported in Eq. 2.6.

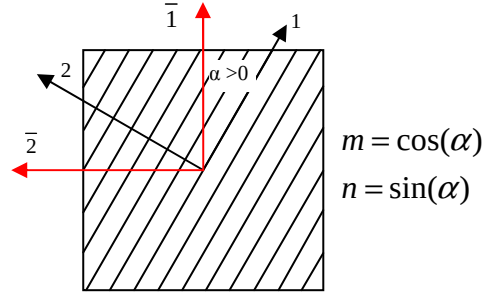


Fig. 2.52: Coordinate system rotation

$$\text{Eq. 2.6} \quad \{\bar{\sigma}\} = [T]\{\sigma\} \quad [T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}$$

$$\text{Eq. 2.7:} \quad [D]_{\alpha} = [T][D][T]^T$$

Furthermore if we consider different laid upon layers

$$\text{Eq. 2.8:} \quad [D]_{\text{tot}} = \begin{bmatrix} \frac{E_{11eq}}{(1 - \nu_{12eq}\nu_{21eq})} & \nu_{21eq}D_{22eq} & 0 \\ \nu_{12eq}D_{11eq} & \frac{E_{22eq}}{(1 - \nu_{12eq}\nu_{21eq})} & 0 \\ 0 & 0 & G_{eq} \end{bmatrix} = \frac{1}{h_{\text{tot}}} \sum_{i=1}^n ([D]_{\alpha i} h_i)$$

Where $[D]_{\alpha i}$ is the matrix $[D]$ for each layer, h_{tot} is the overall laminated thickness and h_i is the thickness of the single layer “i”.

Once computed the $[D]_{\text{tot}}$ it is possible to determine the equivalent properties of the composite material using the following relationship:

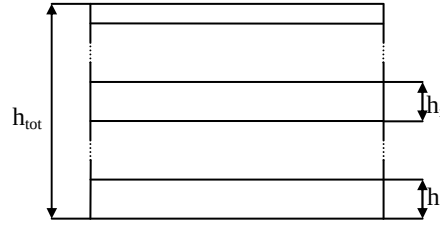


Fig. 2.53: Thickness of the laminated composite material

$$[D_{tot}] = \begin{bmatrix} D_{tot11} & D_{tot12} & 0 \\ D_{tot21} & D_{tot22} & 0 \\ 0 & 0 & D_{tot33} \end{bmatrix} = \begin{bmatrix} \frac{E_{1eq}}{(1 - \nu_{12eq} \nu_{21eq})} & \nu_{21eq} D_{22eq} & 0 \\ \nu_{12eq} D_{11eq} & \frac{E_{2tot}}{(1 - \nu_{12eq} \nu_{21eq})} & 0 \\ 0 & 0 & G_{eq} \end{bmatrix}$$

$$\text{Eq. 2.9: } \Rightarrow \begin{cases} \nu_{12eq} = \frac{D_{tot21}}{D_{tot22}} \\ \nu_{21eq} = \frac{D_{tot12}}{D_{tot22}} \\ E_{1eq} = D_{tot11} \left(1 - \frac{D_{tot12}}{D_{tot22}} \frac{D_{tot21}}{D_{tot22}} \right) \\ E_{2eq} = D_{tot22} \left(1 - \frac{D_{tot12}}{D_{tot22}} \frac{D_{tot21}}{D_{tot22}} \right) \end{cases}$$

2.3.1 Composite material design

It has been chosen a pitch based “URN 300” from SK Chemicals© whose properties are reported in Tab. 2.5.

Different solutions have been evaluated varying the thickness of the layers in order to obtain a good compromise from the contribute linked to each lying.

We have selected the configuration that maximizes the bending stiffness increment:

- 85% of the overall thickness with $\alpha = 0$
- 5% of the overall thickness with $\alpha = 5$
- 5% of the overall thickness with $\alpha = -5$
- 5% of the overall thickness with $\alpha = 90$

In Tab. 2.5 the properties of the composite prepreg are reported

<i>Property</i>	<i>Value</i>
E_1 (GPa)	380
E_2 (GPa)	5.1
G_{12} (GPa)	5.5
ν_{12}	0.29
ρ (kg/m ³)	1750
ζ	20.7e-3

Tab. 2.5: prepreg properties

And using the Eq. 2.9 the following equivalent properties have been obtained:

$$\text{Eq. 2.10: } \left\{ \begin{array}{l} \nu_{12eq} = \frac{D_{tot21}}{D_{tot22}} = 0.31 \\ \nu_{21eq} = \frac{D_{tot12}}{D_{tot22}} = 0.27 \\ E_{1eq} = D_{tot11} \left(1 - \frac{D_{tot12}}{D_{tot22}} \frac{D_{tot21}}{D_{tot22}} \right) \cong 305000 \text{ MPa} \\ E_{2eq} = D_{tot22} \left(1 - \frac{D_{tot12}}{D_{tot22}} \frac{D_{tot21}}{D_{tot22}} \right) \cong 31000 \text{ MPa} \end{array} \right.$$

2.3.2 Numerical simulations

The undamped modal frequencies of the spindle with the steel shaft are reported in Tab. 2.6.

The model used to evaluate the spindle performance has been depicted in Fig. 2.2.

#mode	Freq.[Hz]
1	507,1
2	622,1
3	1007,96
4	1297,83
5	1571,34
6	2145,26

Tab. 2.6: Undamped modal frequencies – steel version

Substituting the shaft properties with the composite ones reported in Tab. 2.7.

Property shaft material	Value
E_{1eq} [MPa]	305000
ν	0.3
ρ [kg / m ³]	1750
shear	1.5

Tab. 2.7: Shaft material properties, carbon-epoxy composite

#mode	Freq.[Hz]	Increment%
1	609,67	20,2
2	762,9	22,6
3	1116,33	10,8
4	1582,15	21,9
5	2023,8	28,8
6	2519,3	-
7	3101,8	44,6

Tab. 2.8: Modal frequencies – increment – undamped analysis

The modal frequencies linked to the composite shaft version, having considered the properties reported in Tab. 2.7, are reported in Tab. 2.8. Moreover the frequency increments have been underlined.

There is a modal flipping between the two alternative solutions. In sixth mode a local compliance occurs, in the steel shaft spindle the analogous eigenmode is located at higher frequency.

In order to evaluate the stability limit it is necessary to compute the tool tip dynamic compliance of these two design solutions.

An important preface regarding the model used to evaluate the tool compliance is indispensable: for this purpose it isn't possible to set modal damping ratios because they would alter the physical behaviour of the system; the damping needs to be introduced into the system by an appropriate matrix. In SpindlePro®, a software suitable for the spindle modelling, this can be done introducing the damping through the “bearing” elements in fact a damping coefficient can be associated to each bearing stiffness.

In order to tune the modal damping ratios estimated from the experimental modal tests formerly reported, a fictitious intermediate bearing has been introduced. This fictitious bearing has a very low stiffness but can be used to adjust the modal damping of the system. The damping introduced by the intermediate bearing can be thought as linked to the damping that is really entered by the rotor.

A first tuning stage was hence performed on the spindle with the steel shaft.

Using this approach it is therefore possible to correctly evaluate the dynamic behaviour modification due to the use of the composite material.

In the performed analysis the intrinsic damping contribute just related to composite material hasn't been considered because it is possible to evaluate it only by experimental tests. Not even in the specific literature accurate data linked to the damping increment can be found.

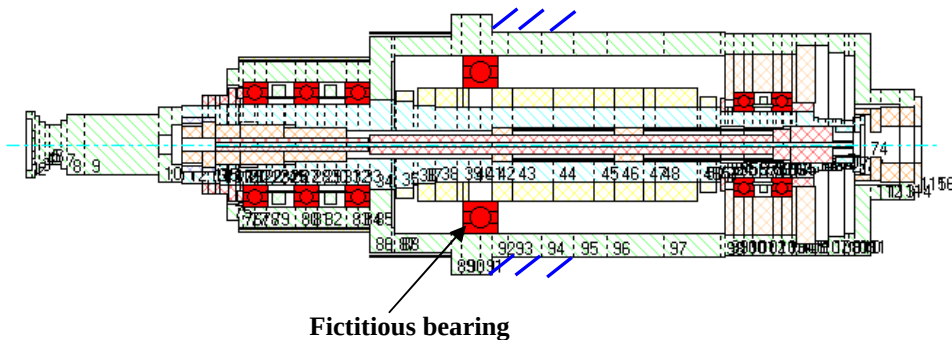


Fig. 2.54: Spindle model – fictitious bearing

The compared FRFs have been reported in Fig. 2.55.

The result is very interesting: nevertheless the carbon-fiber composite has a higher specific stiffness than the steel, it hasn't brought to a better dynamic behaviour and consequently this tested design solution seems absolutely not promising. An interpretation will be provided.

Basically it depends on the effect of the stiffness increment and the mass reduction on the modal parameters especially on the modal damping ratios. In Fig. 2.56 are reported respectively the effect of the stiffness increment and the mass reduction on the asymptotic dynamic compliance.

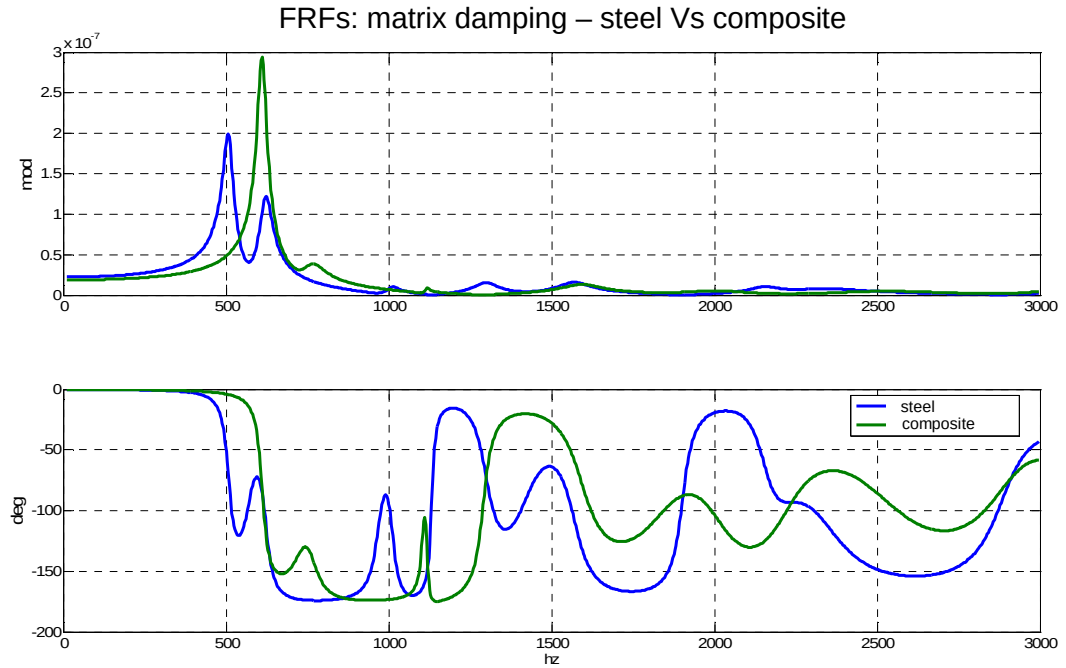


Fig. 2.55: Tool tip dynamic compliance – composite shaft Vs steel shaft

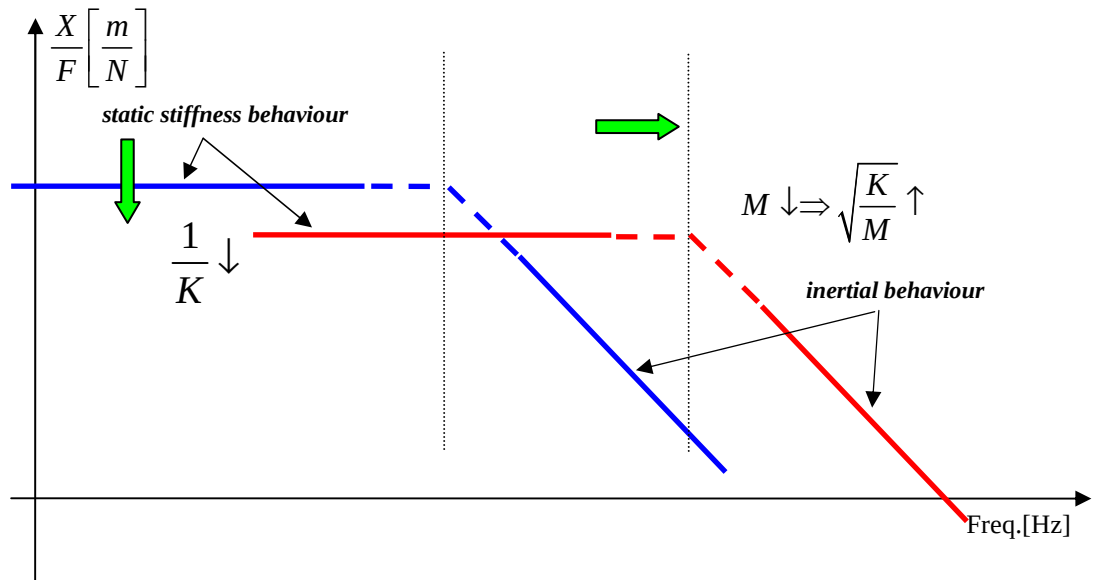


Fig. 2.56: Stiffness increment and mass reduction effects – single dof system

If we consider the relationships reported in Fig. 2.57 for a single degree of freedom we can assert that the compliance peak depends on the modal stiffness and the damping ratio.

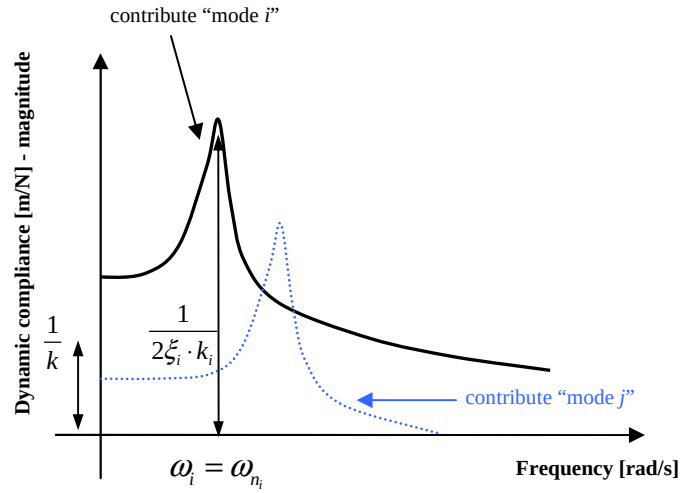


Fig. 2.57: Modal parameters – 1d.o.f.

$$\omega_{n_i} = \sqrt{\frac{k_i}{m_i}} : \text{natural frequency} \left[\frac{\text{rad}}{\text{s}} \right]$$

$$\xi_i = \frac{C_i}{2\sqrt{m_i \cdot k_i}} : \text{damping ratio}$$

Eq. 2.11:

$$k_i : \text{stiffness} \left[\frac{\text{N}}{\text{m}} \right]$$

$$m_i : \text{mass} [\text{kg}]$$

$$C_i : \text{damping} \left[\frac{\text{N}}{\text{m/s}} \right]$$

So it is necessary to evaluate separately the two effects even if the spindle case is more complex than a single degree of freedom system.

#mode	Freq.[Hz]	Modal mass [kg]	Modal stiffness [N/m]	ζ %
1	508,2	7,9	80939243,5	3,3
2	621,4	9,3	142123240,6	3,1
3	1101,2	80,9	3264343204,2	1,8

Tab. 2.9: Modal parameters, steel version

Considering the same damping introduced in the system by “bearing elements” the identified modal parameters linked to both the solutions are reported in Tab. 2.9 and Tab. 2.10

#mode	Freq.[Hz]	Modal mass [kg]	Modal stiffness [N/m]	ζ %
1	610,9	4,8	71671602,7	2,4
2	762,9	18,3	421659475,7	4,8
3	1116,1	279,3	13739681910,5	5,8

Tab. 2.10: Modal parameters carbon-epoxy composite

Using the modal parameters it is hence possible to approximately evaluate the height of the first resonance peak in both the solutions. Fig. 2.58 shows that the first peak is mostly influenced by the first modal contribute so we can outline some conclusions directly from the modal parameters linked to the first mode.

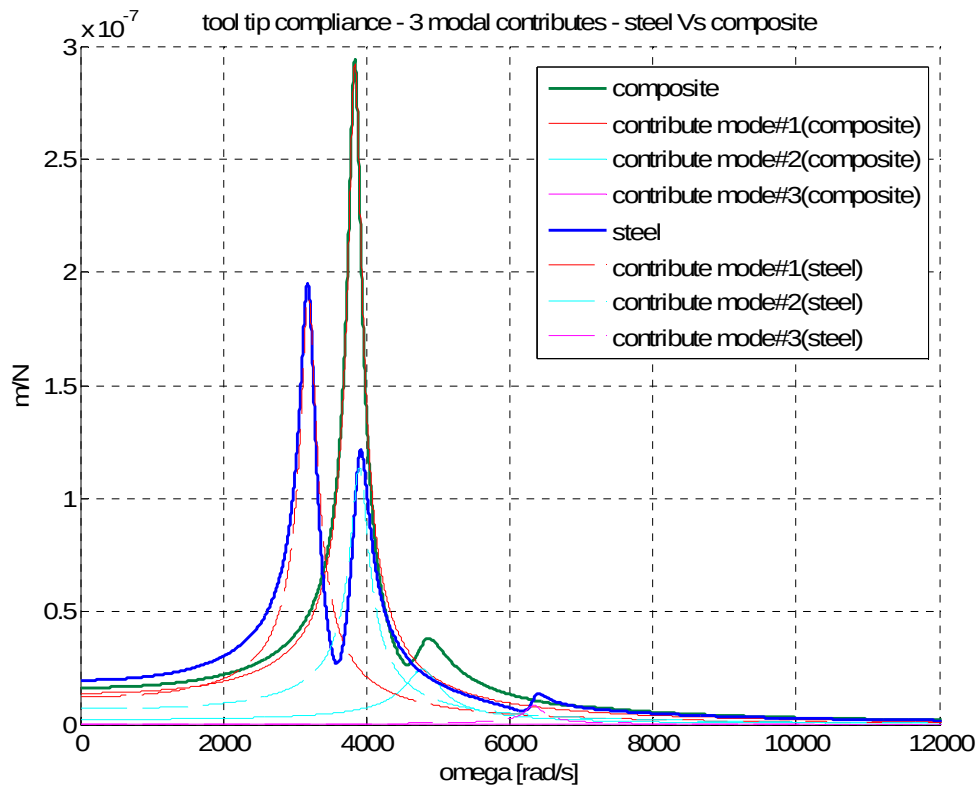


Fig. 2.58: Tool tip compliance - modal contributes

It can be observed that the damping reduction is the most responsible to the increment of the resonance peak.

It is interesting to understand which contribute is predominant in the observed behaviour that is if this effect is more influenced by the stiffness increment or mass reduction.

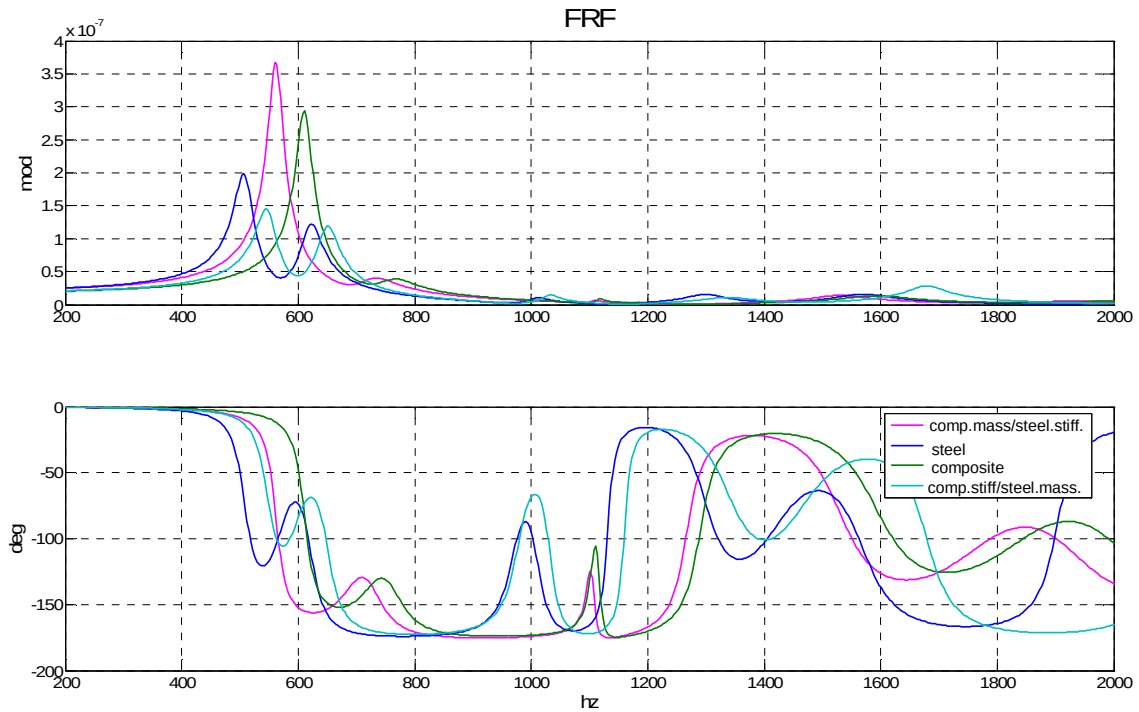


Fig. 2.59: Tool tip dynamic compliance

In order to draw some outlines two hybrid and fictitious spindle models have been created: in the first one the shaft has the composite density and the Young's module of the steel, conversely the second hybrid model has the density of the steel and the carbon-epoxy stiffness.

These two hybrid models allow to evaluate separately the interested contributes. Fig. 2.59 underlines that the observed increment of the first and dominant resonance peak is basically lead by the mass reduction linked to the introduction of carbon-epoxy composite.

#mode	Freq.[Hz]	ζ %
1	561,71	2,37
2	729,1	4,86
3	1106,39	5,99
4	1530,4	3,41
5	1933,8	4,96
6	2475,84	7,36
7	2798,99	2,03

Tab. 2.11: Fictitious case – composite density/steel stiffness

#mode	Freq.[Hz]	ζ %
1	547,51	3,39
2	650,18	3,02
3	1031,75	1,43
4	1340,73	3,51
5	1678,95	2,11
6	2278,29	5,36
7	2494,08	1,66

Tab. 2.12: Fictitious case – composite stiffness/steel density

This can be appreciated also in Tab. 2.11 and Tab. 2.12: the damping reduction is basically caused by the mass reduction.

The following tables shows the properties of the fictitious materials used to obtain the results depicted in Fig. 2.59.

Shaft material properties	Value
E_{1eq} [MPa]	198400
ν	0.3
$\rho \left[\frac{kg}{m^3} \right]$	1750
shear	1.5

Tab. 2.13: Hybrid shaft material – ρ composite/E steel

Shaft material properties	Value
E_{1eq} [MPa]	305000
ν	0.3
$\rho \left[\frac{kg}{m^3} \right]$	7718
shear	1.5

Tab. 2.14: Hybrid shaft material – ρ steel/E composite

3 Spindle-machine dynamic interaction

As formerly explained in High Speed Machining the design of high speed spindles is very important, because it often has a strong influence on the resulting tool/workpiece compliance and, thereafter, on process stability. Several CAE software packages are available to build spindle structural models, based on the finite element approach, but even with these sophisticated tools, the estimate of spindle behaviour in operating condition can be quite unsatisfactory, because modeling information on appropriate boundary conditions, corresponding to the specific machine tool, are unavailable, because spindles and machines are usually designed separately: the first is modeled considering the connection flange as restrained to an ideal ground Gangol [33], Cao [31], Movahhedy [36], Erturka [37], while, in typical machine tool structural models, the spindle is represented as a lumped rigid mass Albertelli [38], Bianchi [39] and Brecher et. al. [40].

In this work the issue has been studied like suggested in Cao [32] and [44] by building a machine simplified structural model that reproduces two eigenmodes that are mostly involved in the coupled dynamics with the spindle. Assembling this modal model to the spindle one, a good reproduction of the coupled dynamic behaviour experimentally identified has been obtained.

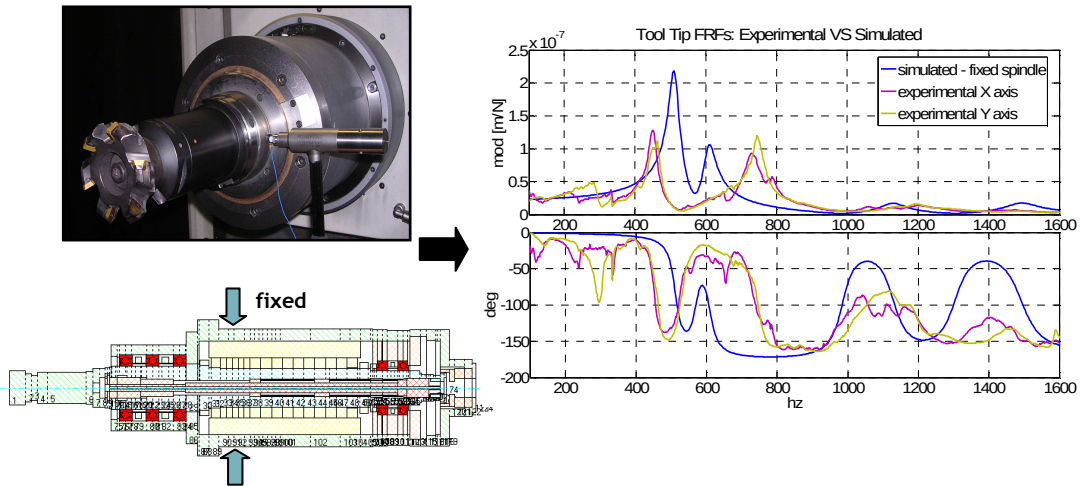


Fig. 3.1: Motor spindle test case and model – test case A

Moreover, in the present work the problem has been investigated developing a full finite element model of a machine tool, including the spindle, and performing experimental dynamic testing on machines and spindles. The obtained data has been evaluated exploiting the chatter theory in order to quantify the significance of the phenomena.

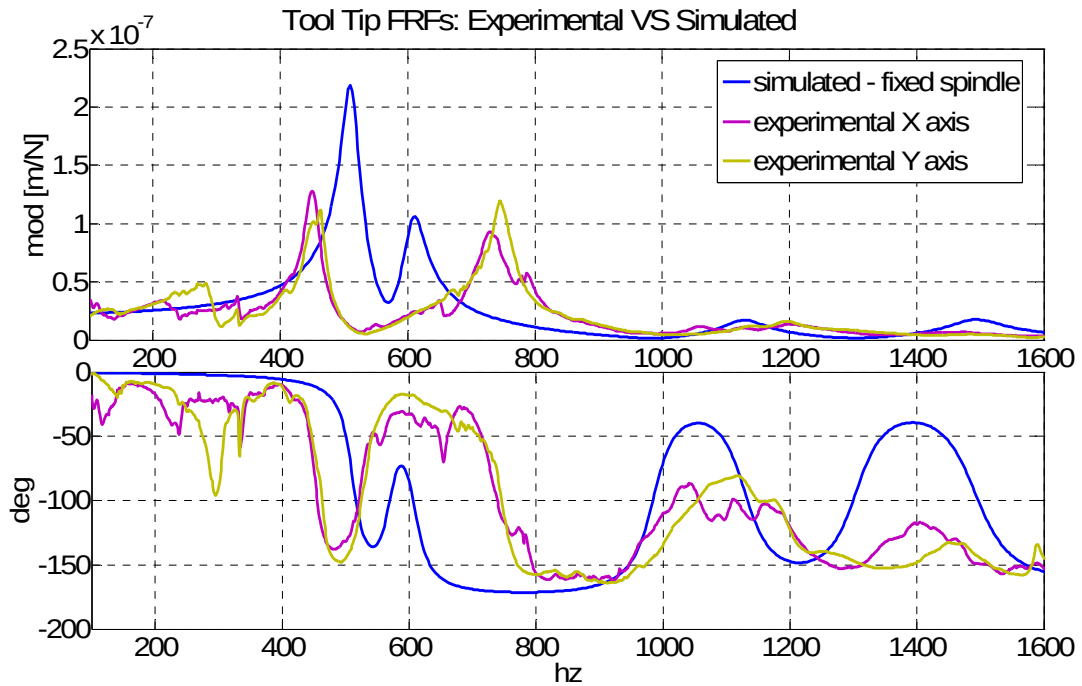


Fig. 3.2: Tool tip dynamic compliance – experimental vs simulated - zoom

The test case motor spindle, depicted in Fig. 3.1, has a built-in electrical motor and five preloaded ball bearings in back-to-back configuration. Bearing preload is applied to the rear bearing pack by a hydraulic system. Tool holder is HSK 100; the tool is locked and unlocked by a draw bar, pulled by a set of springs. In order to evaluate the cutting process stability a Finite Element spindle model has been developed as described in the previous section 2.

The model has been updated, as formerly explained, using the information gathered by an experimental modal analysis campaign and conceived in order to characterize the contribution of the spindle motor rotor, the tool holder, the clamping system and the bearing preload hydraulic piston.

In order to evaluate the cutting process stability limit it is necessary to define a boundary condition for the spindle model: lacking more realistic information on the machine tool, two possible opposite constraint conditions are flange fixed to an ideal ground and free-free conditions.

Even if the real boundary condition, when the spindle is assembled on machine, can be considered as an intermediate case, the spindle dynamic is often evaluated by restraining the spindle model to ground, as depicted in Fig. 3.1 that shows the updated model overestimates the most flexible eigenmode frequency and the corresponding dynamic compliance.

In order to quantify how these discrepancies would influence the material removal analysis, it is useful to investigate the real part of the same FRFs, Fig. 3.3, that is roughly inversely proportional to the maximum stable depth of cut. In the simulated data the two most significant modes show higher modal

compliance and nearer eigen frequencies, producing therefore higher interaction between them and a lower absolute minimum of the real part.

The approximation illustrated in Fig. 3.2 and Fig. 3.3 has been then transferred on the cutting process limit estimation. dynamic compliance at the tool tip for the mentioned spindle, obtained by the described numerical models and by experiments conducted with the spindle mounted on a real machine. The limits of the considered modelling approaches can be evaluated by the difference between simulated and measured tool tip Frequency Response Function along the X axis and Y axis: the fixed

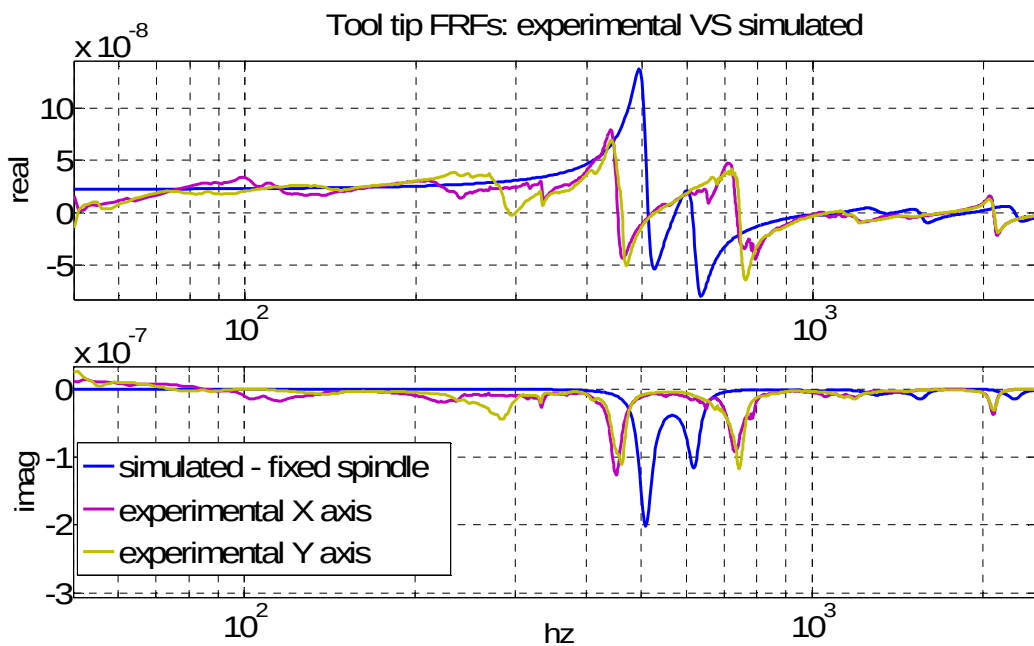


Fig. 3.3: Tool tip FRFs - real and imaginary part

The corresponding analytical lobes diagram have been computed considering face milling machining with radial tool immersion of 65%, by a 80 mm diameter tool with 6 cutting edges. The diagrams, computed using measured and simulated tool tip compliances, are depicted in Fig. 3.4. Differences on both the value of the unconditioned stability axial depth of cut and the frequency structure of the plot are clearly visible.

The presented results have detected some important limitations in the traditional spindle design approach. It is necessary, in order to correctly evaluate spindle performances in operating conditions, to take into account the machine tool role. In this paper two different scenarios are presented: the first suggests to build a simplified machine tool model to be used as boundary condition for the spindle model (mostly suited for spindle designers), while the second one proposes to mount the beam spindle model on machine FE model (mostly suited for machine designers).

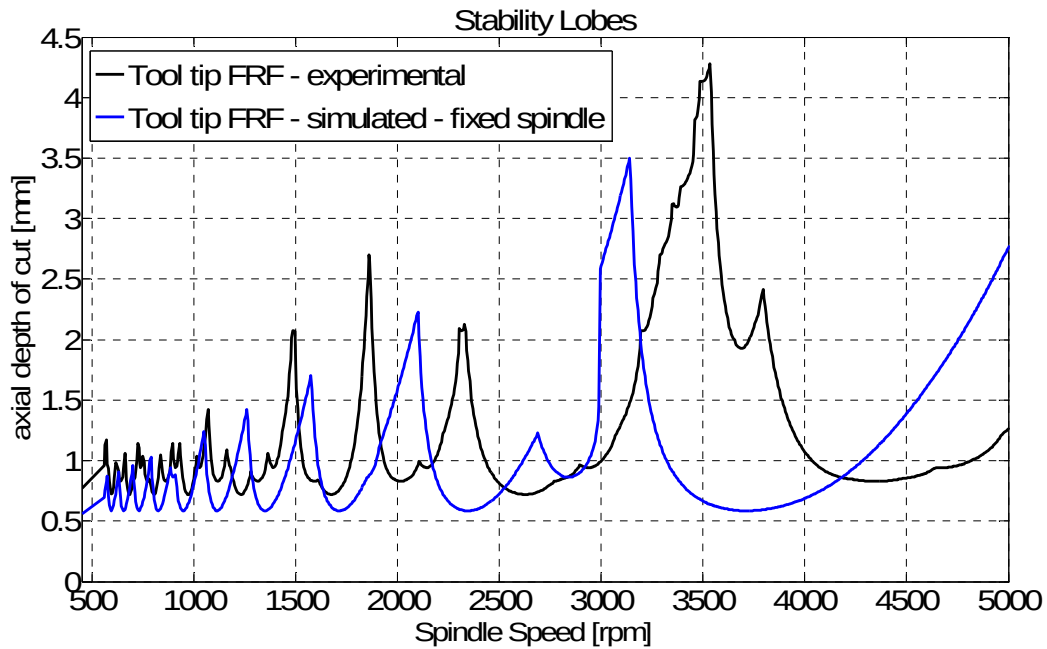


Fig. 3.4: Analytical Stability Lobes - experimental VS simulated

3.1 Simplified Machine model

As presented in [32], a simplified machine model has been developed. For test case (A) this simplified machine, depicted in Fig. 3.5, has been proposed. The beam spindle model is connected by spring elements to a flexible ram, with a lumped mass at one end.

Operating on mass and spring values and on beam properties, a fitted spindle model has been obtained. It is a sort of modal tuning, as done in [32].

Fig. 3.6 illustrates how the simplified model can reproduce the dynamic spindle behaviour when the spindle is mounted on the machine head.

Using this updating strategy we have fitted the experimental Frequency Response Function both for X and Y axis. Certainly a future development of this approach will be to investigate if it is possible to establish simple rules to build the simplified model starting from typical machine tool specifications, like static stiffness, first resonance frequency etc.

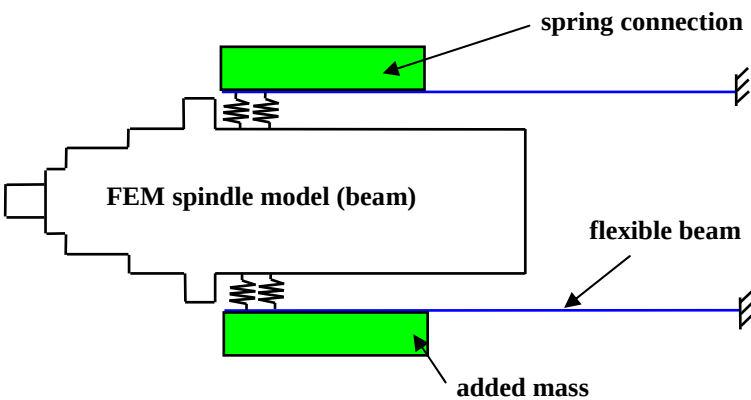


Fig. 3.5: Spindle and simplified machine model

Using this modelling approach it is necessary to build two different simplified models (for the X and Y axis respectively) due to the asymmetry introduced by the machine structure.

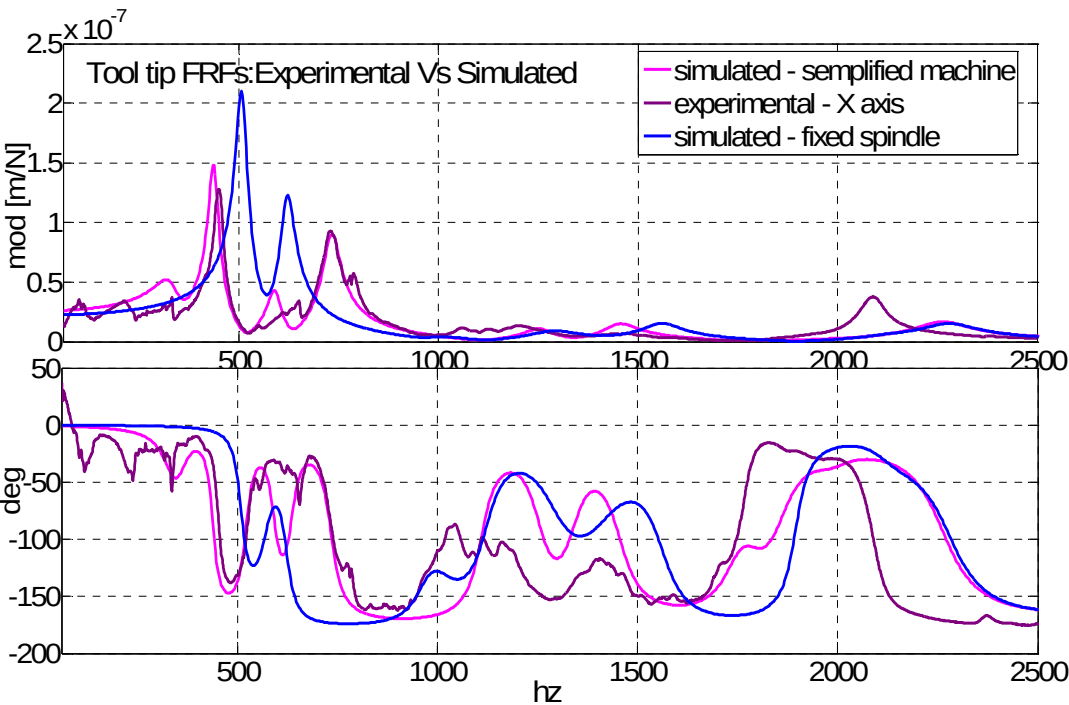


Fig. 3.6: Tool tip FRFs - fitted simplified machine model

3.2 Full machine model

To evaluate the machine tool influence on the spindle dynamics, a full FEM machine model has been used, assembling on it the beam spindle model. Fig. 3.9 describes the three axis machining centre used (the “MODULA” machine, produced by Linea srl, Italy). The spindle has six ball bearings in back-to-back configuration and an ISO 50 tool holder.

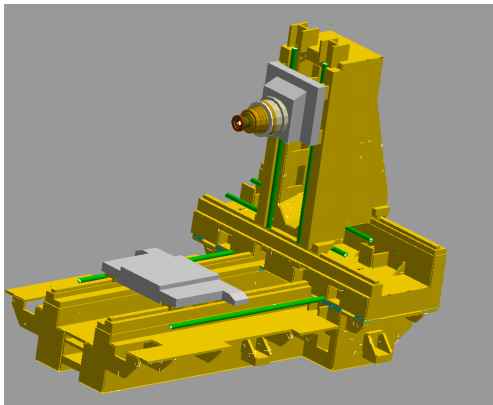


Fig. 3.7: Test case B

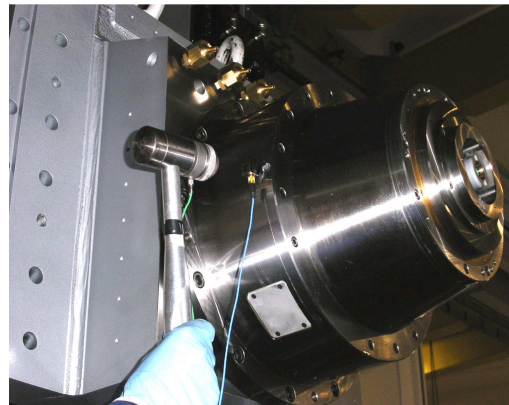


Fig. 3.8: Electro spindle & spindle head

In the following figure the finite element model of the whole machine is presented. The beam model of the spindle, Fig. 3.10 was added to the machine model.

The beam model has been created following the formerly defined modelling guidelines.

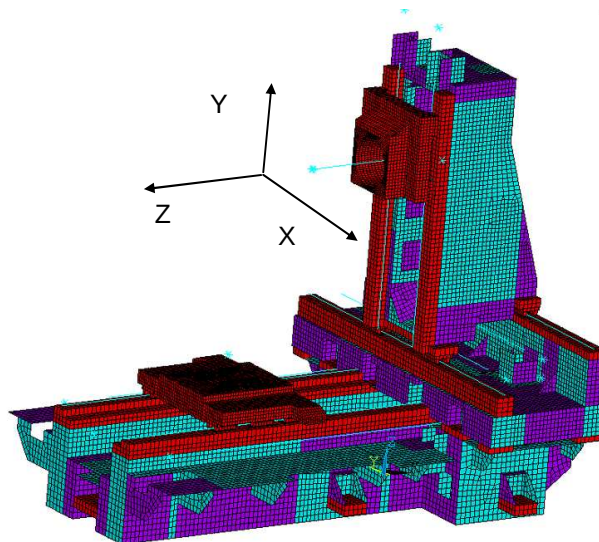


Fig. 3.9: FEM machine model (machine+spindle)

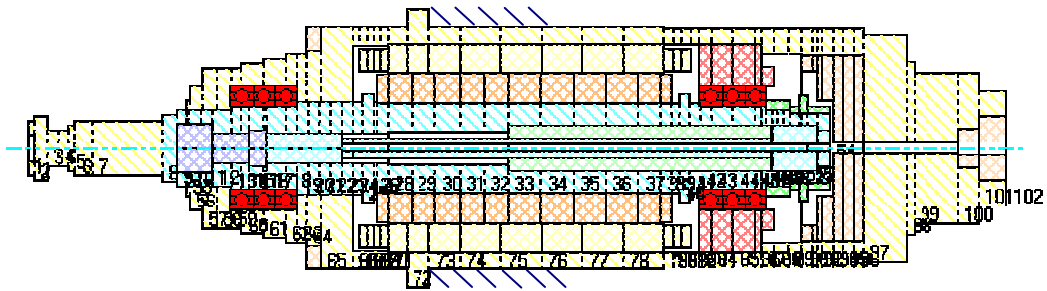


Fig. 3.10: Beam spindle model

The complete machine and spindle model has been tuned using data from an experimental modal analysis campaign. The comparison between simulated and experimental spindle nose Frequency Response Functions is depicted in Fig. 3.1: the updated model shows a good agreement with experimental measurements over a wide range of frequencies.

By adding the beam model of the spindle to the full machine model it is possible to investigate (see Fig. 3.12) how the spindle modifies the dynamic behaviour at the tool tip, especially in the higher frequencies region.

From the cutting process stability point of view, considering a low speed high torque milling operation, and referring to this specific machine (test case B), the dominant eigenmode that limits the Material Removal Rate is the 65 Hz one.

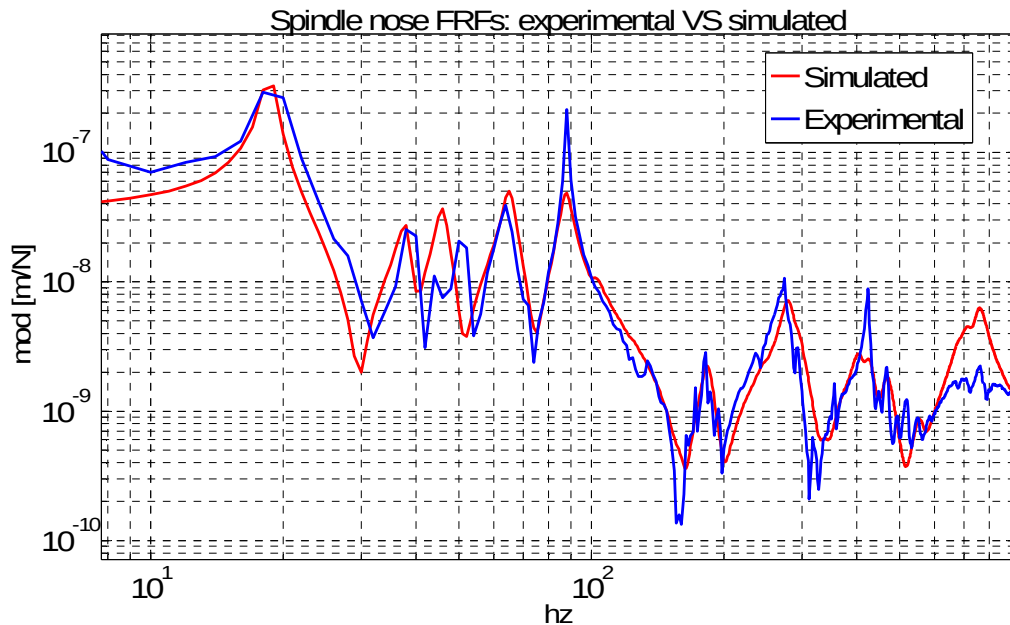


Fig. 3.11: Spindle nose FRFs - simulated/experimental comparison

It is important to notice that the 18 Hz mode, which seems the most flexible mode (see Fig. 3.11) doesn't limit the cutting process stability because the whole machine moves on its supports to ground and therefore is not excited by

the relative forces applied to tool and workpiece by the cutting process. The 65 Hz eigenmode instead provokes a relative displacement between tool tip and workpiece (see Fig. 3.12) and can be easily excited by cutting forces

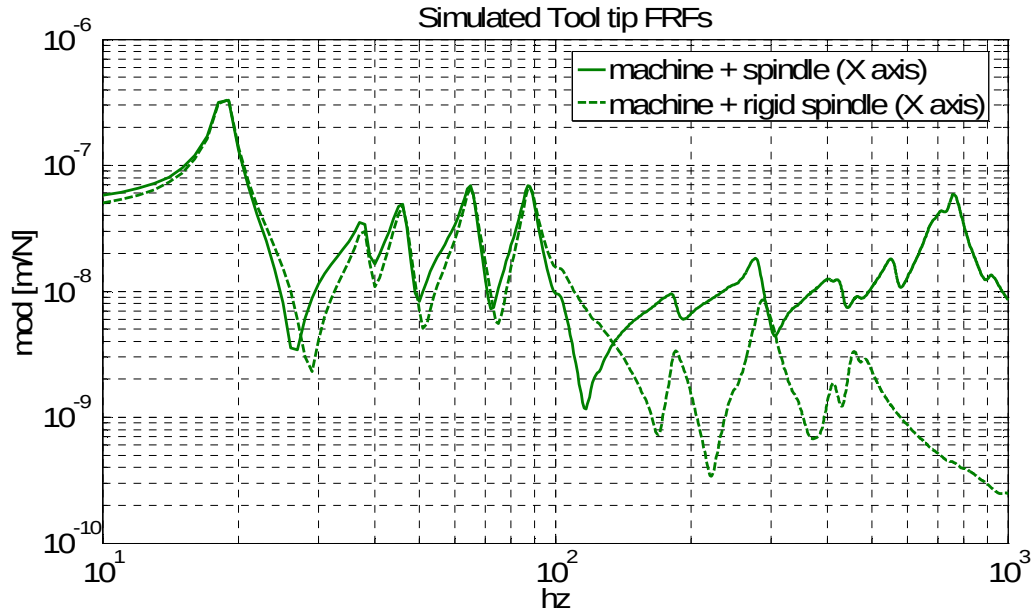


Fig. 3.12: Simulated tool tip FRFs - X axis

In this specific test case the spindle dynamic behaviour only slightly influences the stability lobes diagram and the cutting stability limit.

In order to study the influence of the machine on the spindle dynamic behaviour we have focused on the higher frequency range, from 200 Hz to 1000Hz, where, as shown in Fig. 3.12, the spindle role is predominant.

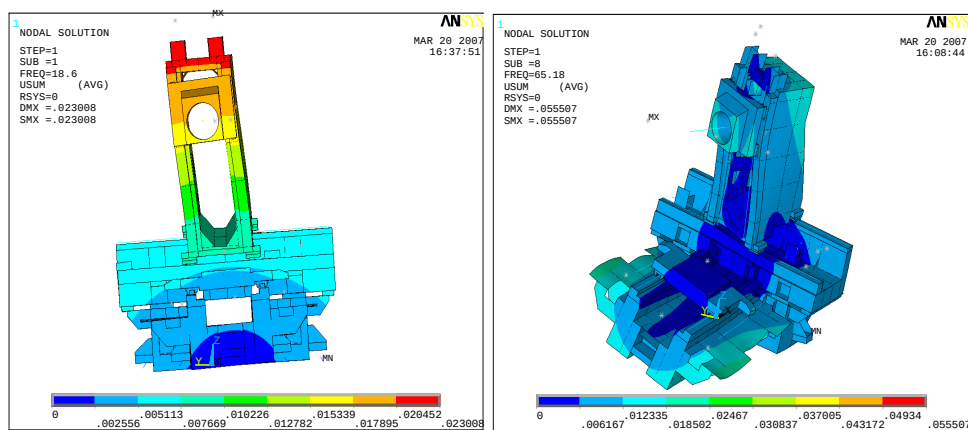


Fig. 3.13: Machine modal shapes

In Fig. 3.14 the tool tip Frequency Response Functions of the spindle fixed to ground and in free-free conditions are shown.

The MODULA (produced by Linea) machine realizes a stiff boundary condition for the spindle that would be difficult to reach on different machine architectures, e.g. in machines with the spindle on a moving ram or with a tilting head. In order to make a first analysis of these different cases, a fictitious machine with a more compliant spindle head has been generated, just dividing by two both the material density and the elastic modulus of the head material.

The resulting effect on the dynamic spindle behaviour is depicted in Fig. 3.14 and in Fig. 3.15.

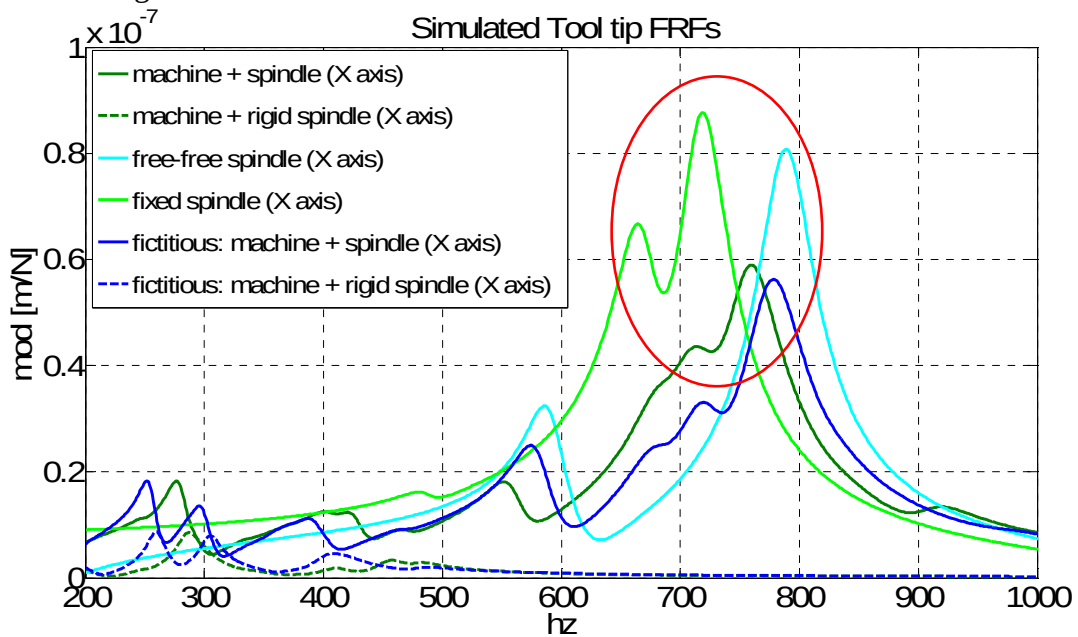


Fig. 3.14: Simulated tool tip FRFs – boundary condition effects

The different influence of the machine on the analytical stability lobes is shown in Fig. 3.16. It is important to remind that the stability lobes have been computed considering the dynamic behaviour over 200Hz because, for the aims of this work, low frequency machine eigenmodes, where the spindle dynamics is not involved, are not considered.

It is useful to physically interpret the phenomena: why the spindle mounted on the machine, with lower static stiffness, can achieve higher axial depths of cut than the spindle fixed to ground, that possesses a higher static stiffness?

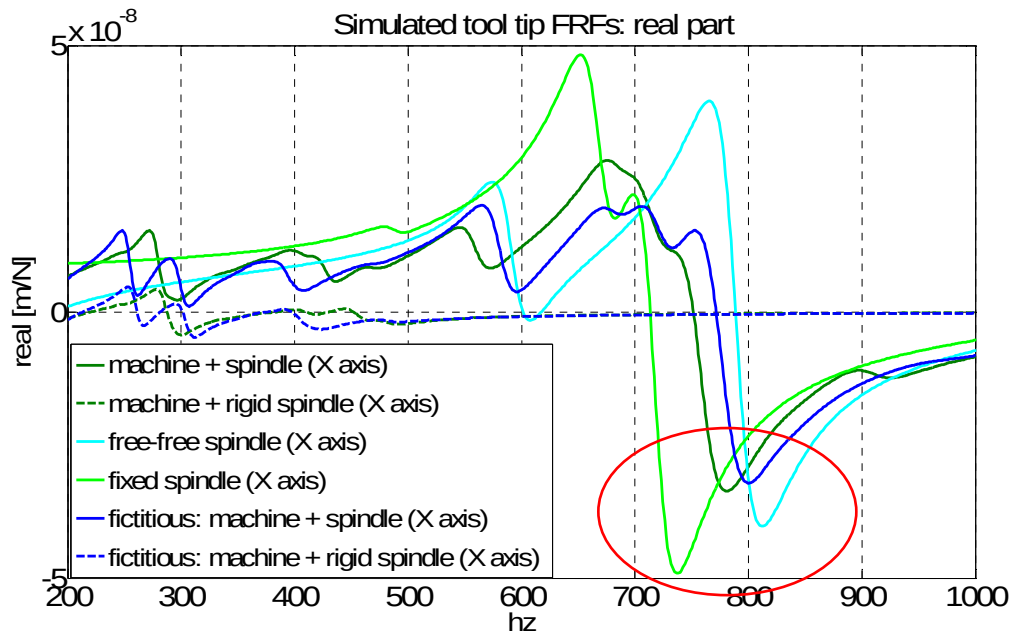


Fig. 3.15: Simulated tool tip FRFs – real part

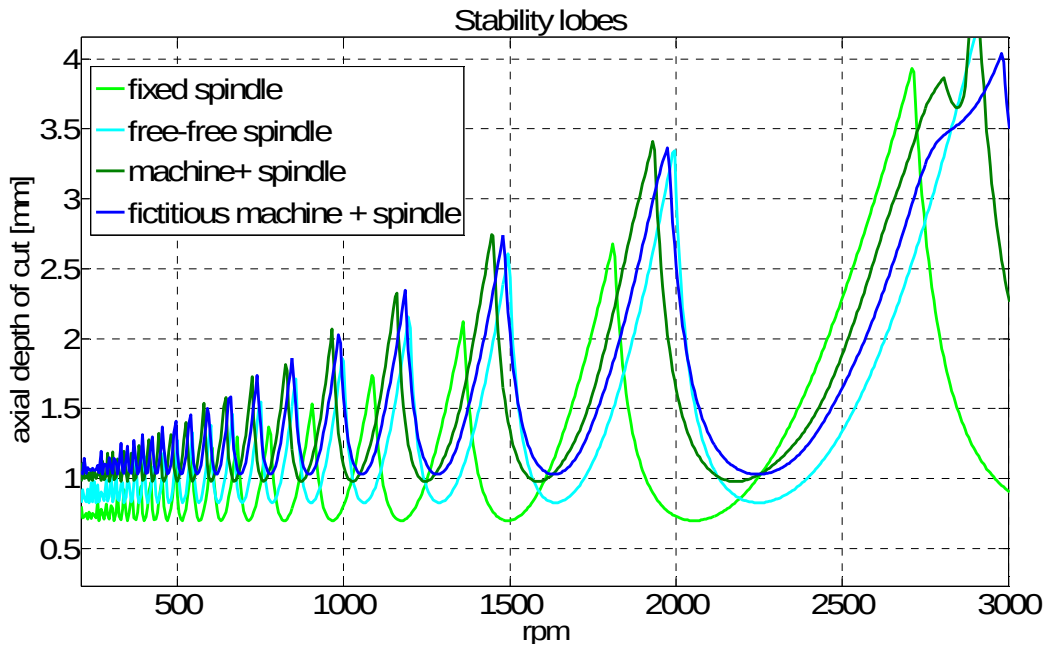


Fig. 3.16: Analytical stability lobes

The spindle mounted on the machine, because of the different boundary conditions, exhibits a different dynamic behaviour. In particular, the eigen modes near to the first resonance frequencies of the spindle fixed to ground involve now a larger motion of the spindle housing, some movement of the machine in general and of its head in particular (see Fig. 3.17). As a

consequence, even if the spindle shaft and the tool are moving at much larger amplitudes than the rest of the machine, given the much larger masses involved, the resulting modal mass at the excitation point is much larger. Modal parameters referred to a single degree of freedom are reported in Fig. 2.57 and in Eq. 2.11; they can be useful to understand the dynamic behaviour change.

In Tab. 3.1 the identified modal parameters of the most flexible eigenmodes, that mostly influences cutting stability, are reported. Passing from the fixed boundary condition to the machine mounting an increment of modal mass and modal stiffness has occurred: this explains the resonance peak shifting and the correspondingly increased cutting capacity.

Going back to Fig. 3.15 and considering the ideal sequence starting from the spindle fixed to ground through the “stiff” and “soft” machine down to the spindle in free-free conditions, it is interesting to note that there are specific boundary conditions that assure the maximal material removal capacity.

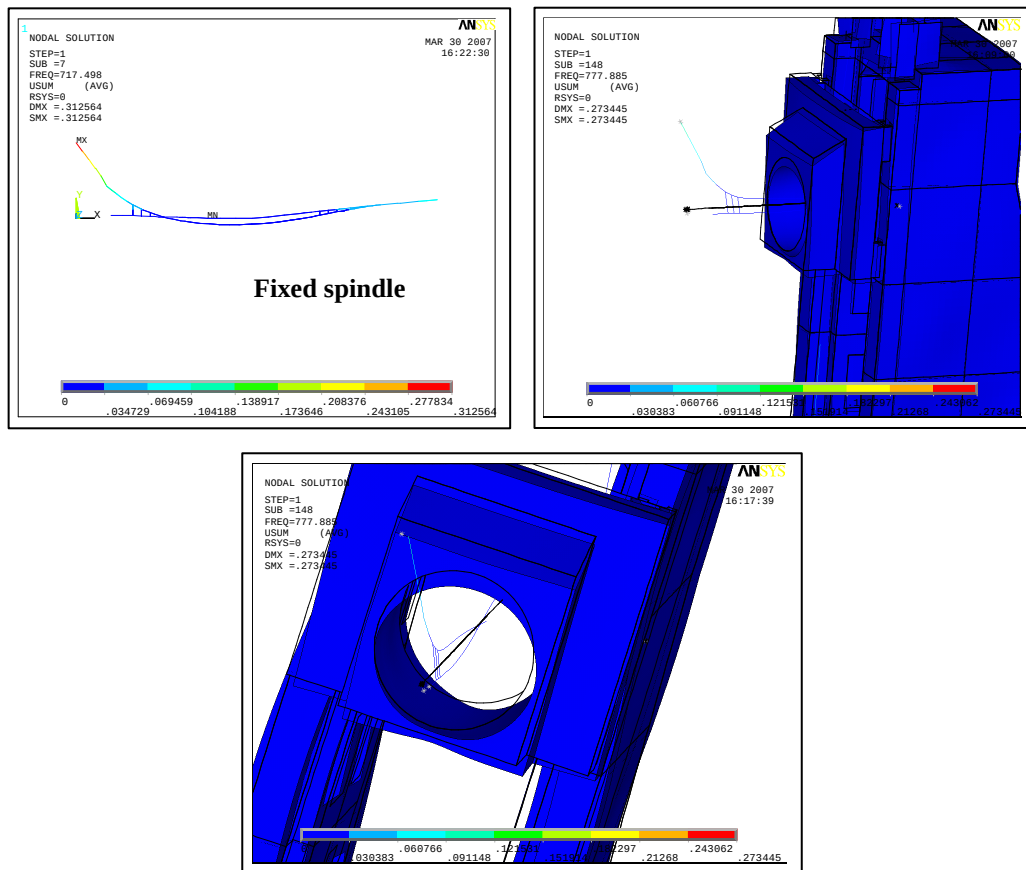


Fig. 3.17: Modal shape comparison

In order to roughly comprehend the interaction between machine tool and spindle a simple 2 dof system has been analyzed, Fig. 3.18.

This analysis is presented to show the complexity of the issue nevertheless a very simple 2 degrees of freedom system has been considered.

The simple system depicted in Fig. 3.19 represents a roughly model of the spindle-machine head system.

modal parameters	fixed spindle	machine + spindle
Frequency [Hz]	718	760
modal damping [%]	2.8	3.1
modal stiffness [N/m]	2.08E8	2.95E8
modal mass [kg]	10.24	12.95

Tab. 3.1: Identified modal parameters of the dominant mode

The analysis shows the influence of the “machine stiffness K_r ” on both the modal frequencies and on the modal shapes of the 2 dof system.

If the spindle is mounted on a rigid machine ($K_r \rightarrow \infty$) only a resonance peak can be observed (Fig. 3.20), the other eigenmode is at very high frequency.

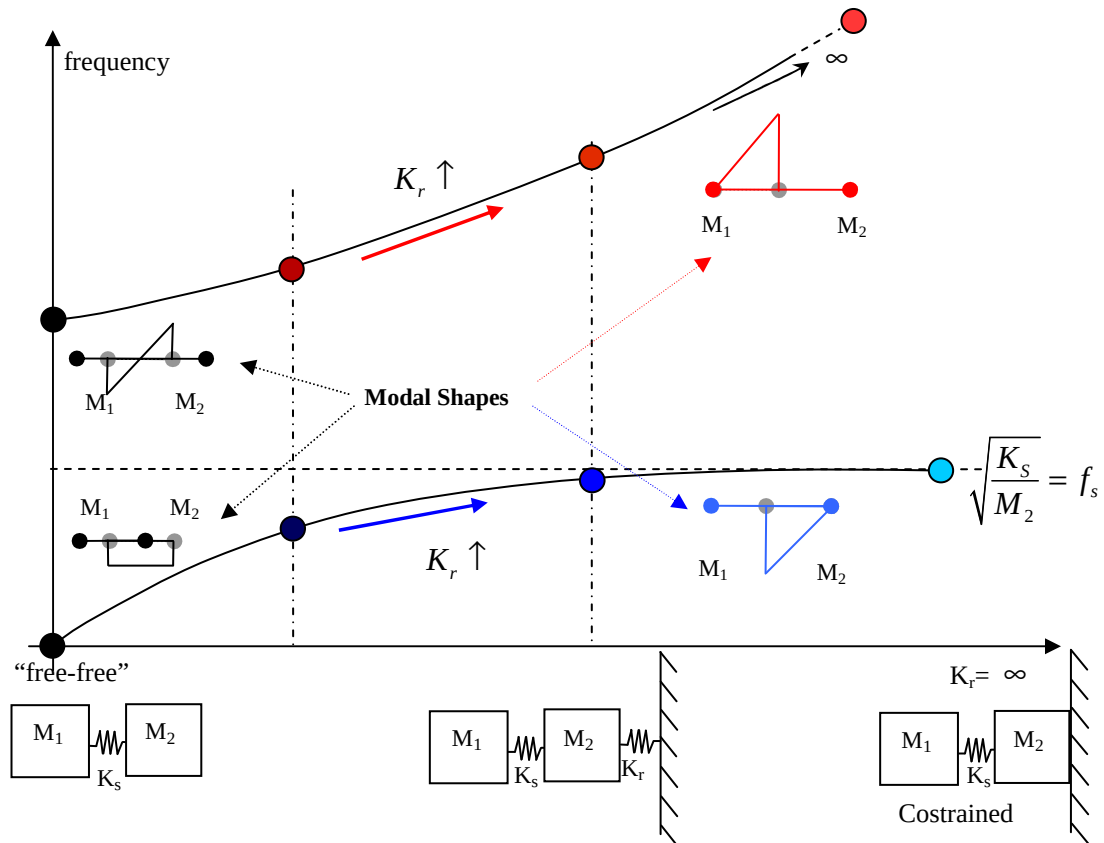


Fig. 3.18: Constraint conditions – effects on a 2 dof system

When the machine is get softened the high frequency mode appears and the frequency of the first eigenmode decreases like shown in Fig. 3.20.

This is very similar to those happened in test case A considering the first spindle mode, Fig. 3.6.

In this case we haven't considered the modal mass change due to the spindle mounting.

Obviously if we consider a more complex systems (i.e more degrees of freedom) both for machine tool and spindle it is very difficult to predict the overall dynamic behaviour.

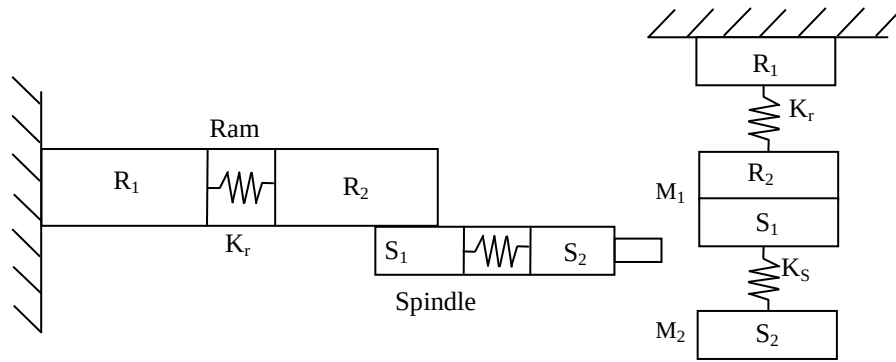


Fig. 3.19: Spindle - machine system - mass/spring model

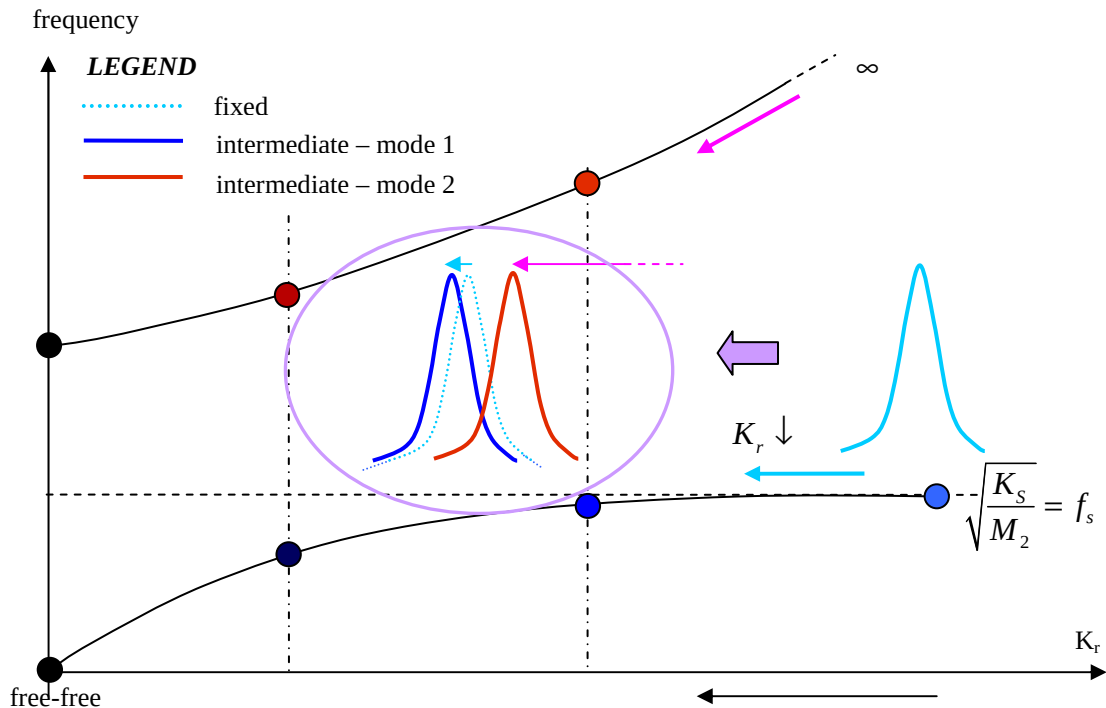


Fig. 3.20: Constraint conditions – machine stiffness influence

The two proposed approaches used respectively in test case A and B can be a valid support to develop a high performance spindle-machine system.

The analysed models, with the help of experimental measurements, have pointed out the relevance of the dynamic coupling of the two subsystems: it is therefore important, both for spindle and machine tool designers, to take into account this interaction during the corresponding design stages.

Interesting developments have been identified concerning both approaches. For the first one, it would be interesting to develop a general methodology to define the simplified machine model using only the most important machine specifications, such as the static stiffness, the first machine resonances and some indicative mass value.

For the second approach it would be interesting to define a design strategy to profit by the spindle - machine interaction in order to maximize cutting process stability.

4 Spindle Speed Variation

The Spindle Speed Variation is one of the techniques that have been studied to suppress the regenerative chatter both in turning and milling operations.

Many scientific works were published about this chatter suppression strategy but any meaningful industrial application to milling hasn't been revealed.

A complete analysis of the state of the art and a detailed study of the technique will be further presented. The goal of this study is to comprehensively understand how the spindle speed modulation interacts with the regenerative effect that can bring the system to instability during the cutting. The importance of the parameters selection on Spindle Speed Variation effectiveness will be moreover presented.

4.1 State of the art Spindle Speed Variation

In order to suppress the chatter phenomenon one of the possible strategies is to continuously vary the spindle speed during the cutting process.

The Spindle Speed Variation (SSV) has been well studied in literature by different authors.

These works have revealed that the modulation of the spindle speed, breaking the regenerative effects, can be used in some cases to stabilize the cutting process.

Application with different speed modulating laws have been studied but the most promising one seems to be the sinusoidal one (SSSV or S^3V Sinusoidal Spindle Speed Variation), this is due basically to the capability of the spindle drive to track the sinusoidal speed profile. Triangular and square profiles were also analyzed Lin [66], Takemura [69].

The first studies were focused on the capabilities of the SSV to suppress chatter in turning machining.

The idea was introduced in 1972 by the Stoferle and Grab [67], later, Sexton and Stone [70] argued previous works showing that the results obtained with the SSV were not as good as predicted before. De Canniere et al. [71] showed that the speed modulation was mostly equivalent to the modulation of the time lag between the cutter and the previous one.

Tsao et al. [73] proposed a methodology to analyze the stability of S^3V machining by using spindle angular position as an independent variable. A finite difference scheme was used in analyzing the stability of the system.

By using spindle angular position as the independent variable, the system dynamics are modelled as a linear periodic time-varying system with fixed delay. This representation is proven much easier to analyze and to numerically simulate than the time-varying delay representation which traditionally uses the real-time as the independent variable. This approach makes possible the quantitative characterization of system stability as a function of variable speed

profiles as well as other system parameters such as stiffness and damping of the cutting process and the tool/workpiece structure. Applications of the method to milling operations were proposed but introducing important simplifications on the machine dynamics; for example only one degree of freedom case has been considered. Simulations and experimental tests were performed to validate the proposed approach but no general rules were defined to select the proper values that describe the speed profile.

An analytical model to predict the chatter stability of variable spindle speed machining is presented by Jayaram et. al. in [76] that developed a Fourier expansion of the turning process equations. This approach is quite similar approach to the sophisticated multi-frequency solution for constant speed. This model is based on transforming the linear differential equations with time varying delay to the solution of an infinite order characteristic equation. The model has been validated using the results of numerical time domain simulations and through experimentation, turning operations were performed. The analytical model employs normalized parameters which permits its use in studying the stability of variable spindle speed machining systems with a wide range of parameters without having to solve the complex system of differential equations. The analytical model is also used to design optimal variable spindle speed machining parameters in the presence of known fixed or varying machining dynamics. Contour plots are used to map the effectiveness of the SSSV varying the spindle speed parameters considering also the uncertainty of the machine dynamics.

As formerly mentioned the S^3V has been the more widely adopted spindle speed modulation technique even if some author have proposed particular solution [84] for turning machining. Using a sinusoidal speed profile tracking troubles can be reduced because infinite jerk or infinite acceleration are not required like in case of the triangular and the square ones.

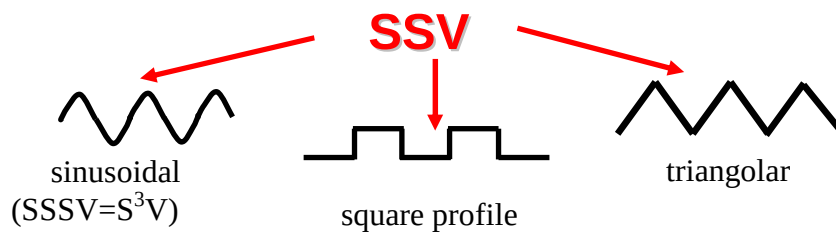


Fig. 4.1: Analyzed spindle speed profiles

The sinusoidal modulation strategy has generally been described with the following parameters that represent respectively the amplitude and frequency of the sinusoidal profile: RVA and RVF. These quantities are non-dimensional ones, typical used values are RVA=0-40% and RVF=0-40% [74], [75].

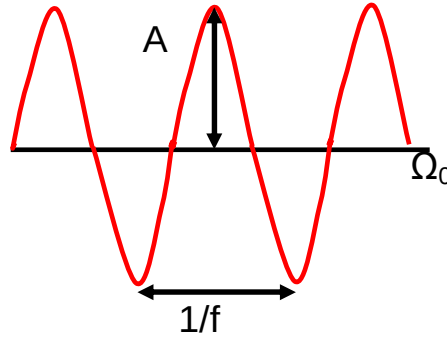


Fig. 4.2: Sinusoidal profile description

Eq. 4.1
$$\Omega(t) = \Omega_0 + (\Omega_0 \cdot RVA) \cdot \sin(\Omega_0 \cdot RVF \cdot t + \psi)$$

Eq. 4.2:
$$RVF = \frac{2\pi f}{\Omega_0}$$

Eq. 4.3:
$$RVA = \frac{A}{\Omega_0}$$

Using a sinusoidal profile it is possible to limit the tracking error: experimental tests revealed that the error is limited to 5%, tracking different profiles can generate higher error (up to 22%) Lin et. al. [66]. In the same work and in [72] a milling cutting process model was developed in order to investigate by simulations the powerful of the SSSV as chatter suppression technique. The machine dynamic was lead by three dominant eigenmodes. Time domain simulations reveal that the RVA has higher stabilizing effect than RVF on the vibration reduction goal. Generally when the instability is brought by an high frequency eigenmode the RVF parameter doesn't strongly influence the vibration suppression. If the dominant mode is at low frequency referring to the nominal cutting speeds, the RVF parameter can't be used to enhance the spindle behaviour. Moreover the SSSV proper parameters selection seems more crucial for high frequency dominant modes.

Finally authors have stated that an increment of RVA and RVF generally produces a vibration level reduction.

In [74] Radulescu et. al. analyzed how the effectiveness of the SSSV in milling operations is influenced by the tool-workpiece dynamics. A model of cutting process that considers the tool-workpiece dynamic compliance was developed. Test cases with different dynamic behaviours were analyzed: a structure with a single and fixed dominant eigenmode, one that changes its dynamic behaviour during the machining and the last one that has a more complex dynamics. Experimental results were performed only for the easiest analyzed case. Some

conclusions have been drawn. The VSM (Variable Spindle Machining) generally flattens the stability lobes so the proper spindle speed selection is not as crucial as in Constant Speed Machining. CSM can be more convenient than VSM if the nominal spindle speed is close to the stability pocket regions of the lobes diagram. Moreover the VSM is especially effective when the tool-workpiece dynamic behaviour has different non-coupled eigenmodes or when the dynamic proprieties change during the cutting. In second part of the work [75], the authors tried to physically understand how the cutting speed modulation is effective to suppress the vibration and to increase the depth of cut. An energetic approach was proposed for a non realistic orthogonal cutting and the work done by the cutting forces was analytically calculated. This shows that the forces in VSM generally introduce less energy in the machine tool than during the CSM cutting; this could be the reason of the enhancements due to the use of the SSSV technique. Some milling simulations were done to analyze the effects of SSSV on the regenerative phenomena and or forced vibrations that correspond to different position on the stability lobes diagram. Some other interesting conclusions have been outlined: the VSM is a robust technique that can be used to suppress the vibration during the cutting especially when precise information about the system dynamics and the real cutting conditions aren't available and the RVA parameter is more effective than the frequency of the speed profile.

An extension of the method introduced in [76] has been developed by Sastry et. al. in [77] and [78] for milling operations. The method has been validated by means of time simulations. Few experimental cutting tests were also performed. The results underline the capabilities of the SSSV to suppress the chatter vibration especially when the milling operations involve complex dynamics.

In [79] Al-Regib et. al. have developed an energetic approach based on the minimization of the work done by the cutting force to select the RVA and RVF parameters in turning operations. Experimental tests reveal the chatter suppression capabilities of the VSM. The heuristic method is suitable also for the on-line chatter suppression implementation.

By adapting existing mathematical techniques, a perturbative method is developed in [80] by Namachchivaya to obtain finite-dimensional equations in order to systematically study the mechanism of spindle speed variation for chatter suppression. The results indicate both modest increase of stability and complex nonlinear dynamics close to the new stability boundary. The approach allows to select, for one-dimensional cutting process, the optimal parameters that of the sinusoidal speed modulation.

Recently, Insperger et al. [86], [87] and Long and Balachandran [15] adapted the semidiscretization method to investigate the effect of variable speed machining. These works are focused basically on the numerical approach to the solution of the one-dimensional cutting process problem: DDE equations.

Kubica and Ismail proposed the on-line determination of the parameters of the sinusoidal spindle trajectory by a set of fuzzy rules based on experimental tests [82], [83].

Recently Bendiaga et.al. [85] presented the results of an experimental campaign on a milling machine tool. The reported results show that the SSSV is a promising technique to suppress chatter especially for low spindle speeds where better vibration reduction properties were observed.

A lot of studies have been developed in the past years on the Spindle Speed Variation as a technique to suppress the chatter vibration but up to now no one has address in a satisfactory way the problem of defining some guidelines to select the proper RVA and RFV values referring to a specific milling operation and a tool-workpiece dynamic behaviour.

In fact the optimum choice of S^3V amplitude and frequency must take into account cutting geometry and dynamics, while some constraints raise from spindle performance such as bandwidth and available power, or machining issues. Moreover the S^3V technique implementation, especially if it is used during a stable milling, can impair tool life and surface quality.

The key of S^3V efficacy is the phase shift between inner and outer modulation when chatter is impending. Chatter can be suppressed or enlarged by provoking different phase shift [88]. Concerning the optimum phase shift, literature is sometimes unsound and a definitive clear treatment is undoubtedly opportune.

In the work that will be further presented, the relationship between phase shift and spindle speed is pointed out: it will be shown that the phase shift corresponding to the worst condition is not constant, but it depends from cutting geometry (end mill entry and exit angles) and from a complex relationship between the dynamics along the two orthogonal DoFs defining the milling plane.

Considering the basic case of a single dominant vibration mode, for the planar displacements dynamics, a function proposed to measures a heuristic distance between the phase shift between inner and outer modulation that would be occurred if the instantaneous value of cutting speed was kept constant and the worst phase shift condition previously identified.

The functional depends on the speed variation law hence, the optimum parameters can be derived solving the following classical optimization problem. The functional J that will be properly described defines the "distance" from the least stable cutting condition.

This approach allows a fast computation of diagrams that point out the stability increases due to S^3V implementation (with various combinations of amplitude and frequency) comparing them with constant speed cutting. Using these diagrams is furthermore very easy to select the best variable speed cutting solution.

The reliability of the proposed methodology has been validated by means of a DoE based on numerical time-domain simulations in Matlab/Simulink© environment.

4.2 SSSV and time domain simulations

As previously mentioned the time domain approach has been selected in order to analyze the SSSV technique and to investigate its capabilities in regenerative chatter suppression during milling operations. The goal of the work is basically to understand in which way the cutting speed modulation can break the regenerative chatter limiting the vibration and enhancing the axial depth of cut. An energetic method has been proposed and it has been used to define some guidelines that help to select the more convenient RVA and RVF parameters of the sinusoidal profile considering a specific milling operation and having defined the tool tip/workpiece dynamic.

4.2.1 Test case description

The analyzed case is taken from a real industrial case. The machine CLOCK TANK MP10 from *m.c.m.* Spa. (Italy) is depicted in Fig. 2.14 and in Fig. 2.15. The tool tip dynamic compliance has been measured using an instrumented hammer and an accelerometer. Details on the procedure used to perform the tests are reported in the Appendix A.

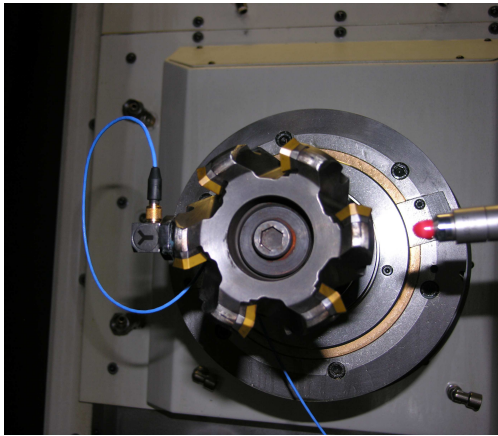


Fig. 4.3 : FRF measurement

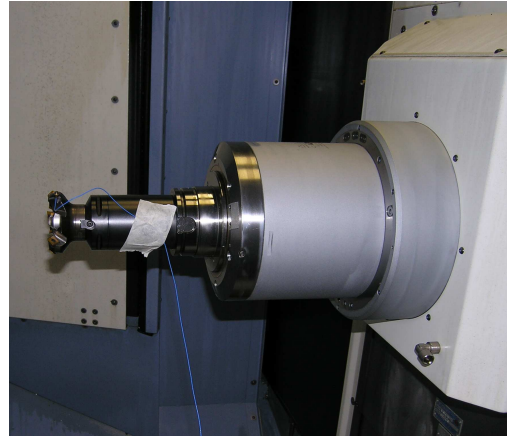


Fig. 4.4: Machine tool - spindle

The estimation of the tool tip dynamic compliance has been computed and it is reported in Fig. 4.5. The dynamic behaviour of the machine tool is dominated by two main eigenmodes: the first one at about 450 Hz and the second one at about 750Hz. Both modes can be associated to the spindle behaviour because they are two high frequency modes and they can be observed in both directions even if some light differences can be pointed out due to the machine-spindle dynamic interaction as explained in the chapter 3.

The main eigenmodes are non coupled ones because the dynamic behaviour of the spindle is almost symmetric considering two orthogonal directions.

The analyzed milling operation is a face milling with a full tool engagement as reported in Fig. 4.6.

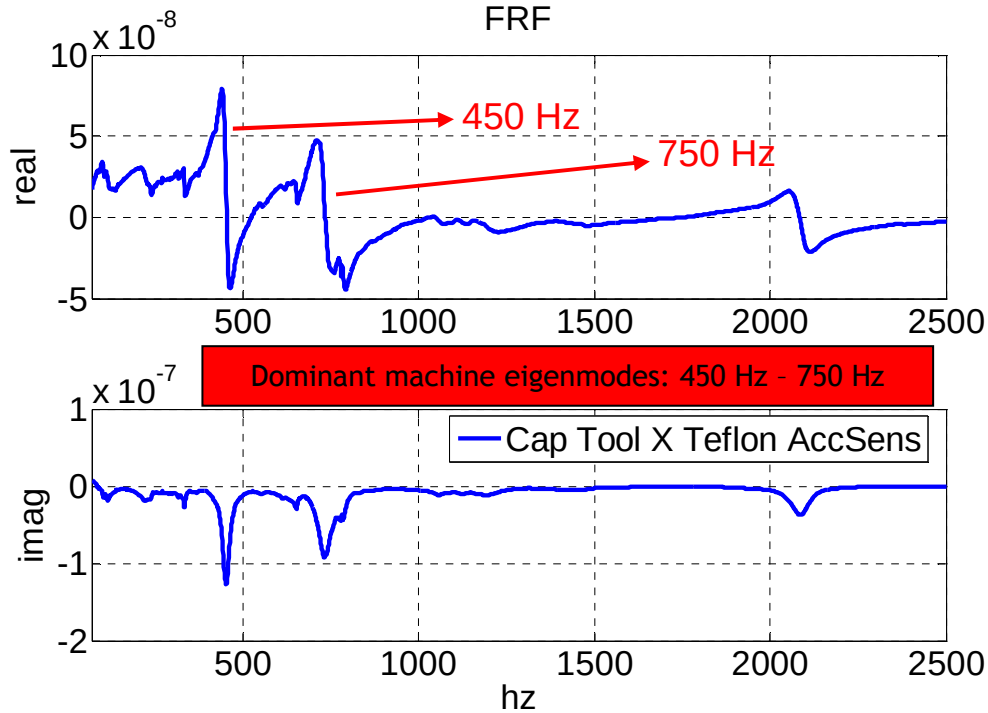


Fig. 4.5: Measured FRF - tool tip dynamic compliance –real & imaginary part- X axis

It is important to underline that the dynamic behaviour of the workpiece-table system has been neglected because it reveals a much more higher dynamic stiffness compared with the one measured at the tool tip.

The cutting parameters used are reported in Tab. 4.1. This face milling is actually used by Capellini S.r.l (Italy), a company that produces electro motor spindle.

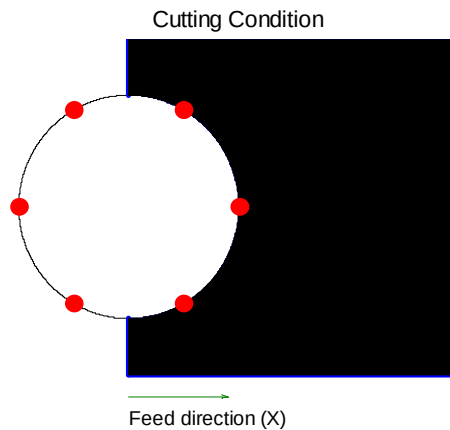


Fig. 4.6: Milling description – cutting geometry

The analyzed test case presents high frequency dominant modes and thus the milling operation described in Tab. 4.1 can be located close to the high order lobes of the stability diagram.

Cutting Parameters	Value
Nominal Spindle Speed	830 rpm
cutter diameter	80 mm
radial immersion	100%
tool length	200 mm
n° cutters	6
feed rate	0.24 mm/tooth/revolution
material	Steel C40

Tab. 4.1: Cutting parameters – steel face milling

4.3 Cutting process model description

A dynamic model of the cutting process that considers the dynamic behaviour of the machine tool has been developed in Matlab-Simulink© environment.

The cutting process model takes into account the regenerative effect so the instantaneous chip thickness related to each cutter depends not only on the instantaneous position of the cutter that is working but on the position assumed formerly by the previous cutters.

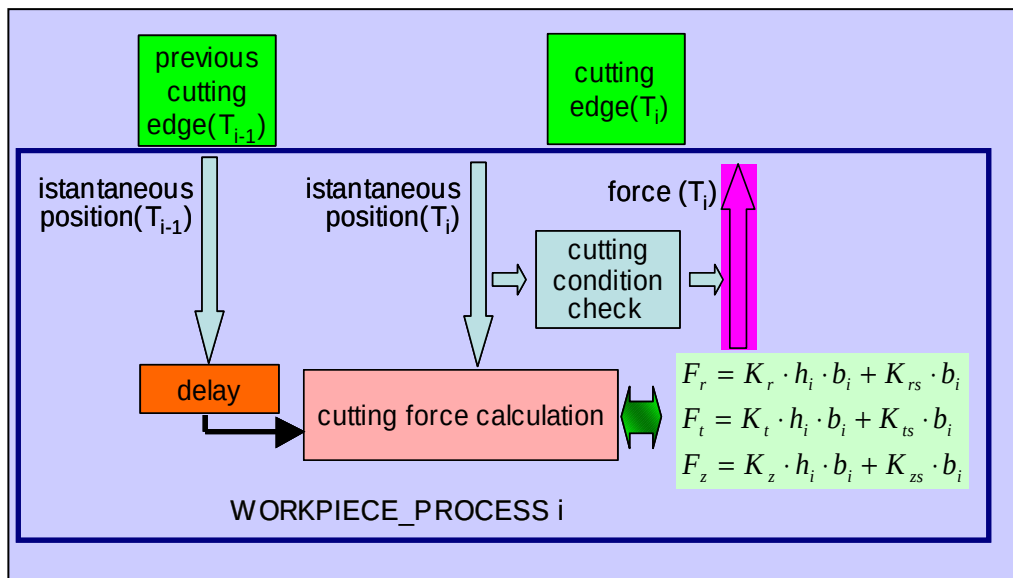


Fig. 4.7: Cutting process model linked to each cutter

A cutting process sub-model for each cutter has been considered so the regenerative effect can be easily modelled, Leonesio [13] and Liu [14].

As described in Fig. 4.7 the sub-model computes the instantaneous chip thickness and consequently the cutting force components using the Kroenenberg model reported in Eq. 4.4.

$$\begin{aligned} F_r &= K_r \cdot h(t) \cdot b + K_{rs} \cdot b \\ F_t &= K_t \cdot h(t) \cdot b + K_{ts} \cdot b \\ F_z &= K_z \cdot h(t) \cdot b + K_{zs} \cdot b \end{aligned}$$

Eq. 4.4

F_t , F_r , F_z are respectively the tangential, the radial and the axial component of the cutting force.

Moreover $h(t)$ is the instantaneous chip thickness, b the axial depth of cut, K_t , K_r , K_z , the cutting force coefficients and K_{rs} , K_{ts} , K_{zs} the cutting force coefficients that allow to compute the component of the force related to the sliding of the cutters on the machined workpiece.

Each sub-model needs the actual position of the “cutting edge i” and the formerly position of the “cutting edge i-1”. This is obtained using a delay block for the position of the previous cutting edge.

Moreover the sub-model checks the “cutting condition” that is it verifies that the cutter is really in the workpiece and thus it is removing material from the workpiece.

In order to obtain a desired cutting process model it is necessary to assembly different sub-models, one for each cutter.

Fig. 4.9 explains the methodology to assembly different sub-models and Fig. 4.10 shows the Simulink© implementation for a four cutters cutting process model.

Any desired face milling, slot milling or edge milling operation can be obtained assembling the right number of sub-models.

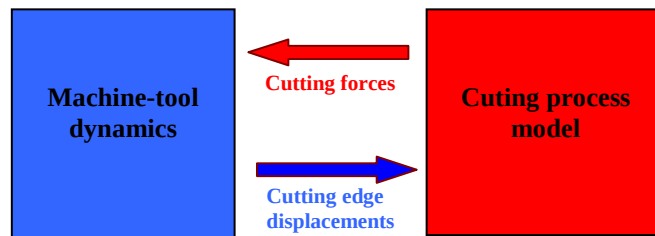


Fig. 4.8: Machine tool – cutting process interaction

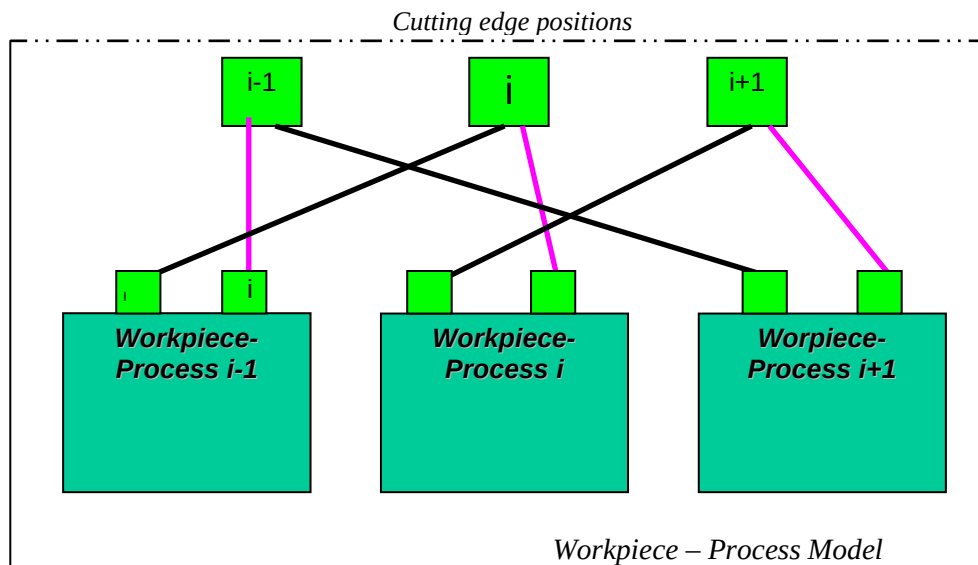


Fig. 4.9: Sub-models connection

Obviously the model can impose to the tool a desired angular speed, in this case a sinusoidal profile has been used to modulate the cutting speed. In this case a sinusoidal modulation of the delay occurs.

The model considers the dynamic interaction between the spindle-machine tool system and the cutting process like described in Fig. 4.8.

The data related to the dynamic behaviour of the machine tool are introduced in the model as modal parameters thus an identification technique has been performed to extract the desired parameters from the measured Frequency Response Function, Fig. 4.5.

Some simplifications have been introduced in the model: the spindle drive dynamics and the axis drive dynamics haven't been considered furthermore a simplified geometry of the cutters has been modelled.

The developed model can be used to analyze the cutting process instability and to understand how the spindle speed modulation can interact with the regenerative phenomenon.

Assuming that each point of the stability lobes diagram corresponds to a specific choice of spindle speed and axial depth of cut and having fixed the other cutting parameters, the stability diagram can be computed from a time domain simulation campaign. It is possible to establish if a specific milling operation is stable or not by analyzing the spectrum of the cutting forces and the spindle vibrations. From literature it is well known that the chatter instability manifests itself at defined and known frequencies that are approximately close to the dominant resonances.

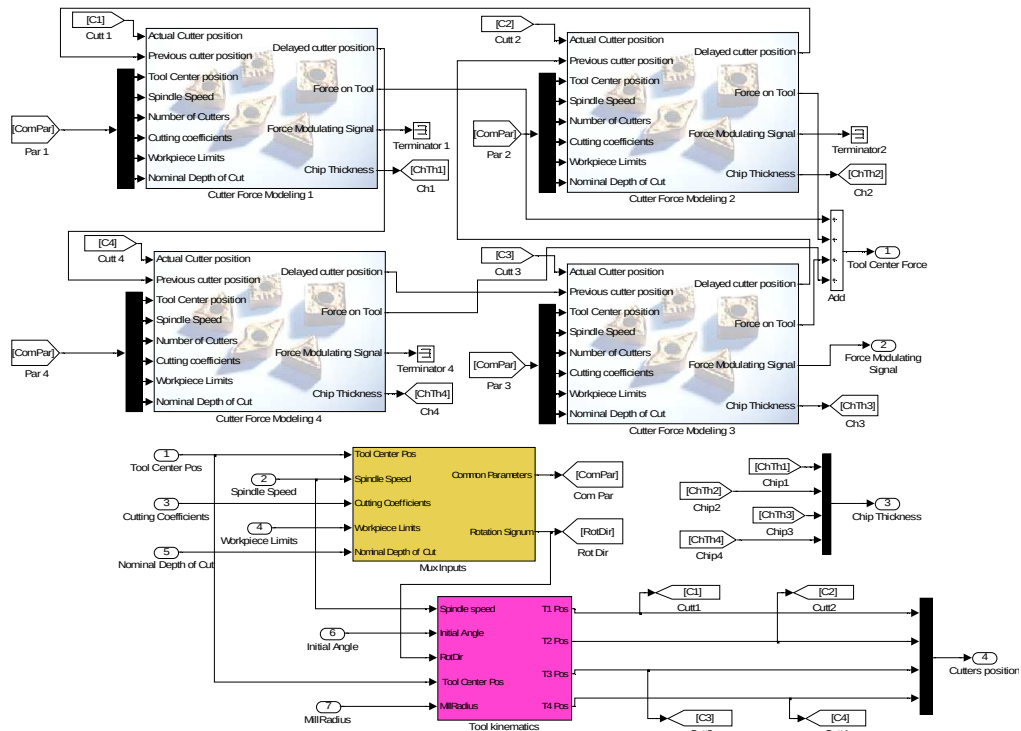


Fig. 4.10: Example of cutting process model for 4 cutters – Simulink©

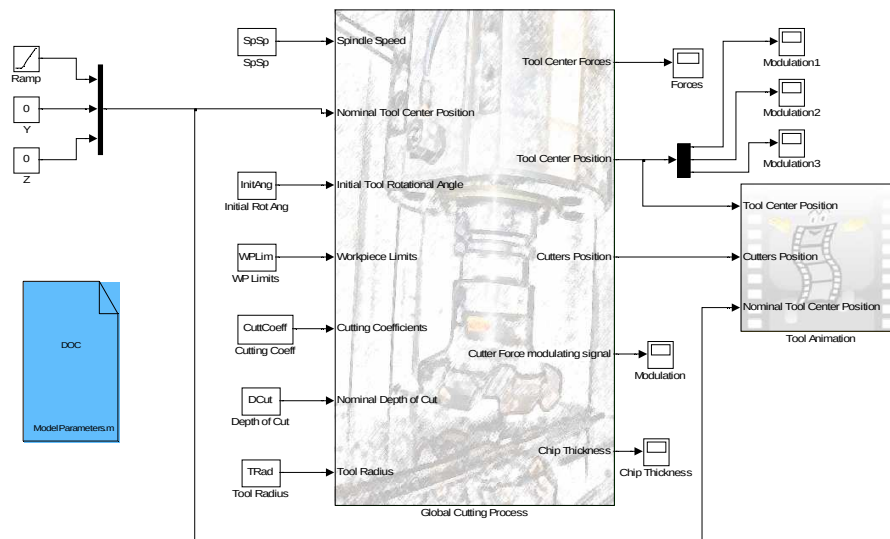


Fig. 4.11: Overall model - cutting process and machine tool model

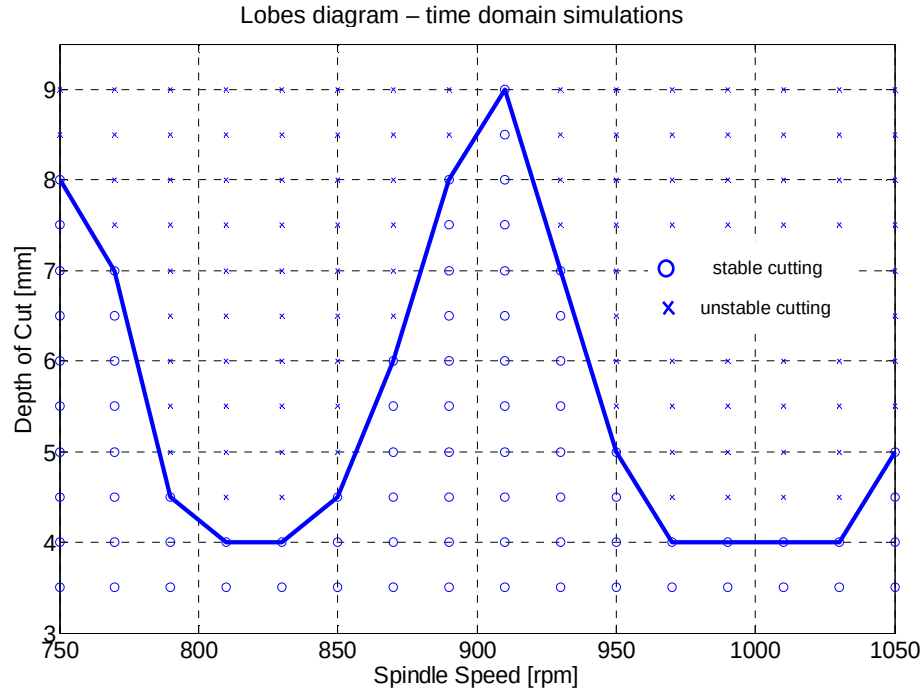


Fig. 4.12: Lobes diagram – Constant Speed Machining (CSM)- time simulations

4.4 SSSV – time domain simulation results

In Fig. 4.13 a stability lobes diagram for Constant Speed Machining (CSM) and for the milling operation defined in Tab. 4.1 is reported. The diagrams have been computed by the well known frequency domain approach proposed by Altintas [10]. It can be observed that the chatter instability is due basically to the main eigenmodes that can be noticed in the tool tip compliance.

The goal of the time domain simulations is to compute a similar diagram for VSM (Variable Speed Machining).

A limited spindle speed range close to the nominal speed has been considered due to the high computational cost of the simulations in time domain.

According with different works from the specific bibliography, the RVA-RVF combinations reported in Fig. 4.14 have been used to investigate the capabilities of the SSSV technique.

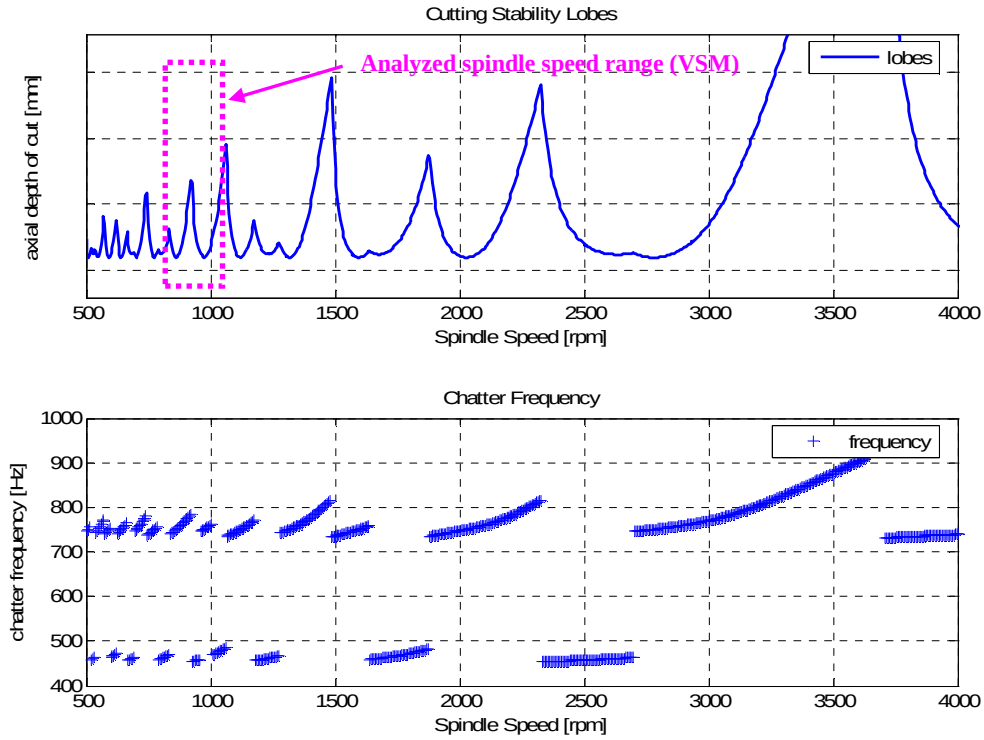


Fig. 4.13: Stability diagram lobes – CSM

Before introducing the results of the simulation it is necessary to state some preliminary remarks about the delay that occurs between two consecutive cutters during VSM.

In many bibliographic references the expression used to compute the “delay” τ is reported in Eq. 4.5:

$$\text{Eq. 4.5} \quad \tau(t) \approx \frac{2\pi}{\Omega(t)} \rightarrow \phi(t) \approx \omega_c \cdot \tau(t)$$

The above relationship is only an approximation as was observed in [73]. The reason of the approximation is graphically explained in Fig. 4.15. The correct time delay can be computed by solving Eq. 4.6.

$$\text{Eq. 4.6:} \quad \int_{t_1}^{t_2} \Omega(t) \cdot dt = \frac{2 \cdot \pi}{N}$$

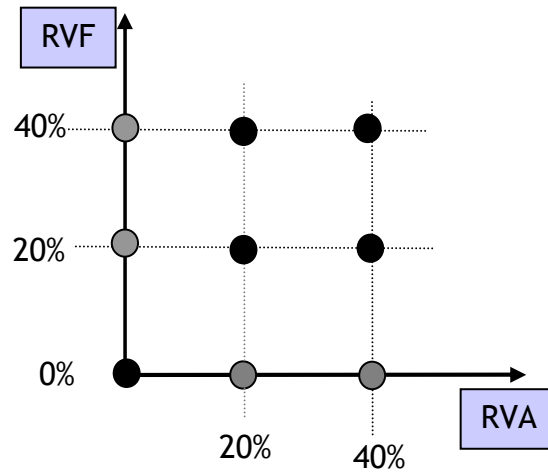


Fig. 4.14: RVA – RVF analyzed cases

Considering the range of SSSV parameters used in the simulations the approximation can be considered valid. The time delay will be used to compute the phase shift $\phi(t)$ as expressed in Eq. 4.5. This quantity will be crucial for the optimization procedure that will be proposed.

The first time domain simulation campaign has been performed considering only a single dominant eigenmode: the one @ 450Hz. The reason of this choice is to understand the effects of the speed modulation in the simplest case, further the results obtained with more complex dynamics will be reported.

The lobes diagram reported Fig. 4.16 is obtained in the way that was previously described.

It is necessary to observe that the spindle speed step and the axial depth of cut step used for the simulations are respectively 20 rpm and 0.5 mm.

In Fig. 4.17 the stability lobes diagram obtained using the classical frequency approach is added to the ones obtained from the time simulations: there is a good agreement between the two diagrams obtained with CSM. This comparison has been used to validate the developed time domain model.

Some important but preliminary conclusions can be drawn. The SSSV is confirmed to be a valid technique to suppress the chatter vibrations and it can be used to increase the axial depth of cut.

If the unconditioned axial depth of cut limit is considered we can underline that the SSSV brings, in the worst case, an increment of 25%.

The SSSV flattens the stability lobes, these results are very similar to the ones reported in [74].

Basically two different zones of the lobes diagram can be individuated: the first one is close to each minimum of the lobes diagram, the other one, where the behaviour of the SSSV seems completely different, is in proximity of each stability pocket.

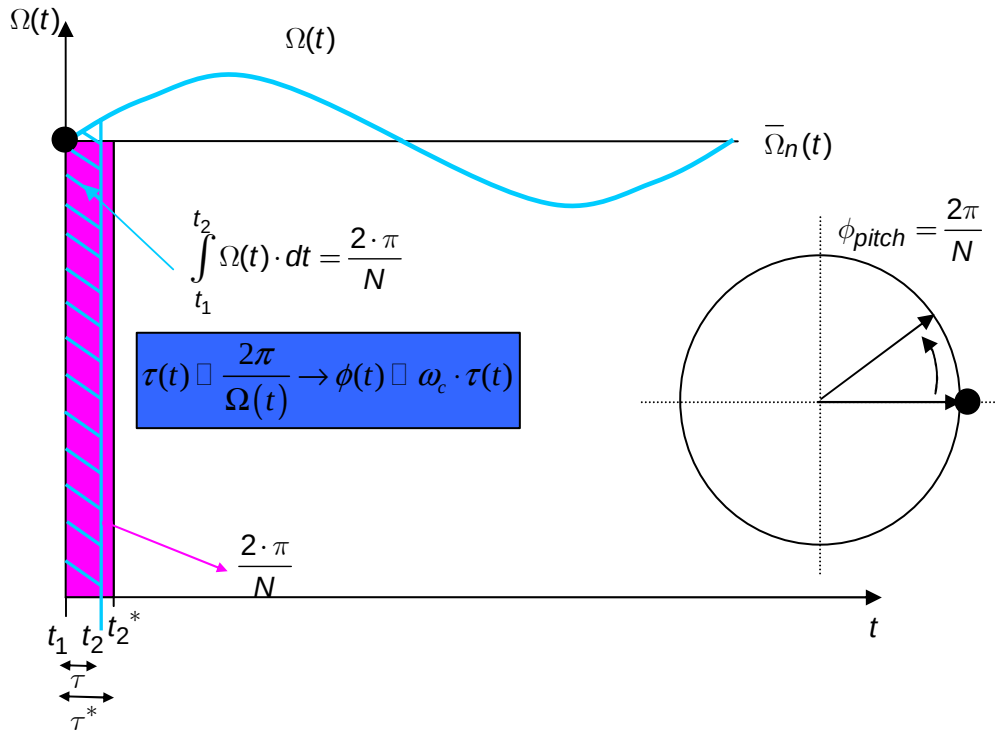


Fig. 4.15: Time delay modulation

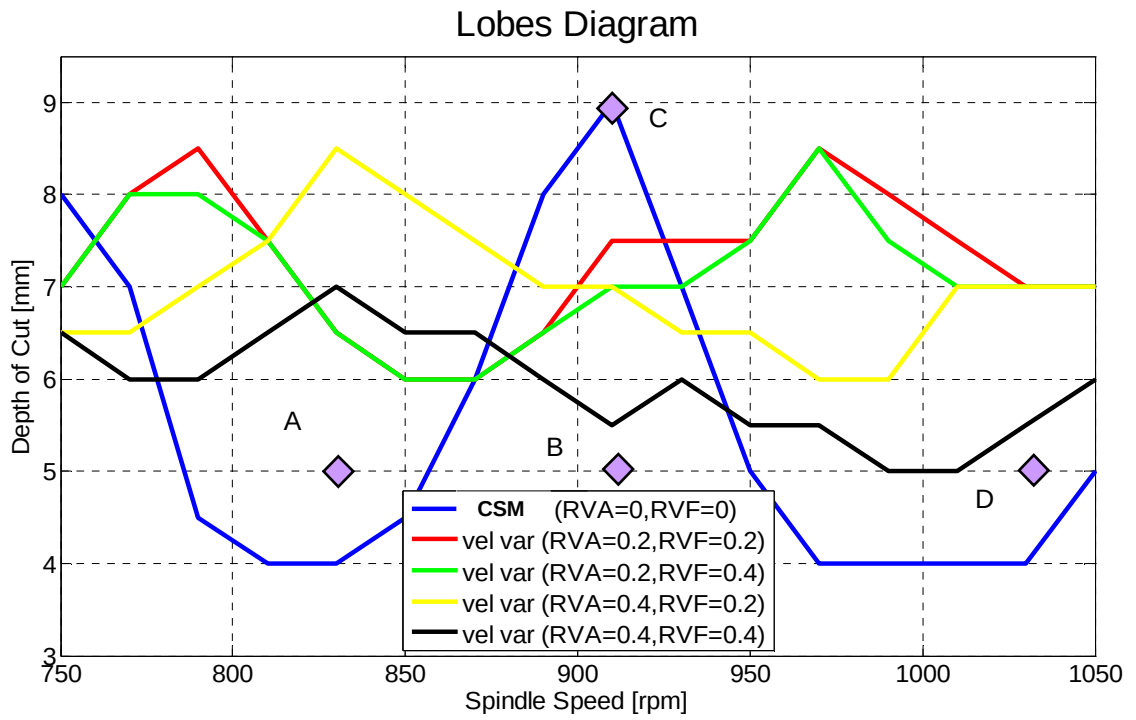


Fig. 4.16: Lobes diagrams – CSM vs SSSV – time simulations (960runs)

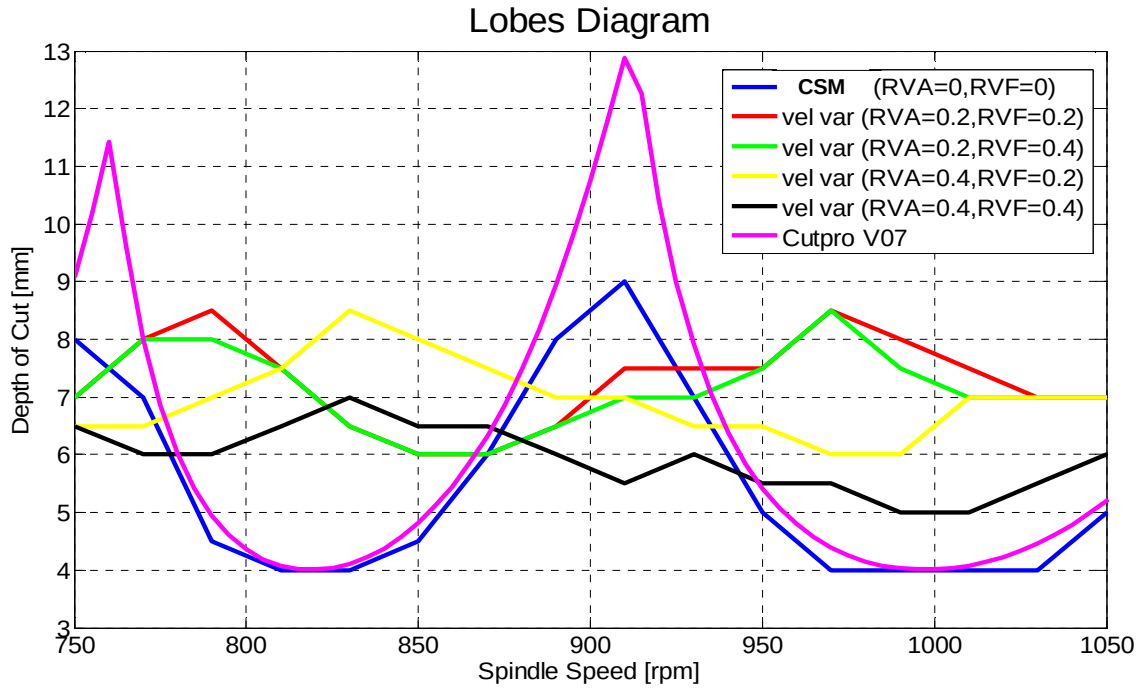


Fig. 4.17: Lobes diagrams - CSM Vs SSSV – time simulations & frequency analysis

In milling operations performed close the first zone we can observe that the SSSV provokes an increment of the stability limit, contrary in the second region a decrement of the stability limit can be observed.

From the lobes diagrams obtained with VSM it can be observed that the spindle law that uses the highest speed modulation ($RVA=RVA=0.4$) is not the better SSSV. This result contrasts with some previous ones reported in literature [75], [74].

If we neglect the small offset between the case $RVA=RVF=0.4$ and the $RVA=0.4/RVF=0.2$ we can affirm that the RVF parameter is not very influent on the trend of the lobes diagram. This strengthens the results reported in [73].

The SSSV seems very promising especially for low cutting speeds, for milling operations that can be located in proximity to the high order lobes of the stability diagram. In fact in this region of the lobes diagram is very difficult to select, assuming to know with a reasonable precision the machine tool dynamic, a proper spindle speed in order to take advance of the stability pocket regions.

It is interesting to understand in a more detailed way what is the behaviour of the system during VSM.

In order to comprehend the non uniform effectiveness of the SSSV considering milling operations in different regions of the lobes diagram some results from time simulations are reported.

The cutting forces and the tool tip displacement will be diagrammed for all the milling operations depicted in Fig. 4.16 and described in Tab. 4.2.

Milling operation	Axial depth of cut [mm]	Nominal spindle speed [rpm]
<i>A</i>	5	830
<i>B</i>	5	910
<i>C</i>	9	910
<i>D</i>	5	1030

Tab. 4.2: Analyzed milling operations

4.4.1 Milling operation A

Considering the milling operation A: we can notice that the introduction of the SSSV allows to stabilize the cutting and to reduce the vibration. This can be well observed in Fig. 4.18: the cutting forces have been limited by the cutting speed modulation and the process hasn't brought to instability.

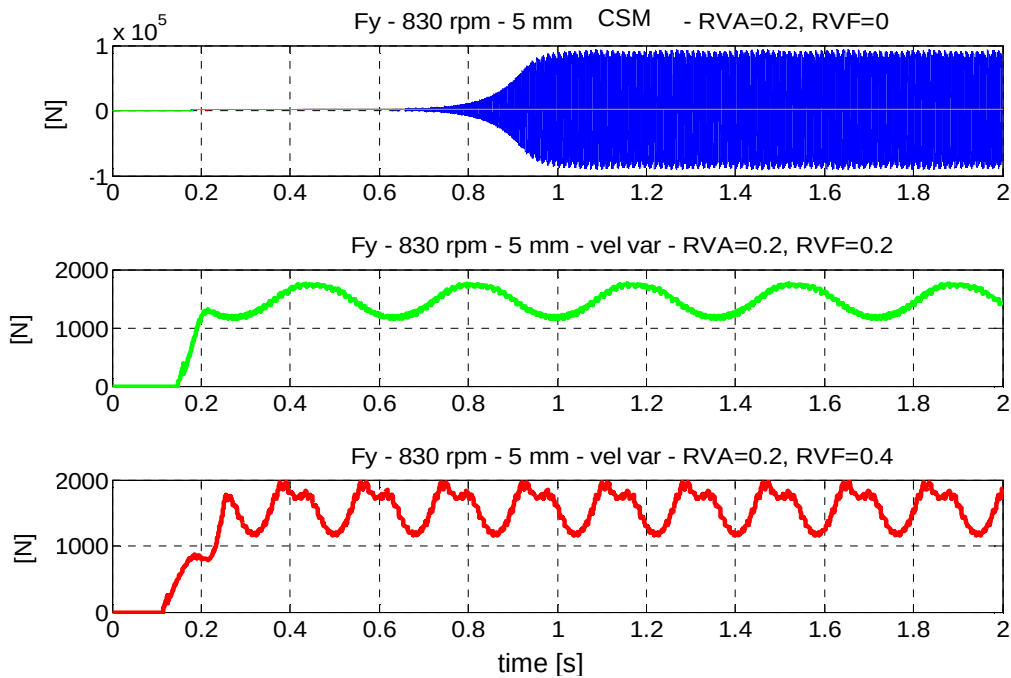


Fig. 4.18: Cutting force comparison – CSM vs SSSV

A more clear comparison of the cutting forces referred to the different cases can be observed in Fig. 4.19.

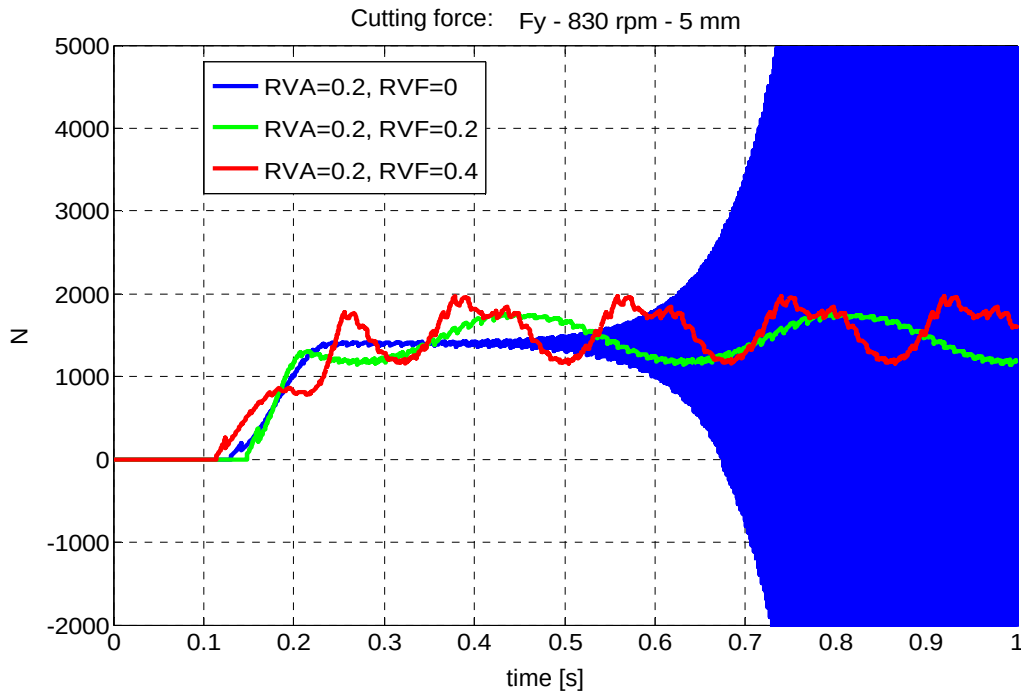


Fig. 4.19: Cutting force comparison

Even if both reported SSSV trajectories ($\text{RVA}=\text{RVF}=0.2, \text{RVA}=0.2/\text{RVF}=0.4$) have the same stability limit the corresponding cutting forces are quite different considering both the maximum value and the frequency content.

The $\text{RVA}=0.2/\text{RVF}=0.4$ presents furthermore a more irregular force profile, Fig. 4.20.

These aspects can be also deduced looking at the Fast Fourier Transforms (FFT) of the cutting force reported in Fig. 4.21.

These differences probably affect also the tool tip displacement and thus the finishing of the machined surfaces.

It can be observed that the $\text{RVA}=\text{RVF}=0.2$ can be considered the preferable combination of parameters and generally not always the SSSV that stresses the speed modulation is convenient. Moreover high speed modulation involves an high consumption of energy.

From this first analysis some general considerations have been outlined but these notes start from the analysis of a specific situation. More general guidelines and a deeper comprehension of the effects of VSM on the regenerative instability is needed.

The simulated tool tip displacements have been also reported.

Fig. 4.22 shows the frequency spectrum of the tool tip displacement of CSM: the chatter frequency that is very close to the resonance of the system can be easily observed as in the first subplot of Fig. 4.20.

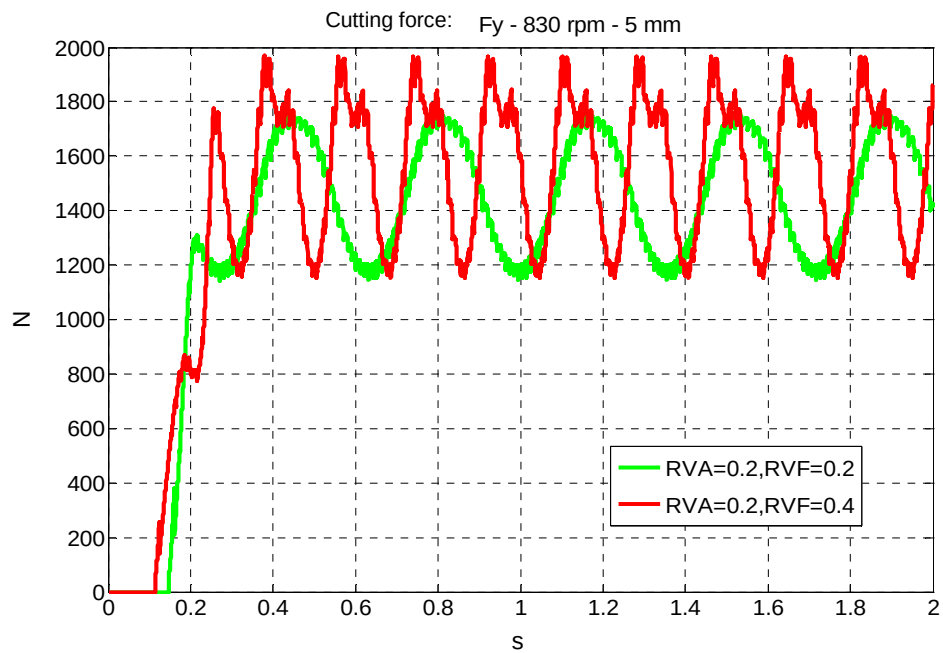


Fig. 4.20: Cutting force comparison – zoom – $RVA=0.2$

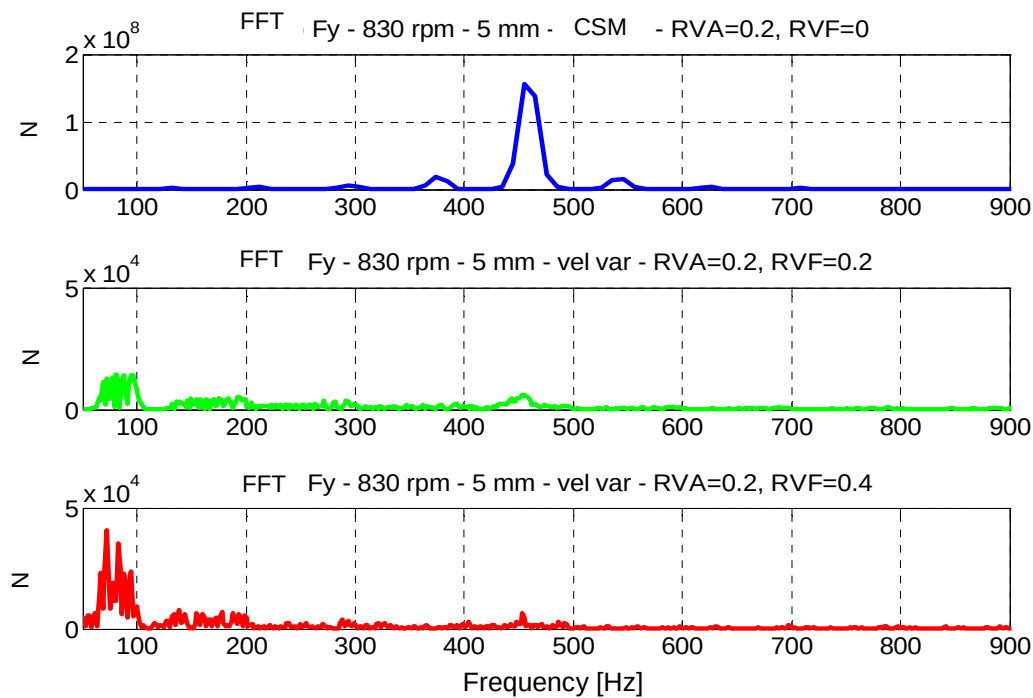


Fig. 4.21: FFT cutting force F_y – SSSV effects – $RVA=0.2$

In Fig. 4.23 the time simulation of the tool tip displacement ($RVA=RVF=0.2$) has been depicted.

We can notice that the SSSV can stabilize the cutting but introduces a modulation of the tool tip position (amplitude= $1 \cdot 10^{-2} mm$), this effect probably can be captured also on the machined surface and can bring some problems if the considered milling is a finishing operation. The obtainment of specific marks on the machined workpiece using VSM was also reported in [79] but referring to turning operations.

It is important to state that VSM has dramatically reduced the vibration level occurred with CSM.

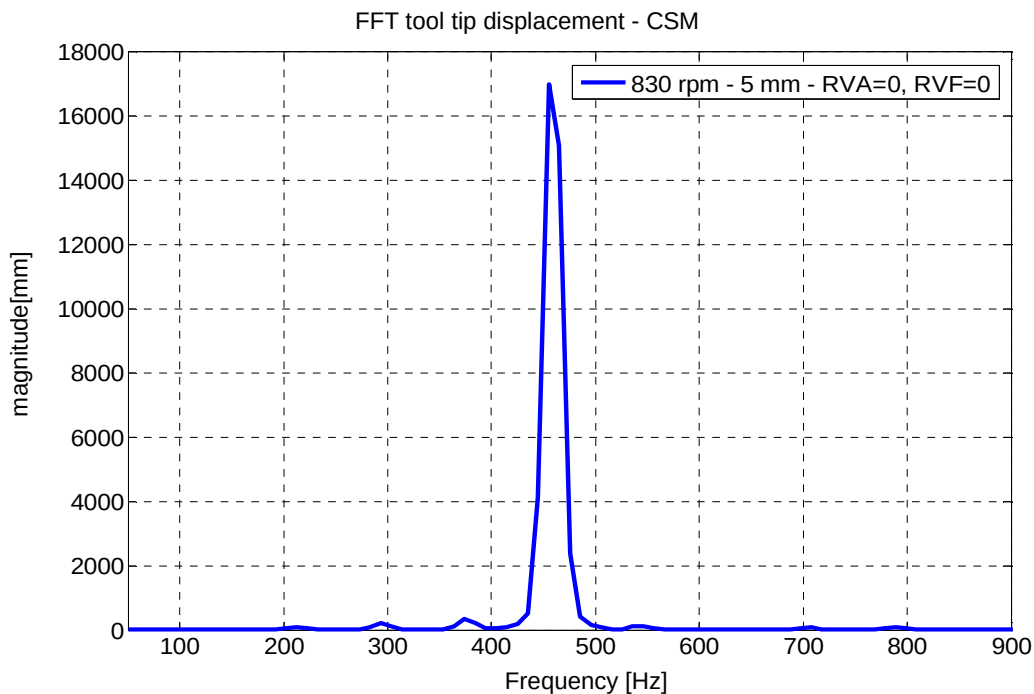


Fig. 4.22: CSM – tool tip displacement frequency spectrum

Analyzing the Fast Fourier Transform (FFT) of the tool tip displacement, Fig. 4.24, we can notice a quite important chatter frequency component but this one is not enough high to bring the instability. I guess it depends on the time window used to compute the FFT.

To complete the analysis of the “case A” the results referred to both $RVA=0.4$ speed modulations are reported in Fig. 4.25, Fig. 4.27 and Fig. 4.26.

From the lobes diagrams reported in Fig. 4.16 it can be pointed out that the $RVA=0.4$ modulations, considering the nominal spindle speed $\Omega_0 = 830 rpm$, should get a higher stability limit than the $RVA=0.2$ one.

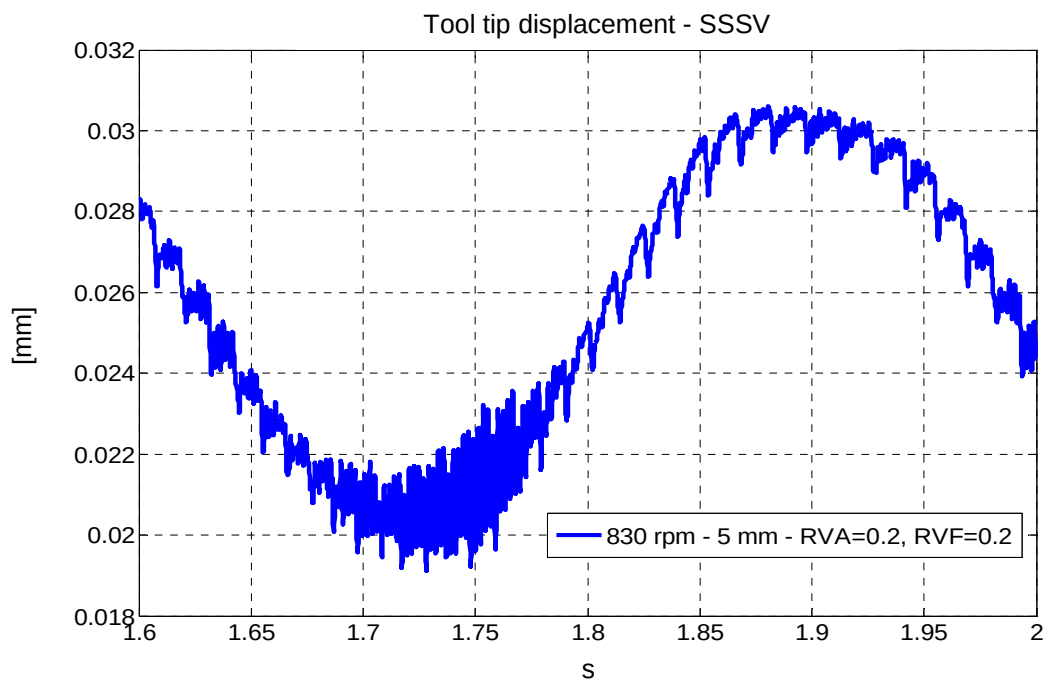


Fig. 4.23: Tool tip displacement - $RVA=RVF=0.2$

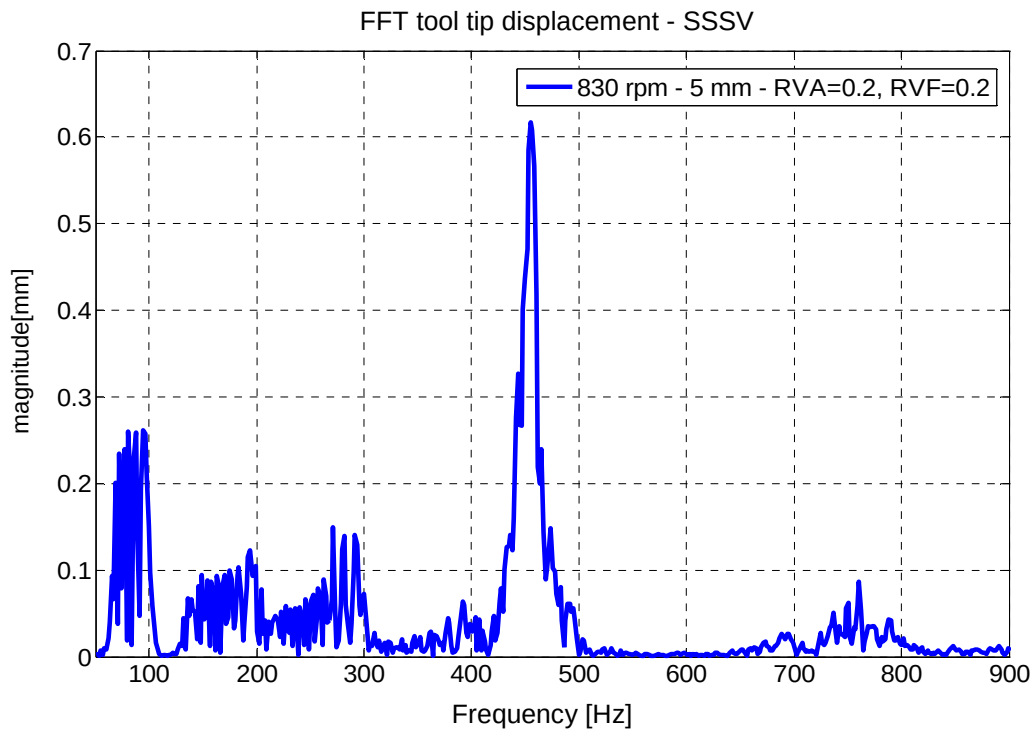


Fig. 4.24: Frequency spectrum tool tip displacement – $RVA=RVF=0.2$

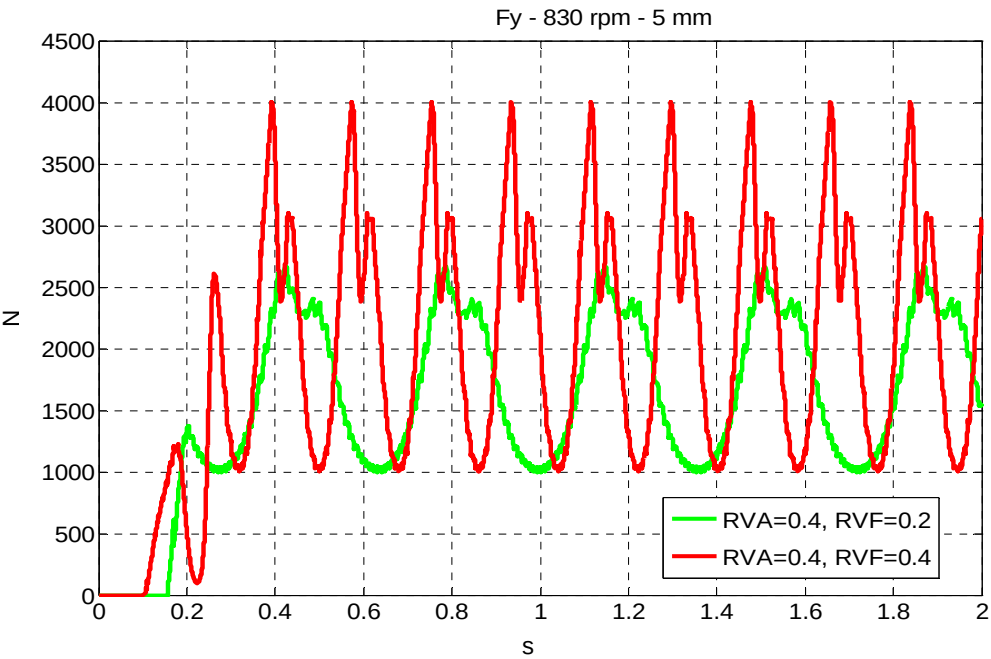


Fig. 4.25: Cutting force comparison – zoom – RVA=0.4

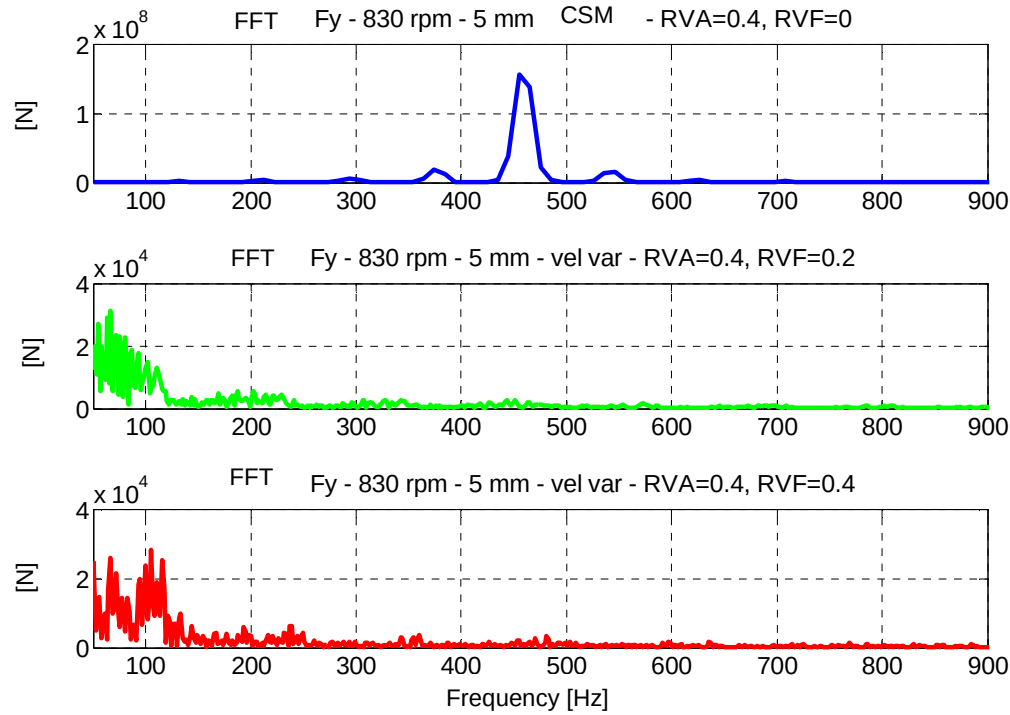


Fig. 4.26: FFT cutting force Fy – SSSV effects – RVF=0.4

Analyzing the diagram reported in Fig. 4.25 it seems that the $RVA=0.4$ modulations involve a higher cutting force than the $RVA=0.2$ ones. An important cutting force increment can be pointed out. Obviously the high level of the cutting forces gets a corresponding high tool tip forced displacement, this could be critical for the surface finishing and for the wear of the cutting edges. Therefore we can deduce that for the analyzed milling operation and for the investigated speed modulations the best SSSV is the $RVA=RVB=0.2$ one.

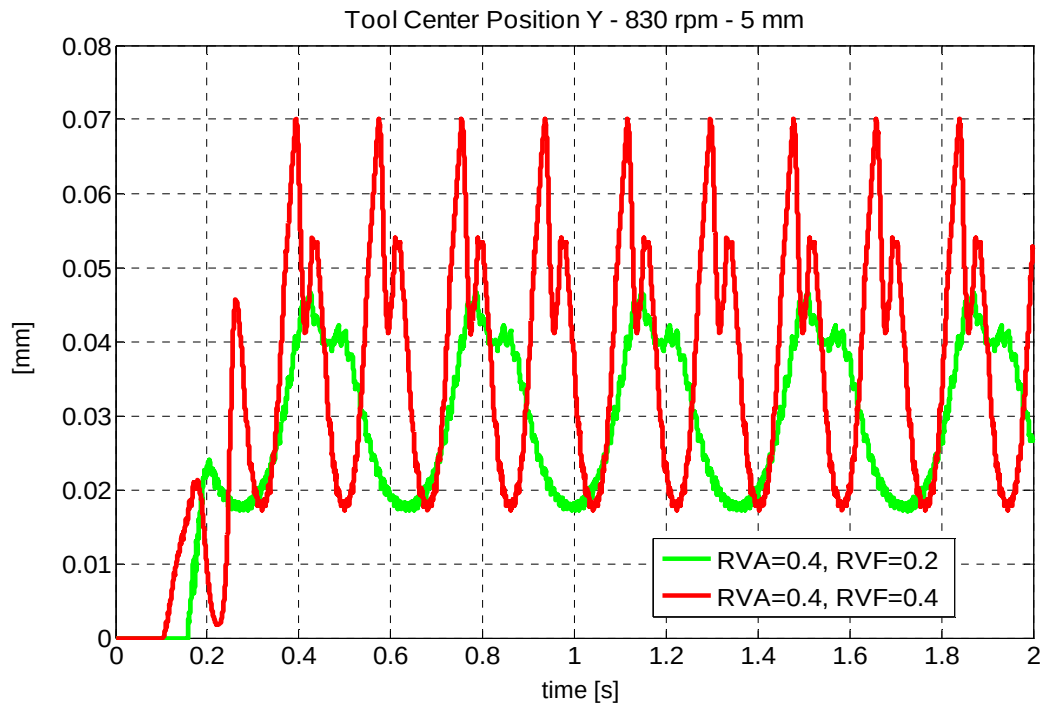


Fig. 4.27: Tool tip displacement $RVA=0.4$

4.4.2 Milling operation B

It would be interesting to analyze a milling operation that can be located in proximity of a stability pocket on the lobes diagram (CSM). From the stability diagram lobes reported in Fig. 4.16 it is clear that the CSM allows higher axial depth of cut than VSM.

It has been states that VSM, especially for low nominal cutting speed, flattens the lobes diagram so an higher asymptotical stability limit occurs.

We are interested to understand the behaviour of the system during VSM considering a milling operations similar to that labelled as “case B”

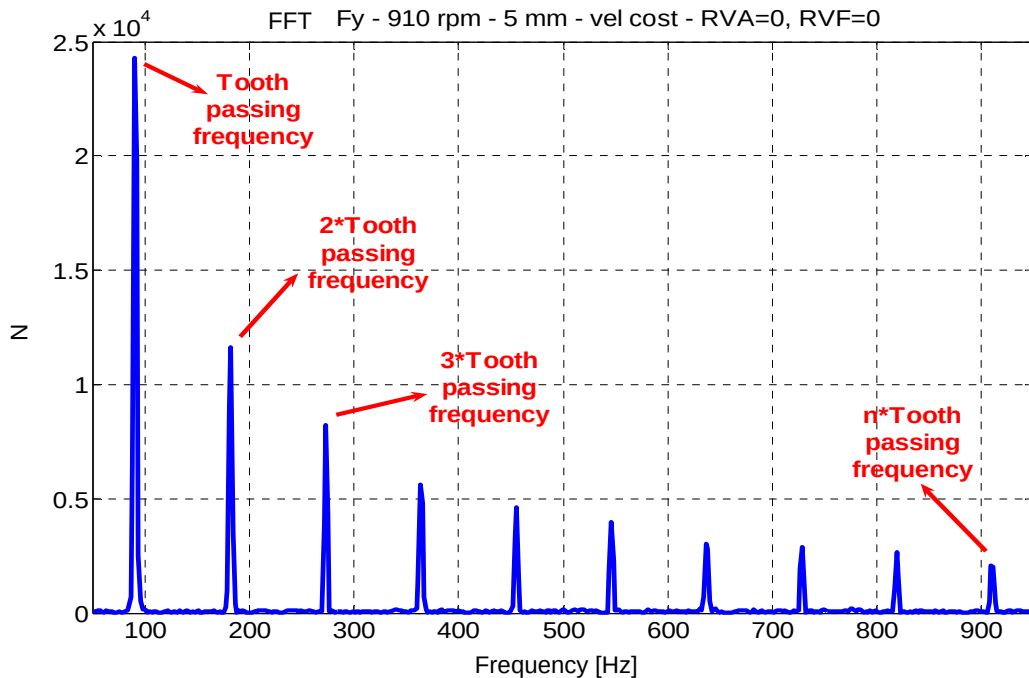


Fig. 4.28: Cutting force frequency spectrum - CSM

Considering a stable cutting at constant speed; the spectrum of the cutting forces, Fig. 4.28, includes components that have frequencies multiple of the tooth passing frequency.

In Fig. 4.29 the comparison between the analyzed cases have been reported: VSM involves higher cutting forces than CSM.

During CSM the cutting forces vary into a limited range of values as depicted in Fig. 4.30, the introduction of SSSV causes an important increment of the variability range therefore a consequent increase of the cutting edge wear and an increment of the tool tip displacement.

In Fig. 4.31 the spectra of the cutting forces considering VSM have been reported: the power of the cutting has been distributed over a wide range of frequencies and the single components can't be found.

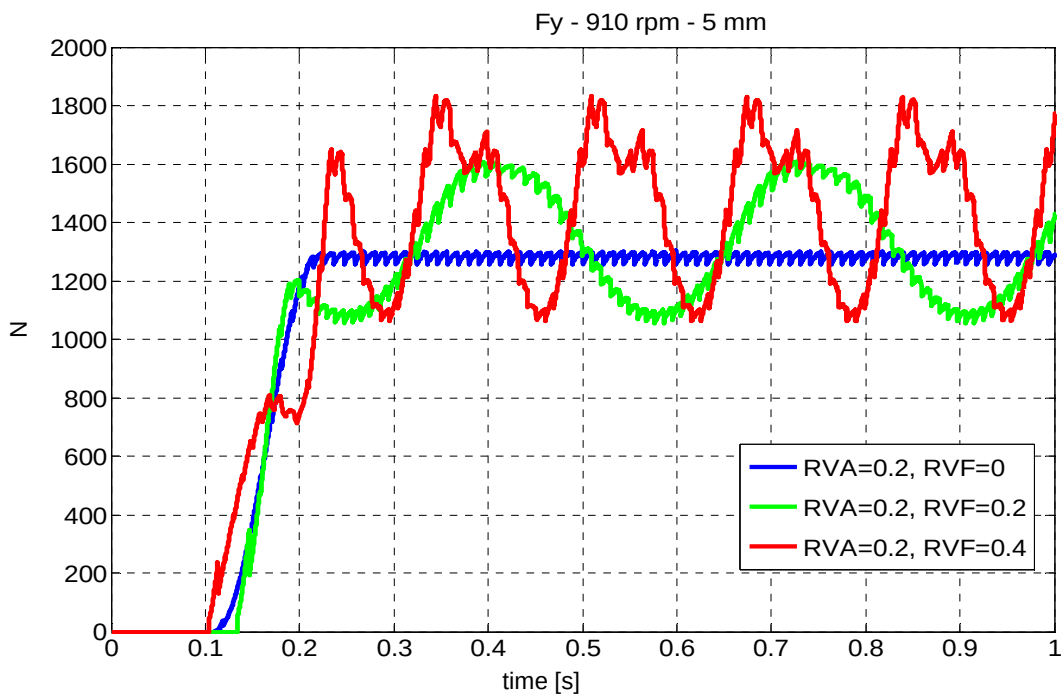


Fig. 4.29: Cutting forces comparison – VSM($RVA=0.2$) Vs CSM - B

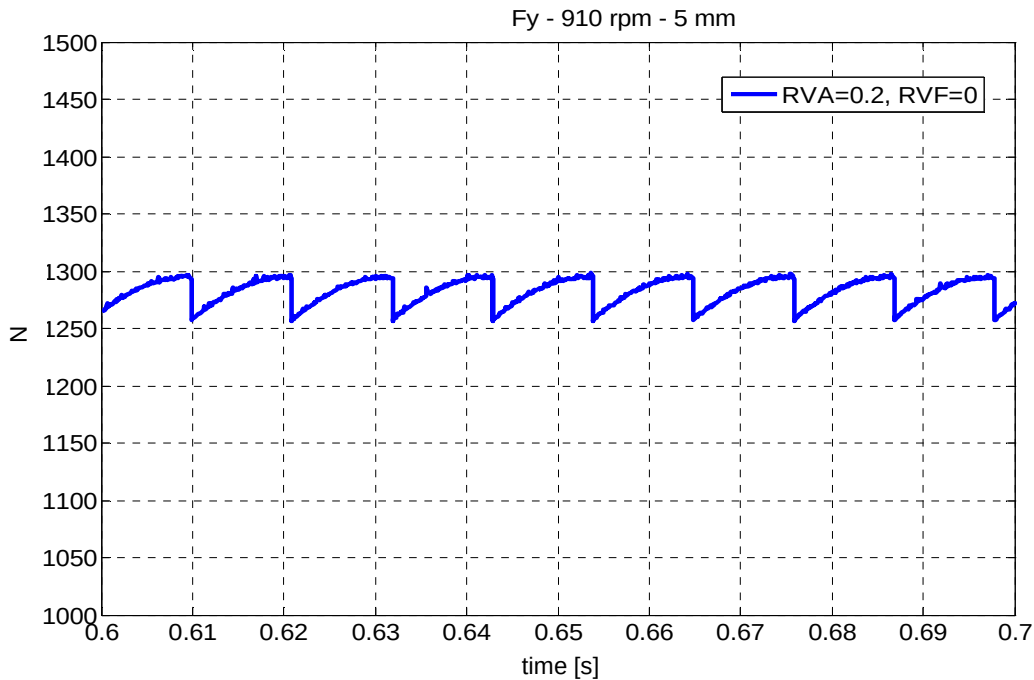


Fig. 4.30: Cutting force F_y – CSM zoom -B

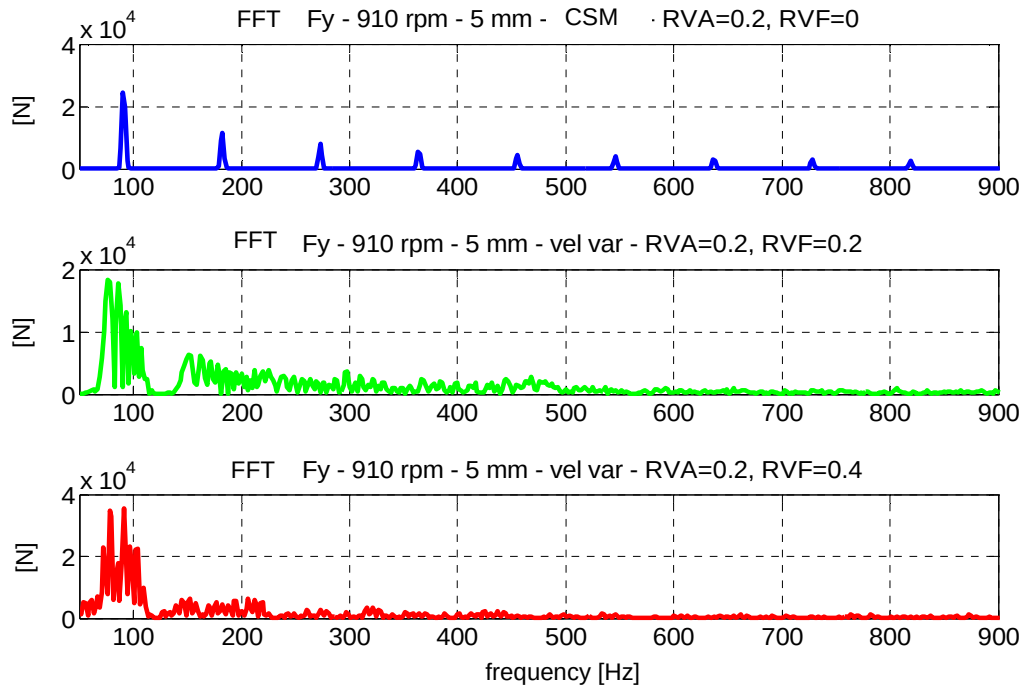


Fig. 4.31: Cutting force frequency domain – VSM ($RVA=0.2$) vs CSM - B

This can be observed also in Fig. 4.34, for different values of RVA/RVF .

A similar behaviour is reported also in [75].

Increasing the amplitude of the sinusoidal speed modulating profile ($RVA=0.2 \rightarrow RVA=0.4$) an increment of the cutting forces can be observed as depicted in Fig. 4.32 and Fig. 4.33 so it is necessary to select the RVA/RVF parameters that allow to increase the stability limit but is necessary at the same time to limit the speed modulation in order to obtain an acceptable surface finishing and to reduce the power consumption.

In Tab. 4.3 are reported the extreme force values occurred during the analyzed VSM cases.

Spindle Speed (rpm)	Axial depth of cut (mm)	RVA/RVF (%)	F_y (N)	RVA/RVF (%)	F_y (N)
910	5	0.2/0.2	1100-1600	0.2/0.4	1100-1800
910	5	0.4/0.2	1000-2500	0.4/0.4	1000-3700

Tab. 4.3: Cutting force range – RVA/RVF effects

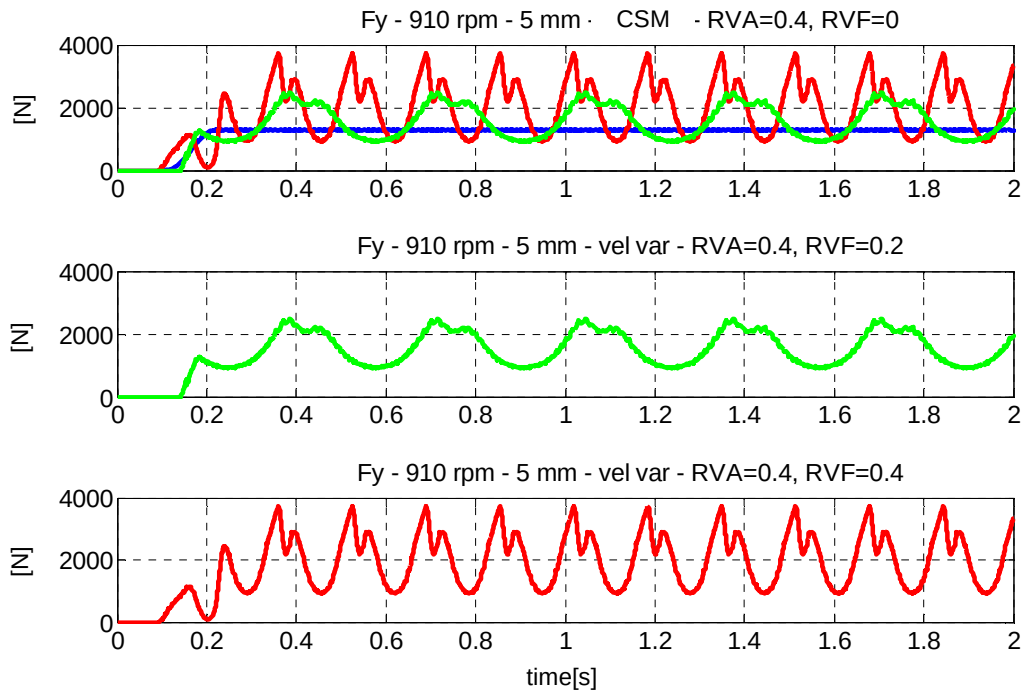


Fig. 4.32: Cutting forces comparison – VSM(RVA=0.4) vs CSM - B

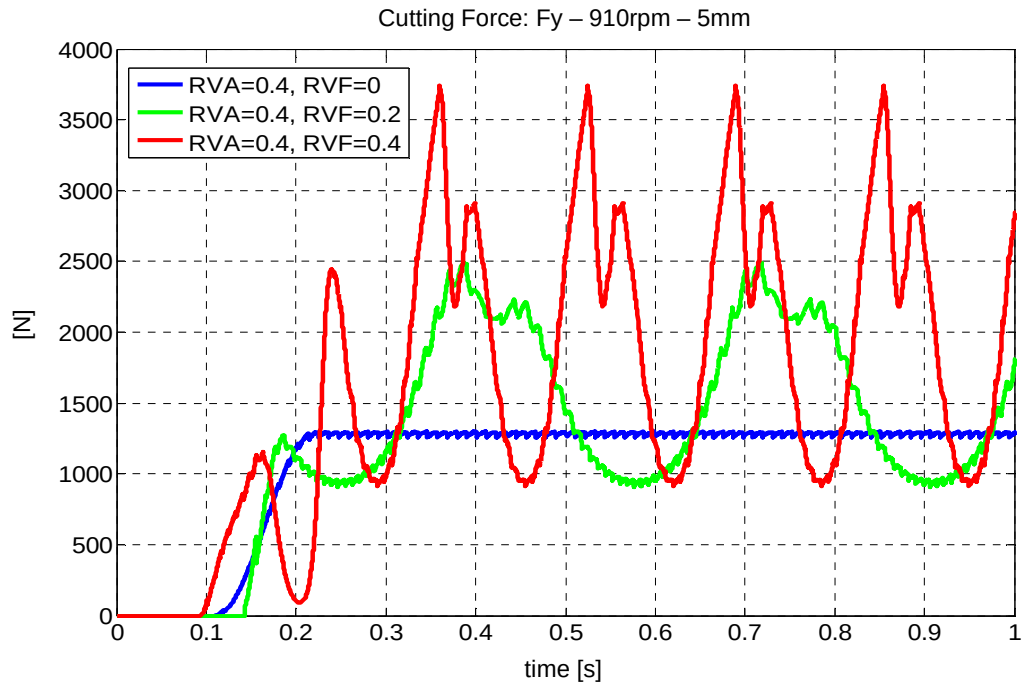


Fig. 4.33: Cutting forces comparison – VSM(RVA=0.4) vs CSM - B

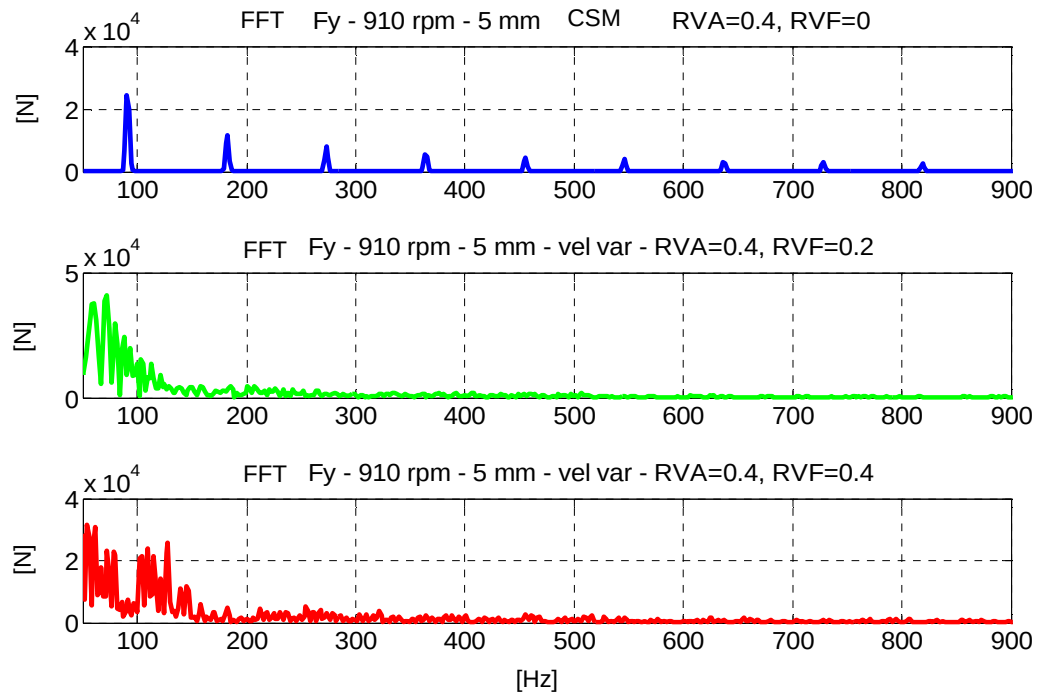


Fig. 4.34: Cutting force - frequency domain – VSM (RVA=0.4) vs CSM - B

4.4.3 Milling operation C

The milling operation C clearly involves cutting instability even if the SSSV technique has been introduced. This can be appreciate looking at cutting forces diagrams reported in the following pictures.

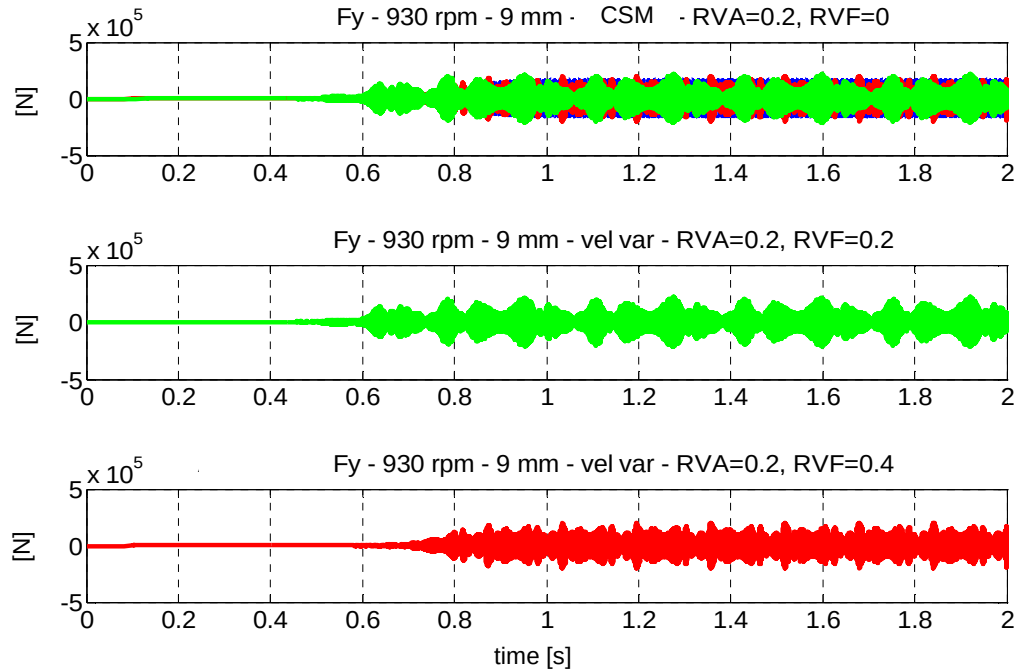


Fig. 4.35: Cutting forces – RVA=0.2 – C

As formerly described the speed modulation distributes the cutting energy over a wide range of frequencies and this can be observed especially for instable cutting, like reported in [75].

Higher is the speed modulation, basically considering the RVA parameter, more distributed is the cutting energy. This can be pointed out comparing Fig. 4.36 and Fig. 4.38.

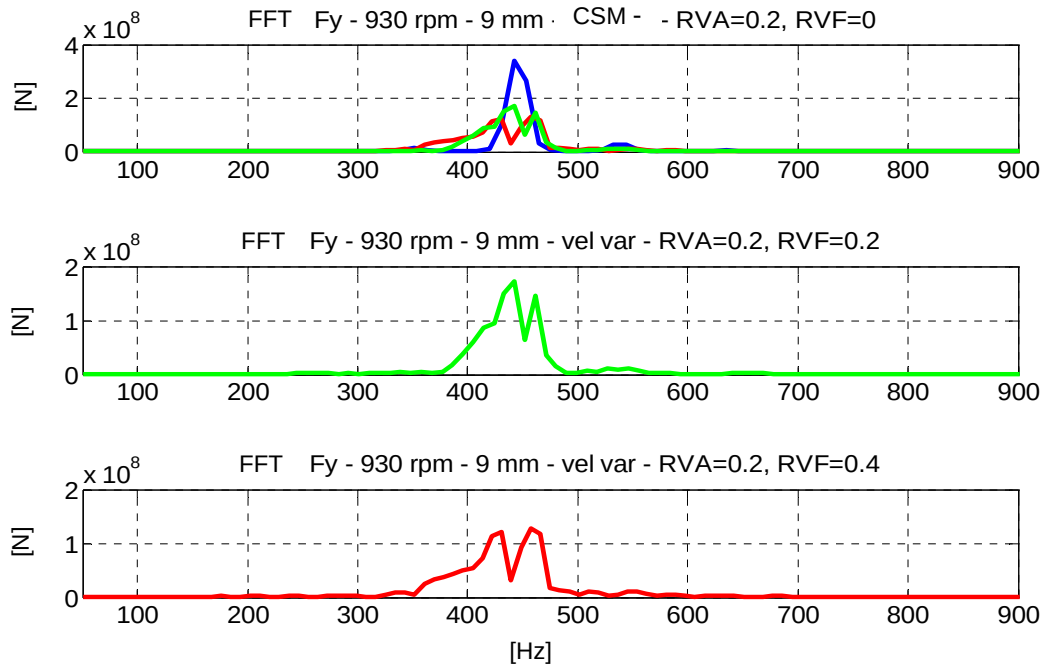


Fig. 4.36: Cutting force - frequency domain – VSM ($RVA=0.2$) vs CSM – C

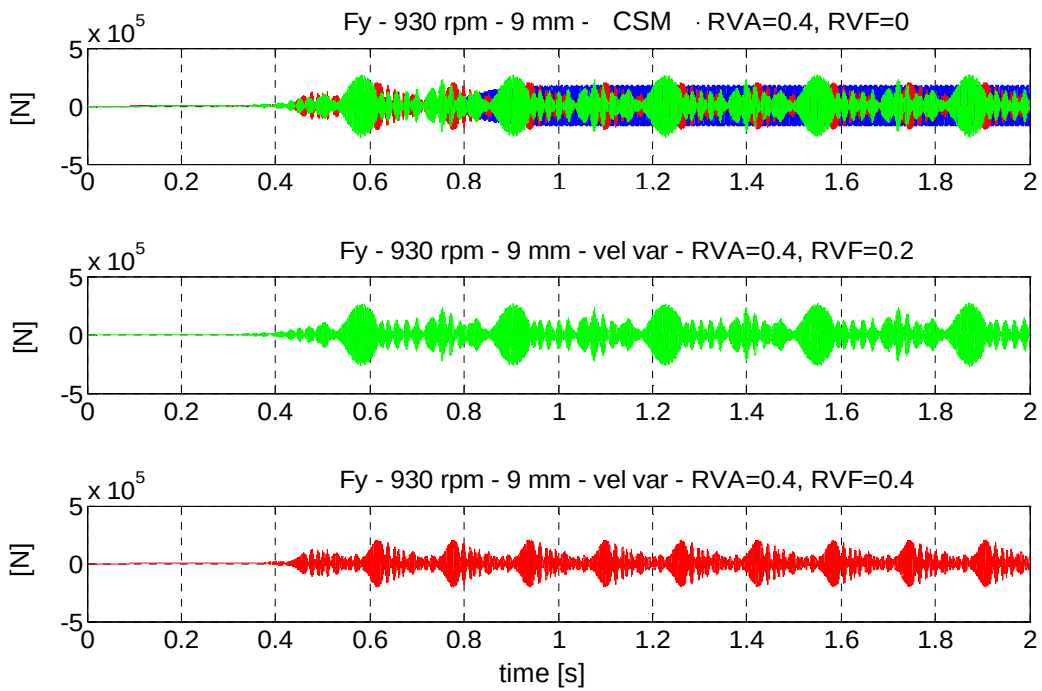


Fig. 4.37: Cutting forces – $RVA=0.4$ – C

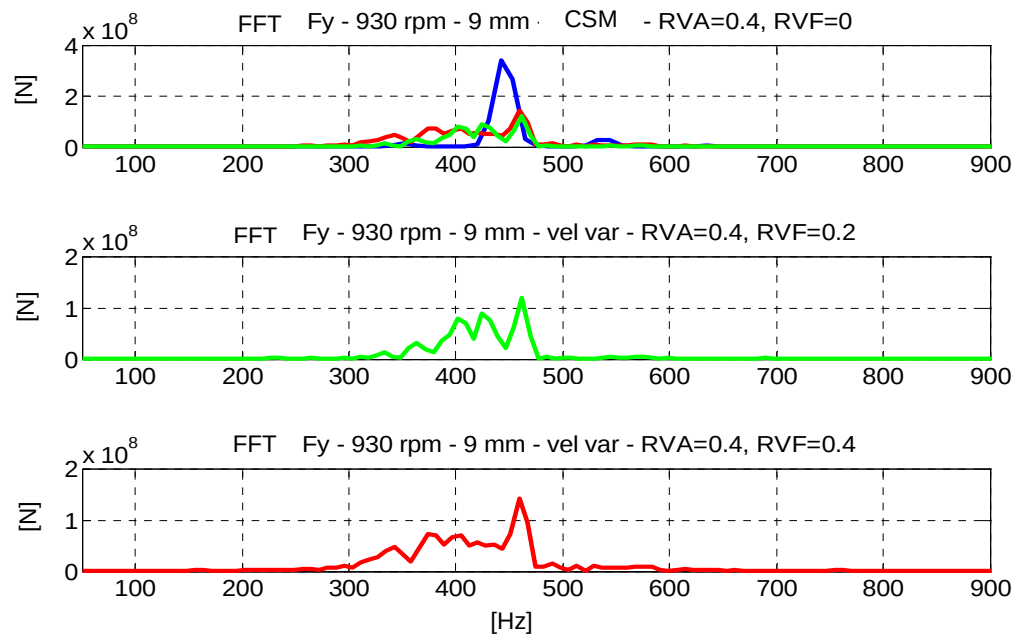


Fig. 4.38: Cutting force - frequency domain – VSM (RVA=0.4) Vs CSM - C

4.4.4 Milling operation D

It is interesting to analyze the effectiveness of the SSSV considering another nominal spindle speed. The chosen nominal spindle speed is $\Omega_0 = 1030 \text{ rpm}$. This milling operation should be very similar to milling A and it is interesting to investigate if the VSM shows the same stabilizing capabilities shown in 4.4.1.

In Fig. 4.39 and Fig. 4.42 the cutting forces F_y (orthogonal to the feed direction) are very similar to these reported in Fig. 4.20 and Fig. 4.25.

In Tab. 4.4 the comparison among the chatter frequency components at CSM has been reported.

Spindle Speed (rpm)	Axial depth of cut (mm)	Chatter frequency (Hz)	Amplitude (N)
830	5	455	$1.558 \cdot 10^8$
1030	5	463.3	$1.452 \cdot 10^8$

Tab. 4.4: Comparison - A vs D milling - CSM

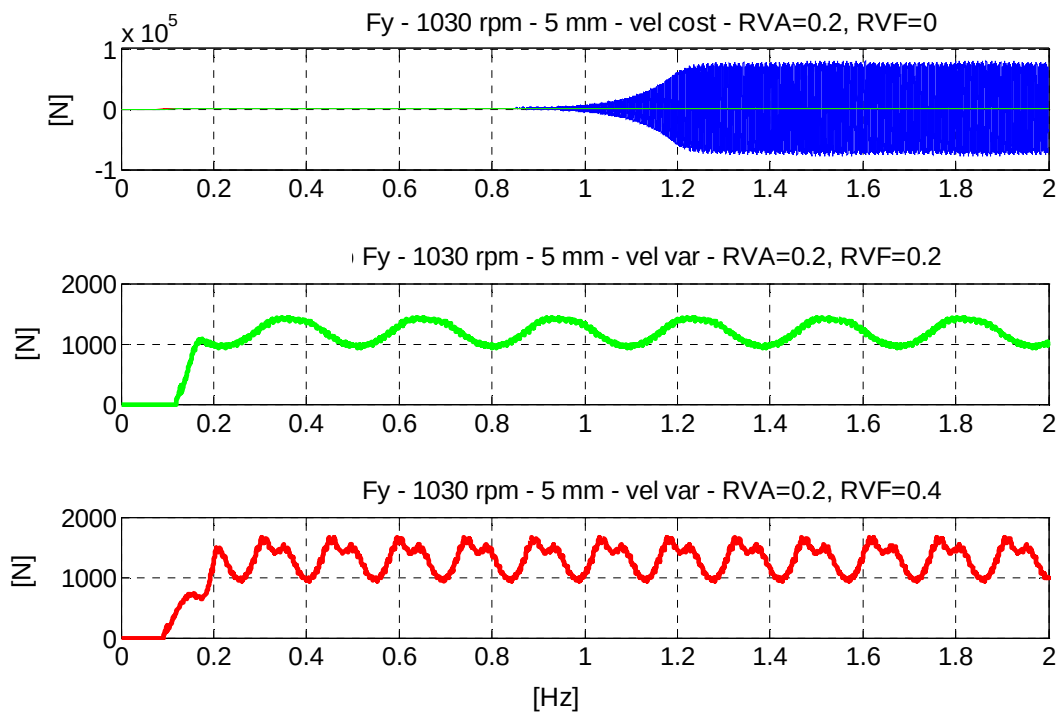


Fig. 4.39: Cutting forces – RVA=0.2 – D

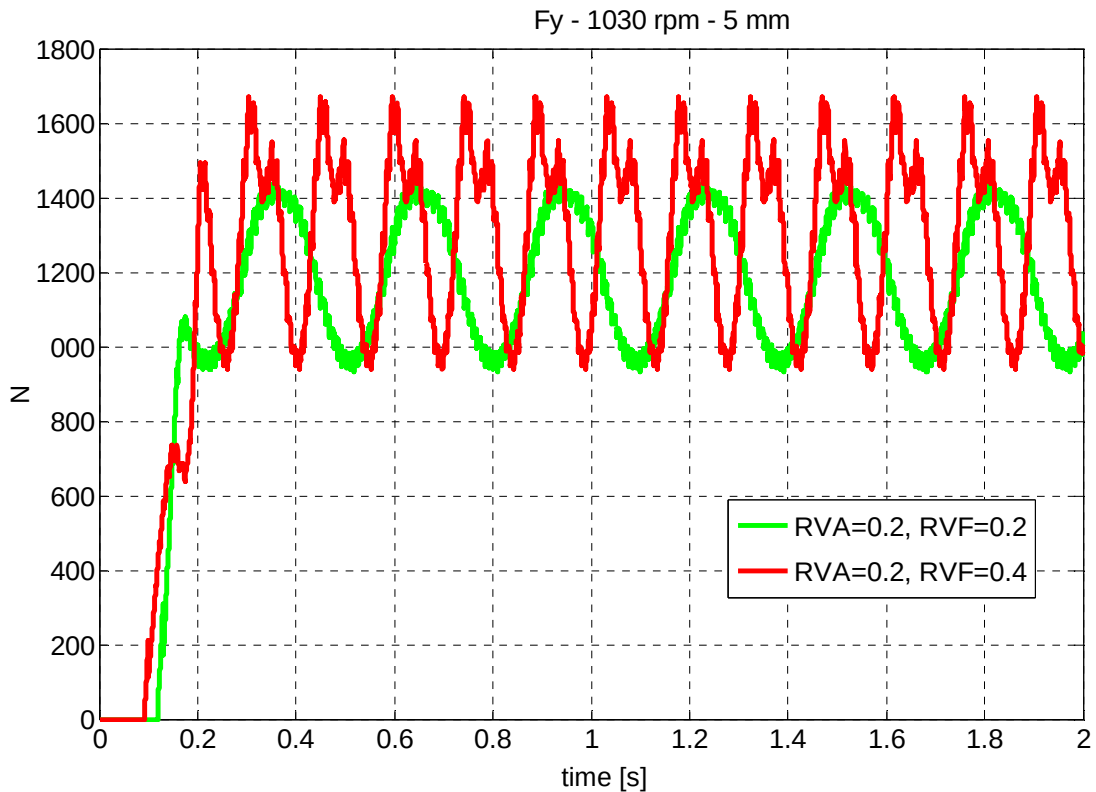


Fig. 4.40: Cutting forces – RVA=0.2 – D – zoom

Moreover, Tab. 4.5 points out the force ranges revealed during the simulations. It seems that milling D involves a smaller force level and probably this justifies a slight increment in the stability limit that the one found in case A.

Spindle Speed (rpm)	Depth of cut (mm)	RVA/RVF (%)	Fy (N)	RVA/RVF (%)	Fy (N)
830	5	0.2/0.2	1200-1800	0.2/0.4	1200-2000
1030	5	0.2/0.2	1000-1400	0.2/0.4	1000-1700

Tab. 4.5: comparison A Vs D milling – VSM – cutting forces – RVA0=.2

In Tab. 4.7 and Tab. 4.6 the capability of the SSSV to suppress the chatter vibration can be observed.

We can conclude that the speed modulation has a similar effect both on the A milling and D milling.

The same considerations can be presented also for the speed modulations with RVA=0.4 as reported in Fig. 4.25, Fig. 4.26, Fig. 4.43, Fig. 4.65 and Tab. 4.1.

Spindle Speed (rpm)	Depth of cut (mm)	RVA/RVF (%)	displacement (mm)	Frequency (Hz)	Amplitude
1030	5	0/0	± 8	463.3	1.546×10^4
1030	5	0.2/0.2	0.016-0.025	449.2	0.4412

Tab. 4.6: SSSV effects – milling D

Spindle speed (rpm)	Depth of cut (mm)	RVA/RVF (%)	displacement (mm)	Chatter freq (Hz)	Amplitude
830	5	0.2/0.2	0.02-0.03	455	0.6178
1030	5	0.2/0.2	0.016-0.025	449.2	0.4412

Tab. 4.7: Comparison A vs D milling – VSM – chatter frequency component

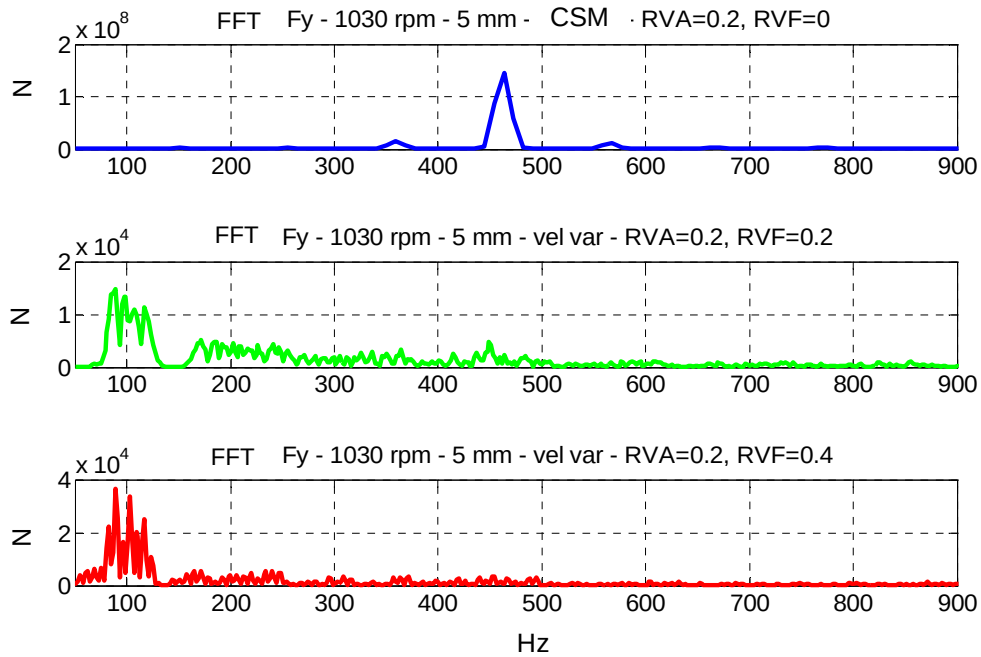


Fig. 4.41: Cutting force - frequency domain – VSM (RVA=0.2) Vs CSM – D

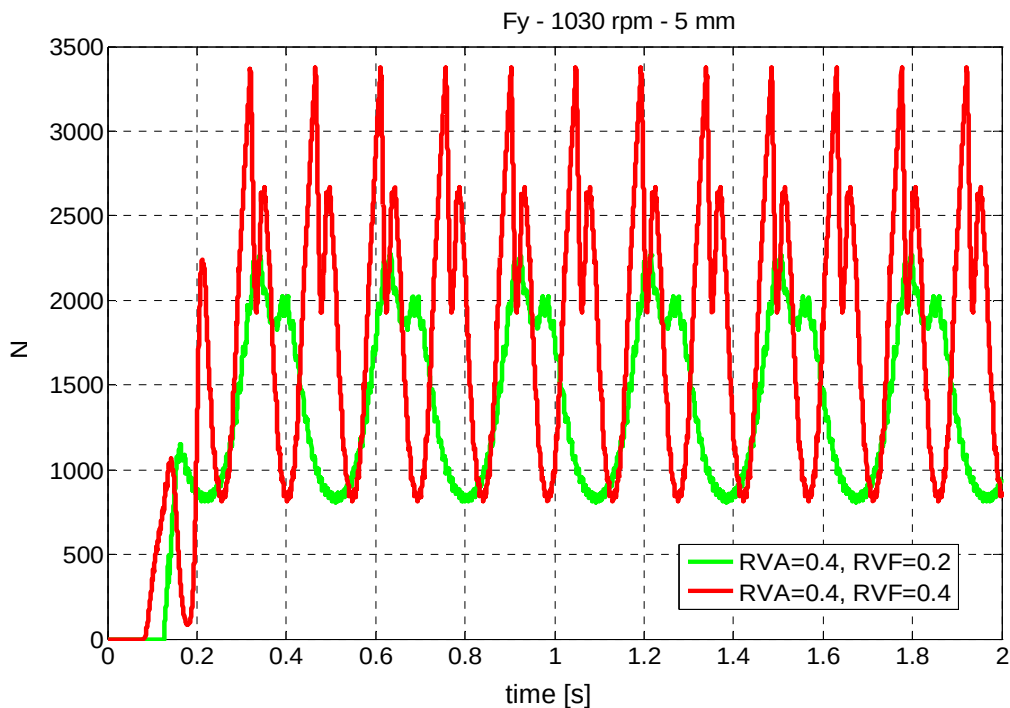


Fig. 4.42: Cutting forces – RVA=0.4 – D – zoom

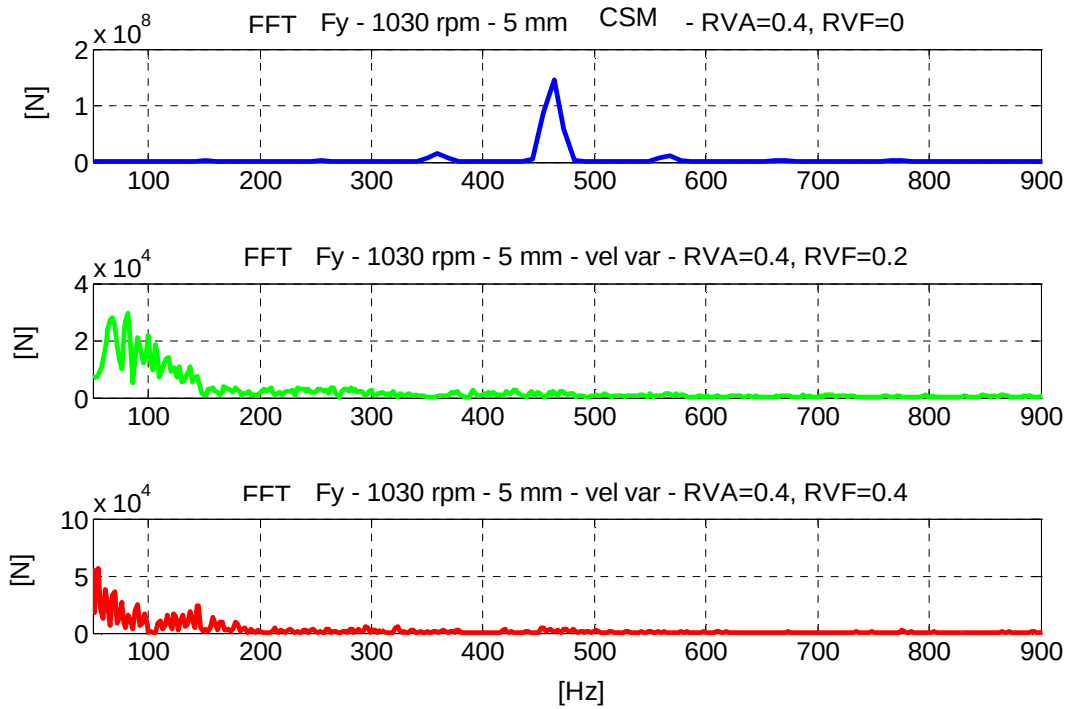


Fig. 4.43: Cutting force - frequency domain – VSM (RVA=0.4) vs CSM – D

Velocità (rpm)	Profondità (mm)	RVA/RVF (%)	Ampiezza F_y (N)	RVA/RVF (%)	Ampiezza F_y (N)
830	5	0.4/0.2	1000-2700	0.4/0.4	1000-4000
1030	5	0.4/0.2	800-2300	0.4/0.4	800-3400

Tab. 4.8: Comparison A vs D milling – VSM – cutting forces – RVA=0.4

4.4.5 Time simulation results: complex dynamic

From the previous reported analyzes it has been pointed out that VSM increase the cutting stability limit if the nominal cutting speed is close to the minimum of the lobes diagram obtained at CSM.

In this region of the lobes diagram the regenerative phenomenon strongly affects the stability limit thus modulation of the cutting speed breaks the regenerative effect therefore enhancing the stability of the cutting. For milling operations located in a region close to the “stability pocket” we can observe an opposite effect: the regenerative phenomenon is very low so there isn’t the possibility to excite the structure and therefore the introduction of the modulation in such a way disturbs this good cutting condition bringing a reduction of the stability limit.

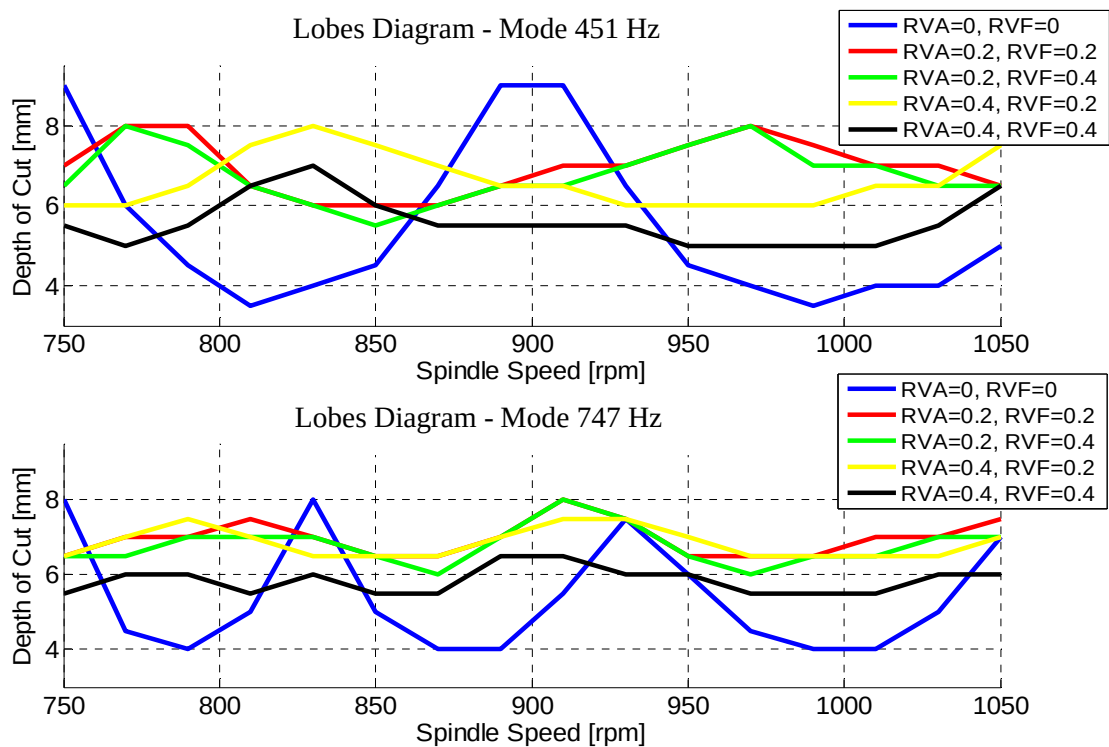


Fig. 4.44: Spindle Speed Variation – contribute of different eigenmodes

The bottom subplot of the Fig. 4.44 shows the stability lobes diagram (VSM) obtained only considering the dominant mode @ 750Hz.

The formerly reported comments can be considered valid also for this latter case.

It is very interesting to analyze the results reported in Fig. 4.45 obtained considering both eigenmodes (@450Hz and @750Hz).

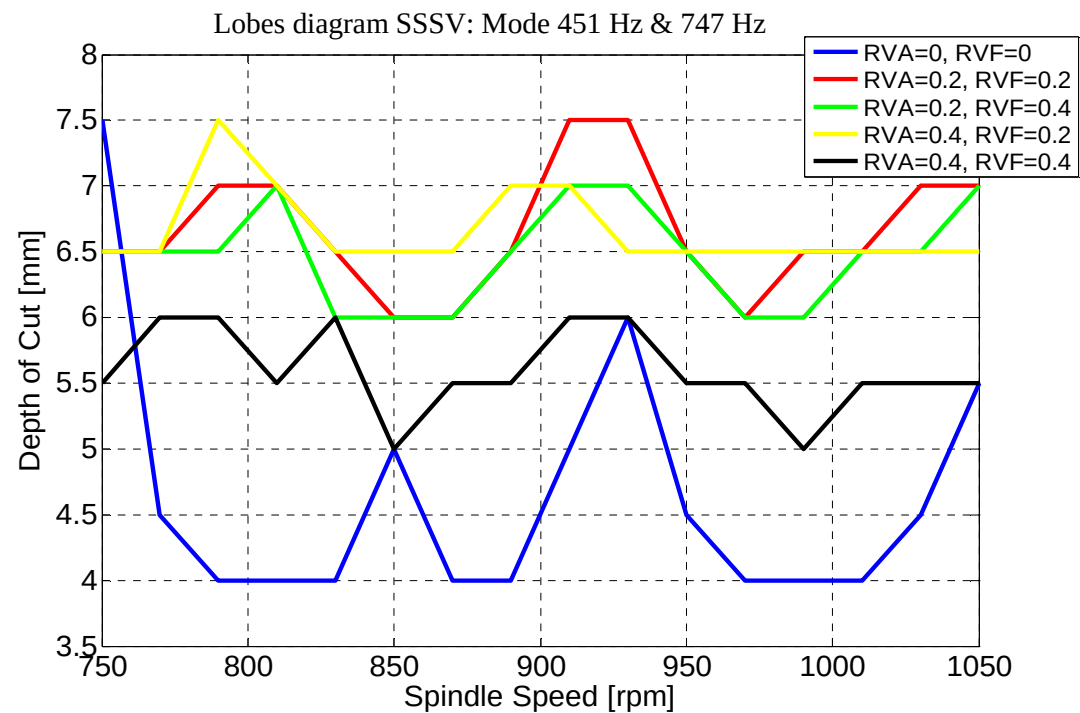


Fig. 4.45: Stability Lobes diagram – VSM – two dominant modes

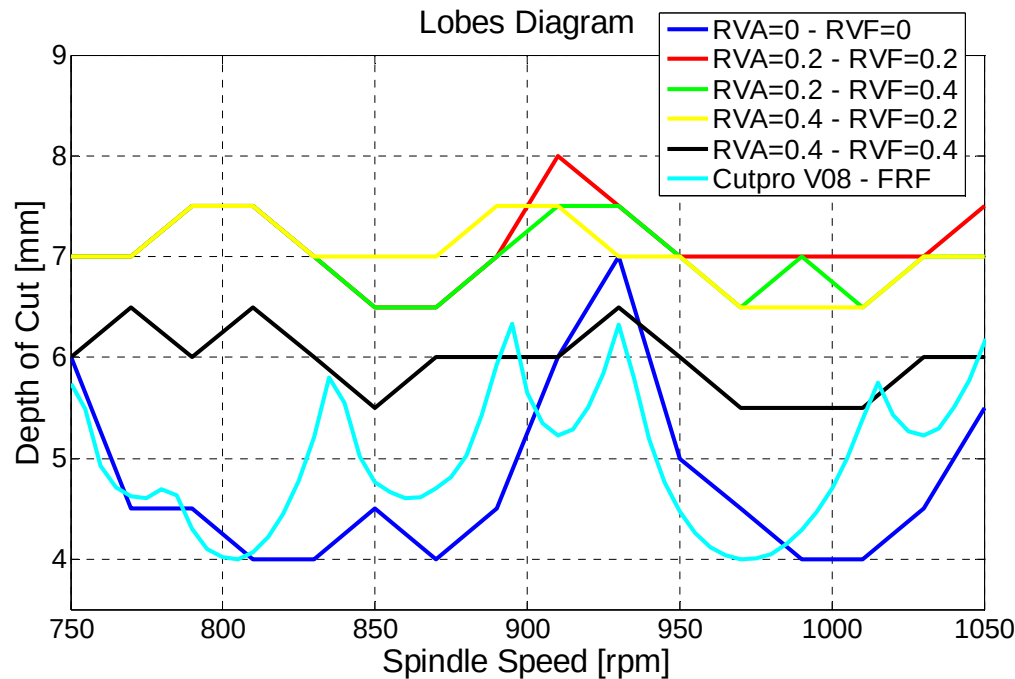


Fig. 4.46: Stability Lobes diagram – VSM – four eigenmodes

It seems that the Sinusoidal Spindle Speed Variation has brought more important stability enhancements to the CSM stability lobes than in the single mode analyzed cases. Analogous considerations were also reported in [74], the authors stated that the SSSV is promising when the machine-workpiece dynamics is very complex.

We would like to interpret this statement. This effect is due to a sort of dynamic interaction between each single mode.

Moreover this phenomenon is quite evident in the analyzed test case.

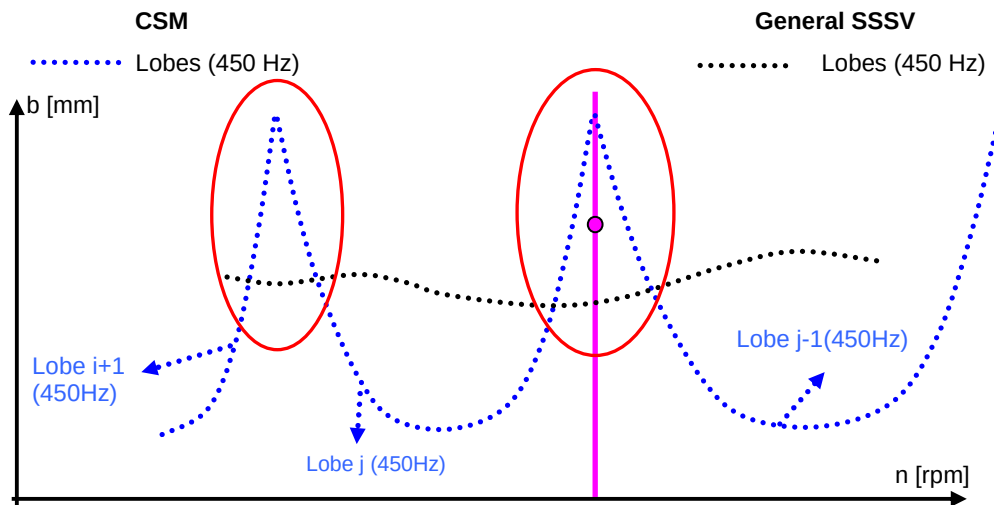


Fig. 4.47: SSSV effects on single mode stability chart

Referring to a schematic and generic lobes diagram, for example obtained considering the mode @450Hz, Fig. 4.47. We have concluded that the SSSV technique can't improve the Material Removal Rate for milling operation located into the underlined regions because the nominal cutting speeds already minimizes the regenerative effect so any modulation makes worse the ideal situations.

Considering a more complicated dynamic, for example introducing the mode @750Hz, as depicted in Fig. 4.48, the analyzed milling operation isn't still allowed due to the regenerative effect that is introduced by the added mode. Considering the mode @750Hz, the nominal cutting speed involves the maximum regenerative contribute for this mode.

Fig. 4.48 shows how the dynamic interaction between different eigenmodes can produce a positive effect on the cutting stability if VSM is used.

Obviously in the analyzed situations this aspect is particularly evident because the considered modes have an antithetic behaviour from the regenerative point of view so that the minima of the lobes linked to one mode corresponds to a stability pocket for the other mode.

Generally we can conclude that more the machine-workpiece dynamic is complex higher is the effectiveness of the SSSV because the probability to have two antithetic modes is higher.

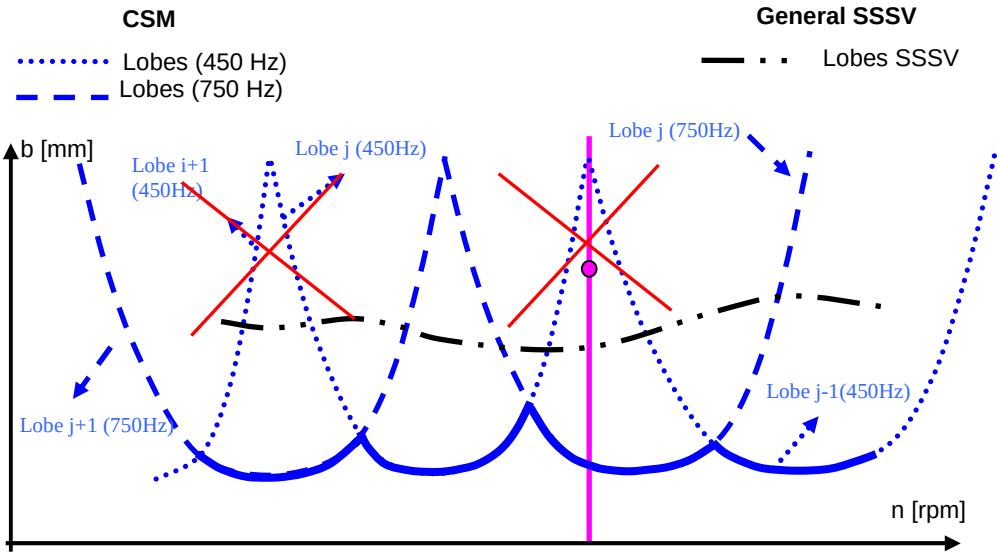


Fig. 4.48: Dynamic interaction between eigenmode& SSSV

In Fig. 4.46 the results obtained considering 4 eigenmodes, obviously for each axis or degree of freedom, have been reported.

4.5 Energetic interpretation of the SSSV technique

Some important conclusions on the effectiveness of the stabilizing mechanism introduced by the use of the SSSV technique have been drawn in the previous paragraph of this chapter.

In order to study in a more detailed manner how the cutting speed modulation interacts with the regenerative phenomenon it is necessary to approach to a simple dynamic system with only one dominant mode. Further conclusions regarding complex dynamics can be outlined considering the “contribute” of each single eigenmode.

Basically we have asserted that system during VSM behaves in different ways depending on the nominal spindle speed that corresponds to a specific position on the stability lobes diagram.

In this section an energetic approach has been proposed to analyze the problem.

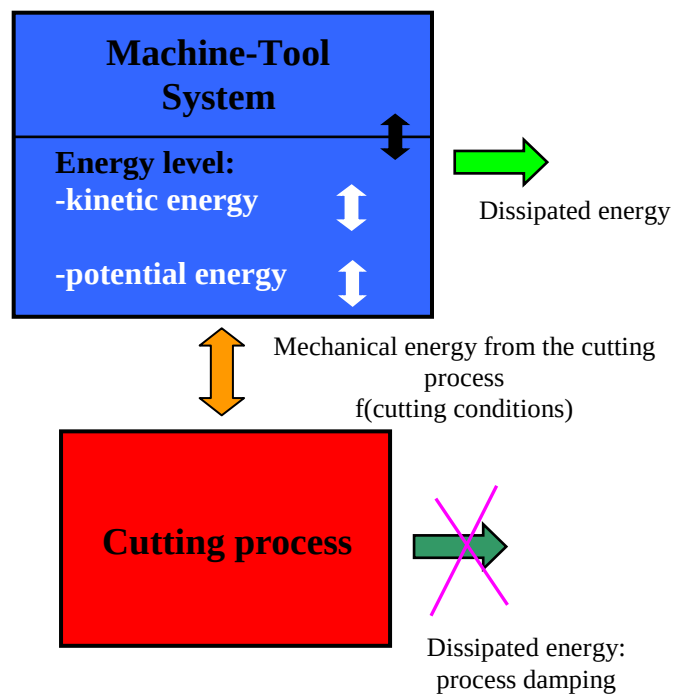


Fig. 4.49: Energetic approach

The energetic approach has been applied to the machine-tool system depicted in Fig. 4.49. The cutting instability mechanism can be explained in a very simple way proposing an energetic interpretation: during an instable milling operation the mechanical energy from the cutting process flows towards the machine tool, in those conditions the system can't adequately dissipate that energy so the stability occurs and the vibrations grow up to an unacceptable level.

The proposed energetic analysis has been focused especially on the energy that comes from the cutting process.

Some simplifications have been introduced in order to deal with the SSSV effect on the regenerative mechanism (i.e. the cutting process damping has been neglected, this damping effect is particularly evident at very low frequency and it is due to the sliding of the back of the cutting edge on the machined surfaces, see 1.1).

When the process damping occurs higher axial depths of cut are allowed.

The first consideration that is necessary to introduce is that the energy that flows from the cutting process to the machine depends basically both on the cutting forces and the corresponding tool kinematics in the defined cutting conditions.

$$\text{Eq. 4.7} \quad E_{IN} = \int_0^t \sum_{i=1}^z g(\varphi_i) \cdot (\overrightarrow{F_{i,T}} \cdot \overrightarrow{v_i}) \cdot dt$$

Where $\overrightarrow{F_{i,T}}$ is the cutting force linked to a generic cutters and transported to the tool centre, $\overrightarrow{v_i}$ is the speed of the tool centre and $g(\varphi_i)$ is a (0-1) coefficient that points out if the cutters is working:

If $\varphi_{IN} < \varphi_i < \varphi_{OUT} \rightarrow g(\varphi_i) = 1$ otherwise $g(\varphi_i) = 0$ and the cutting edge is not still working.

An important key to approach to the problem is to analyze the cutting forces trend during different cutting conditions.

We have firstly decided to study the CSM case: it is very interesting to analyze the worse condition from the stability point of view or rather the milling operations that can be placed close to the minimum of each stability lobe.

The key to forecast the cutting force trend linked the worse condition is to analyze how the instantaneous chip thickness is modulated by the “time delay” that is which is the relationship between the actual cutter position and the track left on the machined surface.

For the CSM it is possible to define a phase shift between the tracks. This phase shift can be computed using the relationship Eq. 4.5. The previous relationship is also approximately valid for VSM and in this case the $\phi(t)$ can be considered as the phase shift that would be occurred if the instantaneous value of the cutting speed was kept constant.

It is important to state that for the analyzed face milling (full tool engagement) the phase shifts corresponding to the two most important regions of the lobes diagram are reported in Fig. 4.50.

The phase shift corresponding to the worse cutting condition from the stability point of view is $\Phi = \pi$.

From the analysis of different milling operations we have noticed that the critical phase shift is not always equal to an half chatter vibration period hence we will further present a comprehensive study of this issue that so far hasn't

been satisfactorily dealt with in the specific literature. Both [88] and [89] have analyzed only turning operations from this point of view.

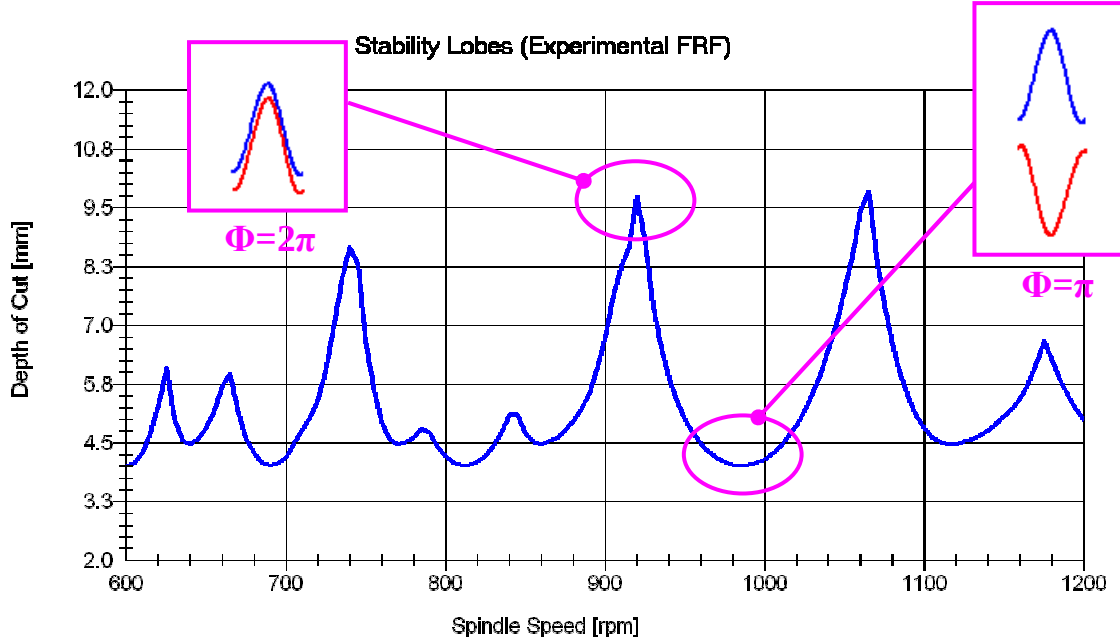


Fig. 4.50 : Generic lobes diagram and phase shift analysis - CSM

As previously stated we are interesting in the analysis of the worse stability condition that is the region where each lobe shows its minimum value.

From the performed time simulations in this low stability zones a particular tool centre behaviour has been observed: the trajectory of the tool is approximately a circle (an ellipse in the more general case). In this cutting condition a phase shift of $\pi/2$ between the orthogonal (X-Y) tool displacements occurs. The analyzed behaviour is well illustrated in Fig. 4.51.

Having described the behaviour of the system and furthermore considering both the chip thickness modulation due to the delay effects and the kinematics of the tool tip it is possible to describe the mechanism that the cutting process uses to introduce energy in the machine tool system.

It is important to assert that only the energy due to the chatter vibration has been considered because this energy is strictly related to the regenerative effect and therefore to the cutting instability.

In order to simplify the analysis the work, done by the force, of a single cutting edge has been considered. Moreover the contribute due the tangential force has basically taken into account.

Looking at Fig. 4.51, we can pointed out two important phases labelled respectively with the letter A and C.

During phase A the maximum positive modulation $h_{\max_A}$ (x axis) of the chip thickness occurs hence the tangential component of the force F_{t_A} (y direction)

introduces the linked energy into the machine tool by means of a tool movement along the orthogonal degree of freedom (y direction) where we can notice the maximum value of speed $\dot{y}_{\max_A}$: a positive quantity of energy therefore flows from the process to the machine tool.

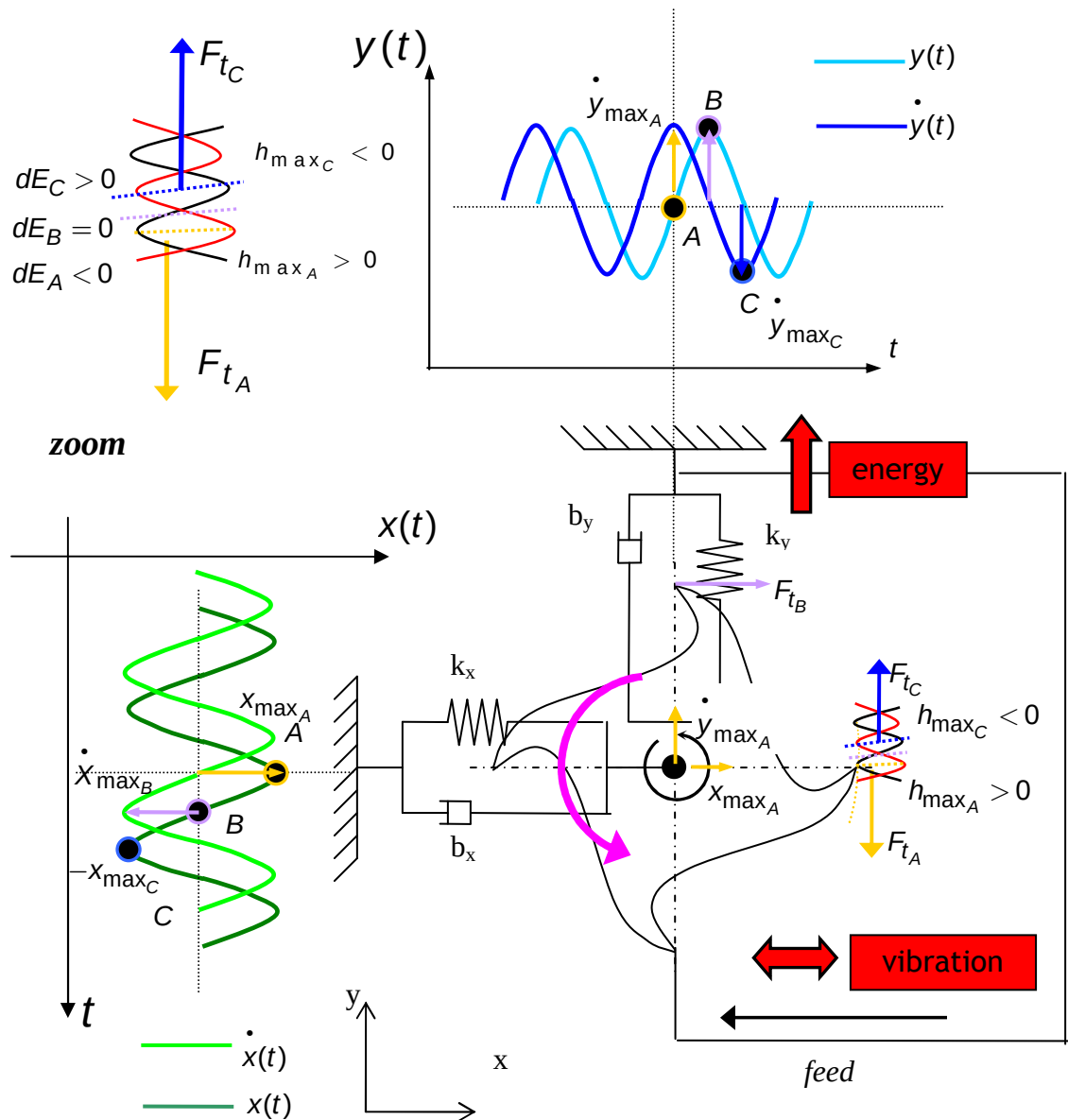


Fig. 4.51: Regenerative effect – energetic explanation

On the contrary during phase C the chip modulation is maximum and negative ($h_{\max_C} \rightarrow F_{t_C}$) but the tool centre speed of the tool along Y axis $\dot{y}_{\max_C}$ has changed verse so a positive quantity of energy flows towards the machine tool. During phase B the regenerative effect doesn't introduce energy in the system because both the chip modulation and the y speed are null: this is true if we consider only the contribute of this cutting edge.

Other cutting edge contributes amplify this effect.

This mechanism brings the system to instability and the typical frequency of the phenomenon is ω_C .

The described mechanism is very similar to the “flutter” phenomenon or instability due to a modal coupling. This unstable phenomenon is well known in Aerospace Engineering.

We can state that the regenerative effect generate cutting forces that introduce energy in the dynamic system doing work by the means of a coupled and synchronized movement of the tool tip in the plane X-Y.

The condition shown in Fig. 4.51 correspond to the worse situation from the stability point of view, this has been also gathered from an energetic interpretation.

It would be interesting to understand how the SSSV modifies this mechanism; this could be useful in order to define some guidelines to maximize the effectiveness of the VSM.

Considering the face milling operation depicted in Fig. 4.6 and using the notes formerly presented, an energetic analytical evaluation has been proposed in a similar way as done for turning operations in [75].

The issue has been approached considering the behaviour of the tool in the plane X-Y as reported in Fig. 4.52.

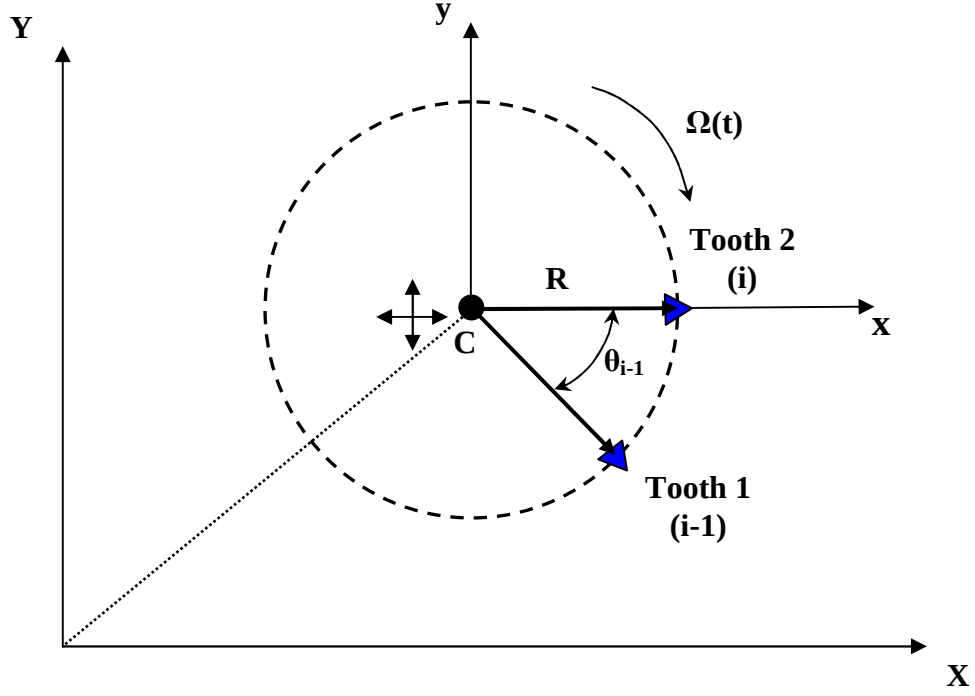


Fig. 4.52: Plane model of the cutting process

The following relationships, Eq. 4.8 and Eq. 4.9 describes the tool kinematic: the generic cutting edge (i) and the previous one (i-1) have been considered.

$$\begin{aligned}
 X_i(t) &= X_c(t) + R \cos \left(\vartheta_i + \int_0^t \Omega(t) dt \right) \\
 Y_i(t) &= Y_c(t) + R \sin \left(\vartheta_i + \int_0^t \Omega(t) dt \right) \\
 X_{i-1}(t - \tau) &= X_c(t - \tau) + R \cos \left(\vartheta_{i-1} + \int_0^{t-\tau} \Omega(t) dt \right) \\
 Y_{i-1}(t - \tau) &= Y_c(t - \tau) + R \sin \left(\vartheta_{i-1} + \int_0^{t-\tau} \Omega(t) dt \right)
 \end{aligned}$$

Eq. 4.8

But if we refer to the situation illustrated in Fig. 4.52 , Eq. 4.9 can be written. Moreover it's necessary to observe that sinusoidal functions have been used to describe the chatter vibration in X-Y plane.

It is important to assert that if we consider a VSM, the chatter frequency varies during the speed modulation thus $\omega_c = \omega_c(t)$. This aspect wasn't considered in [75]. Definitely we can write the following relationships:

Eq. 4.9:

$$\begin{aligned}
 X_c(t) &= Va \cdot t + A \sin \left(\int_0^t \omega_c dt - \frac{\pi}{2} \right) \\
 Y_c(t) &= B \sin \left(\int_0^t \omega_c dt \right) \\
 X_c(t - \tau) &= Va \cdot (t - \tau) + A \sin \left(\int_0^{t-\tau} \omega_c dt - \frac{\pi}{2} \right) \\
 Y_c(t - \tau) &= B \sin \left(\int_0^{t-\tau} \omega_c dt \right)
 \end{aligned}$$

Where:

- Va : feed [mm/sec]
- t : time [sec]
- ω_c : chatter frequency [rad/sec]
- A, B : vibration amplitudes of tool tip centre (X and Y direction) [mm]
- R : tool radius [mm]
- $\Omega(t)$: spindle speed [rad/sec]
- τ : time delay [sec]
- ϑ_2 : initial angular position cutter (i) [rad]
- ϑ_1 : initial angular position cutter (i-1) [rad]

We can assume $\vartheta_i = 0$ and obviously $\vartheta_{i-1} = -\frac{2\pi}{z}$ where z is the number of cutting edges.

$X_i(t)$ contains a term linked to the feed (Va), a term related to the vibration of the tool at chatter frequency (ω_c) and finally a term connected to the angular rotation of the tool.

$Y_i(t)$ contains only two terms: the first one linked to vibration of the tool tip and the second one related to the rotation of the tool at $\Omega(t)$.

$X_{i-1}(t-\tau)$ and $Y_{i-1}(t-\tau)$ take into account the same contributes but a time delay τ has been considered in order to model the regenerative effect.

Furthermore the composition of $X_{i-1}(t-\tau)$ and $Y_{i-1}(t-\tau)$ describes the shape of the track left by the cutting edge (i-1) that is the previous one, on the machined surface.

It is so possible to write the expression for both the tracks but it is necessary to consider the radial projection of both the cutting edge coordinates.

This can be done using the following relationships:

$$Track_i = X_i \cdot V_x + Y_i \cdot V_y + Z_i \cdot V_z$$

Eq. 4.10:

$$Track_{i-1} = X_{i-1} \cdot V_x + Y_{i-1} \cdot V_y + Z_{i-1} \cdot V_z$$

where V_x , V_y , V_z allow to project the cutter coordinates to the radial direction. Eq. 4.10 is the more complex relationship because it considers the 3-dimensional problem.

The third contribute has been neglected

We can write the following relationships:

$$\begin{aligned} V_x &= \cos\left(\vartheta_i + \int_0^t \Omega(t) dt\right) \\ V_y &= \sin\left(\vartheta_i + \int_0^t \Omega(t) dt\right) \end{aligned}$$

Eq. 4.11:

Hence Eq. 4.13 can be written.

Where $\Omega(t)$ is the sinusoidal profile used to modulate the spindle speed.

$$\text{Eq. 4.12} \quad \Omega(t) = \Omega_0 + \Omega_0 \cdot RVA \cdot \sin(\Omega_0 \cdot RVF \cdot t + \psi) \quad [\text{rad/sec}]$$

dove Ω_0 is the nominal spindle speed, RVA and RVF are the parameters that describe the spindle speed modulation technique as formerly described.

Eq. 4.13:

$$\begin{aligned} Track_i &= \left[Va \cdot t + A \sin\left(\int_0^t \omega_c dt - \frac{\pi}{2}\right) + R \cos\left(\vartheta_i + \int_0^t \Omega(t) dt\right) \right] \cdot \cos\left(\vartheta_i + \int_0^t \Omega(t) dt\right) + \\ &\left[B \sin\left(\int_0^t \omega_c dt\right) + R \sin\left(\vartheta_i + \int_0^t \Omega(t) dt\right) \right] \cdot \sin\left(\vartheta_i + \int_0^t \Omega(t) dt\right) \\ \\ Track_{i-1} &= \left[Va \cdot (t - \tau) + A \sin\left(\int_0^{t-\tau} \omega_c dt - \frac{\pi}{2}\right) + R \cos\left(\vartheta_{i-1} + \int_0^{t-\tau} \Omega(t) dt\right) \right] \cdot \cos\left(\vartheta_{i-1} + \int_0^{t-\tau} \Omega(t) dt\right) + \\ &\left[B \sin\left(\int_0^{t-\tau} \omega_c dt\right) + R \sin\left(\vartheta_{i-1} + \int_0^{t-\tau} \Omega(t) dt\right) \right] \cdot \sin\left(\vartheta_{i-1} + \int_0^{t-\tau} \Omega(t) dt\right) \end{aligned}$$

We can consider only the terms related to the chatter frequency:

$$\text{Eq. 4.14} \quad \begin{aligned} \text{Track}_i = & \left[A \sin \left(\int_0^t \omega_c dt - \frac{\pi}{2} \right) \right] \cdot \cos \left(\vartheta_i + \int_0^t \Omega(t) dt \right) + \\ & + \left[B \sin \left(\int_0^t \omega_c dt \right) \right] \cdot \sin \left(\vartheta_i + \int_0^t \Omega(t) dt \right) \end{aligned}$$

$$\begin{aligned} \text{Track}_{i-1} = & \left[A \sin \left(\int_0^{t-\tau} \omega_c dt - \frac{\pi}{2} \right) \right] \cdot \cos \left(\vartheta_i + \int_0^t \Omega(t) dt \right) + \\ & + \left[B \sin \left(\int_0^{t-\tau} \omega_c dt \right) \right] \cdot \sin \left(\vartheta_i + \int_0^t \Omega(t) dt \right) \end{aligned}$$

Eq. 4.14 has been obtained only admitting a phase shift of $\pi/2$ between the Y and X vibration of the tool centre. This consideration has come from the performed time simulations in chatter condition.

It would be interesting to verify the phase shift between the tracks and if it is possible, after having computed the energy introduced in the machine tool, optimize the parameters of the SSSV.

Considering the easiest “Constant Speed Machining” CSM:

$$\text{Eq. 4.15} \quad \Omega(t) = \Omega_0 \quad [\text{rad/sec}]$$

$$\text{Eq. 4.16:} \quad \int_0^t \omega_c dt = \omega_c \cdot t \quad \int_0^{t-\tau} \omega_c dt = \omega_c \cdot (t - \tau)$$

$$\text{Eq. 4.17:} \quad \int_0^t \Omega(t) dt = \Omega_0 \cdot t$$

The following can be obtained, assuming moreover A=B:

$$\begin{aligned} \text{Eq. 4.18:} \quad \text{Track}_i = & -\cos(\omega_c \cdot t) \cdot \cos(\vartheta_i + \Omega_0 \cdot t) + \\ & + \sin(\omega_c \cdot t) \cdot \sin(\vartheta_i + \Omega_0 \cdot t) \\ \text{Track}_{i-1} = & -\cos(\omega_c \cdot (t - \tau)) \cdot \cos(\vartheta_i + \Omega_0 \cdot t) + \\ & + \sin(\omega_c \cdot (t - \tau)) \cdot \sin(\vartheta_i + \Omega_0 \cdot t) \end{aligned}$$

If we consider a milling operation like that represented on the stability chart we obtain the tracks pointed out Fig. 4.54 that agree to the ones obtained in time domain simulations.

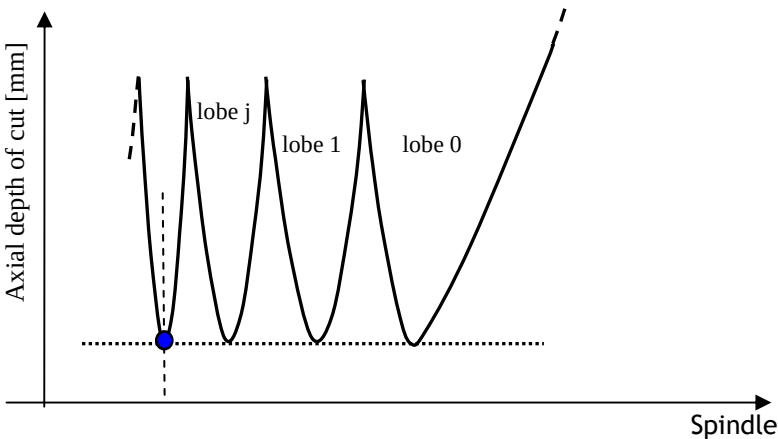


Fig. 4.53: Milling operation – worse stability condition

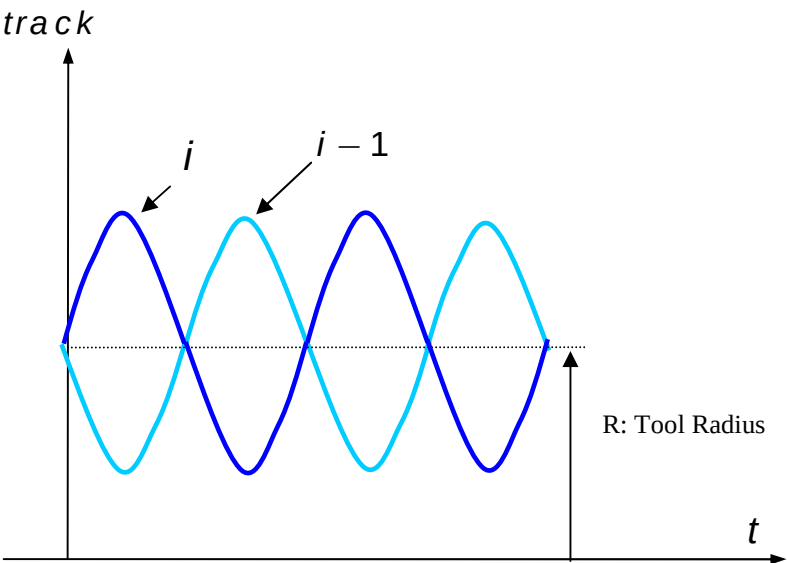


Fig. 4.54: Tracks due to the chatter vibration component

And using some trigonometric relationships:

$$\begin{aligned} \text{Eq. 4.19:} \quad & \text{Track}_i = -\cos(\omega_c \cdot t + \Omega_0 \cdot t + \vartheta_2) \\ & \text{Track}_{i-1} = -\cos(\omega_c \cdot (t - \tau) + \Omega_0 \cdot t + \vartheta_2) \end{aligned}$$

So the instantaneous chip thickness

$$\begin{aligned} \text{Eq. 4.20:} \quad & h(t) \propto \text{Track}_2 - \text{Track}_1 = \cos(\omega_c \cdot t + \Omega_0 \cdot t + \vartheta_2) - \dots \\ & -\cos(\omega_c \cdot (t - \tau) + \Omega_0 \cdot t + \vartheta_2) \end{aligned}$$

And finally the

$$\text{Eq. 4.21:} \quad \Phi = (\omega_c \cdot t + \Omega_0 \cdot t + \vartheta_2) - (\omega_c \cdot (t - \tau) + \Omega_0 \cdot t + \vartheta_2) = \omega_c \tau$$

The previous expression was obtained also using the classical frequency stability analysis, Altintas et. al. [11].

And for CSM:

$$\text{Eq. 4.22:} \quad \Phi = 2\pi f_c \cdot \frac{2\pi}{z \cdot \Omega_0} \quad [\text{rad}]$$

Where the chatter frequency values depends on the instantaneous value of the spindle speed. z is the number of cutting edges.

For milling operations close to the minimum of each lobe the $\Phi \approx \pi$.

It is also possible to analyze the phase shift considering the speed modulation technique but the relationship is approximated as formerly mentioned.

$$\text{Eq. 4.23:} \quad \Omega(t) = \Omega_0 + \Omega_0 \cdot RVA \cdot \sin(\Omega_0 \cdot RVF \cdot t + \psi) \quad [\text{rad/sec}]$$

Substituting $\Omega(t)$ in Eq. 4.21, the following can be obtained

$$\text{Eq. 4.24:} \quad \Phi = \omega_c(t) \cdot \frac{2\pi}{z \cdot [\Omega_0 + \Omega_0 RVA \cdot \sin(\Omega_0 RVF \cdot t + \psi)]}$$

Starting from an energetic approach for turning proposed in a previous work [75] we have decided to extend this approach to milling operations. The goal is to use this approach to choose the SSSV parameters mix that maximize the chatter suppression effect of VSM.

The goal is to minimize the energy that comes from the cutting process and flows towards the machine tool system.

In this first analysis, the full engagement face milling has been considered.

Eq. 4.23 describes how to compute the work done from the cutting process.

It is possible to further simplify the issue by considering the single cutting edge contribute and we can take into account, even if it isn't a realistic cutting condition, that the selected cutting edge is cutting for all time considered T .

Eq. 4.25:
$$E_{IN} = \int_0^{T=2n\pi/\omega} (\vec{F}_{i,T} \cdot \vec{v}_i) dt \quad [J]$$

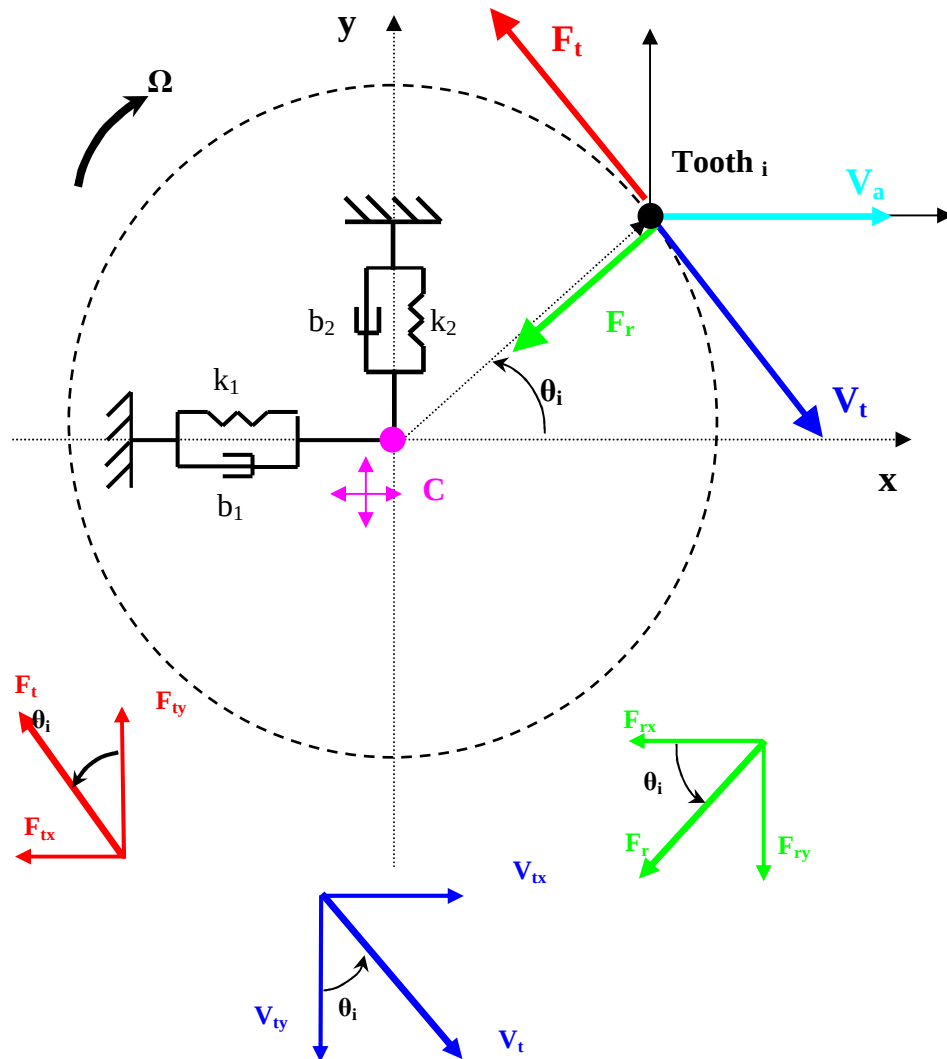


Fig. 4.55: Cutting force plane resolving

Where n is the number of cycles of the tool tip vibration. In order to compute the quantity described in Eq. 4.25 it is necessary to resolve the cutting forces into their components as reported in Fig. 4.55

It is important to remember that only the forces on the plane X-Y have been considered because the regenerative mechanism occurs basically in that plane

The $F_{i,T}$ can be resolved using Eq. 4.28:

$$\text{Eq. 4.26} \quad F_{i,T} = F_{i,TX} + F_{i,TY} + F_{i,TZ} \quad [\text{N}]$$

Where:

$$\begin{aligned} F_{i,TX} &= F_{t,iX} + F_{r,iX} \\ \text{Eq. 4.27:} \quad F_{i,TY} &= F_{t,iY} + F_{r,iY} \quad [\text{N}] \\ F_{i,TZ} &= 0 \end{aligned}$$

And

$$\begin{aligned} F_{t,iX} &= -F_{t,i} \cdot \sin(\vartheta_i + \int_0^t \Omega(t) dt) \\ F_{t,iY} &= F_{t,i} \cdot \cos(\vartheta_i + \int_0^t \Omega(t) dt) \\ \text{Eq. 4.28:} \quad F_{r,iX} &= -F_{r,i} \cdot \cos(\vartheta_i + \int_0^t \Omega(t) dt) \\ F_{r,iY} &= -F_{r,i} \cdot \sin(\vartheta_i + \int_0^t \Omega(t) dt) \end{aligned} \quad [\text{N}]$$

As described in 4.3 the cutting forces can be evaluated using the following relationship:

$$\text{Eq. 4.29:} \quad F_{t,i} = K_{t,c} \cdot b \cdot h(t) + K_{t,e} \cdot b \quad [\text{N}]$$

$$\text{Eq. 4.30:} \quad F_{r,i} = K_{r,c} \cdot b \cdot h(t) + K_{r,e} \cdot b \quad [\text{N}]$$

The instantaneous chip thickness depends on how the tracks left by the cutters are modulated:

$$\text{Eq. 4.31} \quad h(t) \approx (X_i - X_{i-1}) \cdot \cos(\vartheta_2 + \int_0^t \Omega(t) dt) - (Y_i - Y_{i-1}) \cdot \sin(\vartheta_2 + \int_0^t \Omega(t) dt)$$

As formerly explained we are interesting only in the components linked to the chatter vibration.

$$\text{Eq. 4.32: } h(t) = \left[Va \cdot \tau + A \sin \left(\int_0^t \omega_c dt - \frac{\pi}{2} \right) - A \sin \left(\int_0^{t-\tau} \omega_c dt - \frac{\pi}{2} \right) \right] \cdot \cos(\vartheta_i + \int_0^t \Omega(t) dt) \\ - \left[B \sin(\omega_c \cdot t) - B \sin(\omega_c \cdot (t - \tau)) \right] \cdot \sin(\vartheta_i + \int_0^t \Omega(t) dt)$$

And differentiating the tool tip position:

$$\text{Eq. 4.33: } v_{i,x} = \frac{dX_C(t)}{dt} = v_{C,X} \quad v_{i,y} = \frac{dY_C(t)}{dt} = v_{C,Y} \text{ [m/s]}$$

So we can compute the power and the corresponding energy

$$\text{Eq. 4.34: } P_{IN} = (F_{t,iX} + F_{r,iX}) \cdot v_{C,X} + (F_{t,iY} + F_{r,iY}) \cdot v_{C,Y} \quad [\text{W}]$$

$$\text{Eq. 4.35: } E_{IN} = \int (P_{IN}) dt \quad [\text{J}]$$

Considering the Functional $E_{IN} = f(RVA, RVB)$ it has been observed that is not possible to minimize it in an explicit way and obtain the best RVA/RVB combination because ω_c depends on $\Omega(t)$ so only a numeric method can be used to compare different SSSV profiles.

We have decided to approach this issue using directly the cutting process model formerly described because this model can deal with more general cutting conditions. The relationship that have been used to compute the energy introduced into the system are Eq. 4.33, Eq. 4.34. Obviously only the component of the cutting forces due to the vibration has been considered.

The different RVA/RVF parameter combinations of the SSSV will be compared using the energy from the cutting process.

Moreover some further considerations about the mechanism the speed modulation uses to change the relationship between the tracks left on the machined surfaces and consequently the introduction of energy in the system will be presented.

In Fig. 4.56 the energetic interpretation of the cutting instability that considers also the nominal component of chip thickness has been presented.

It is necessary to observe that the analyzed cutting condition is similar to that presented in Fig. 4.53 that is a high order lobe has been considered.

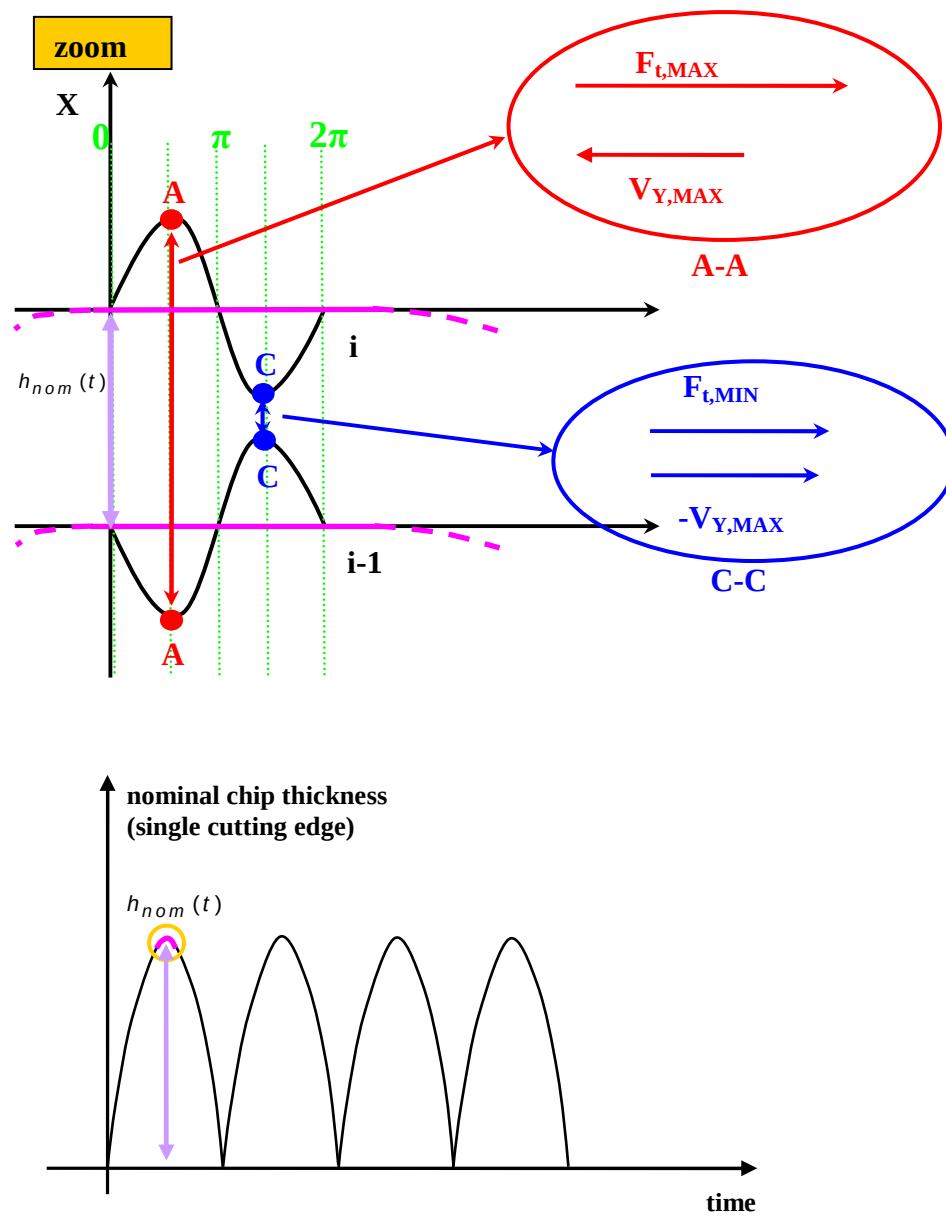


Fig. 4.56: Regenerative effect – energetic explanation – nominal chip thickness contribute

4.6 Energetic simulations analysis

As formerly stated there is a relationship between the mechanism that modulates the instantaneous chip thickness and the vibrational energy that flows from the cutting process through the machine-tool dynamic system.

In this paragraph the effects of the Sinusoidal Spindle Speed variation on both the vibrational component of the instantaneous chip thickness and the related cutting energy will be investigated comparing different RVA/RVF parameters combinations.

Furthermore a more detailed analysis of the interaction between the eigenmodes, obviously considering the case of complex machine dynamics, will be interpreted.

4.6.1 Instantaneous chip thickness modulation – SSSV effects

In order to analyze the effects of the SSSV on the chip shape modulation a meaningful milling operation was simulated.

The analyzed milling is represented on the stability chart obtained from time domain simulations.

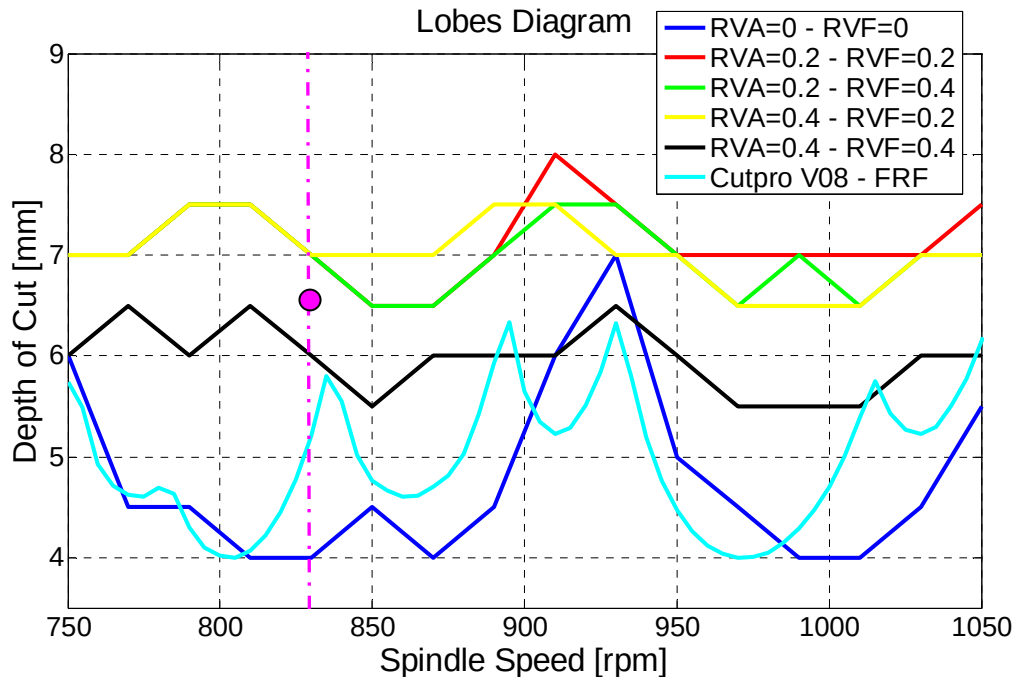


Fig. 4.57: Stability Lobes diagram – VSM –four eigenmodes

The nominal spindle speed $\Omega_0 = 830 \text{ rpm}$ and the axial depth of cut $b = 6.5 \text{ mm}$ are the used cutting parameters.

The stability lobes diagrams have been obtained considering four eigenmodes for both the axis (X-Y).

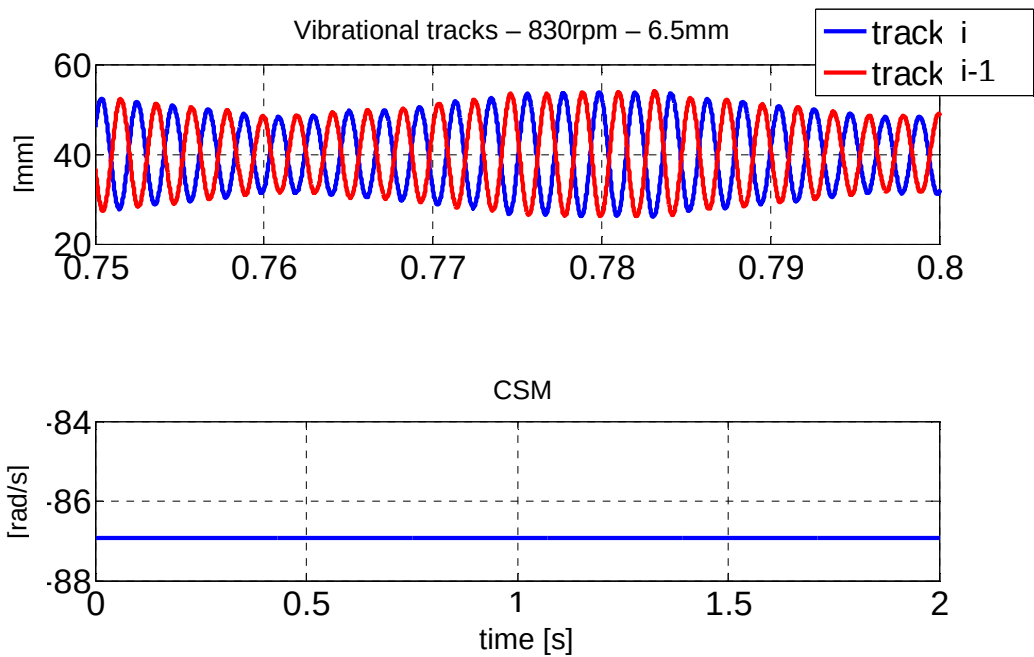


Fig. 4.58: Instantaneous chip thickness modulation - CSM

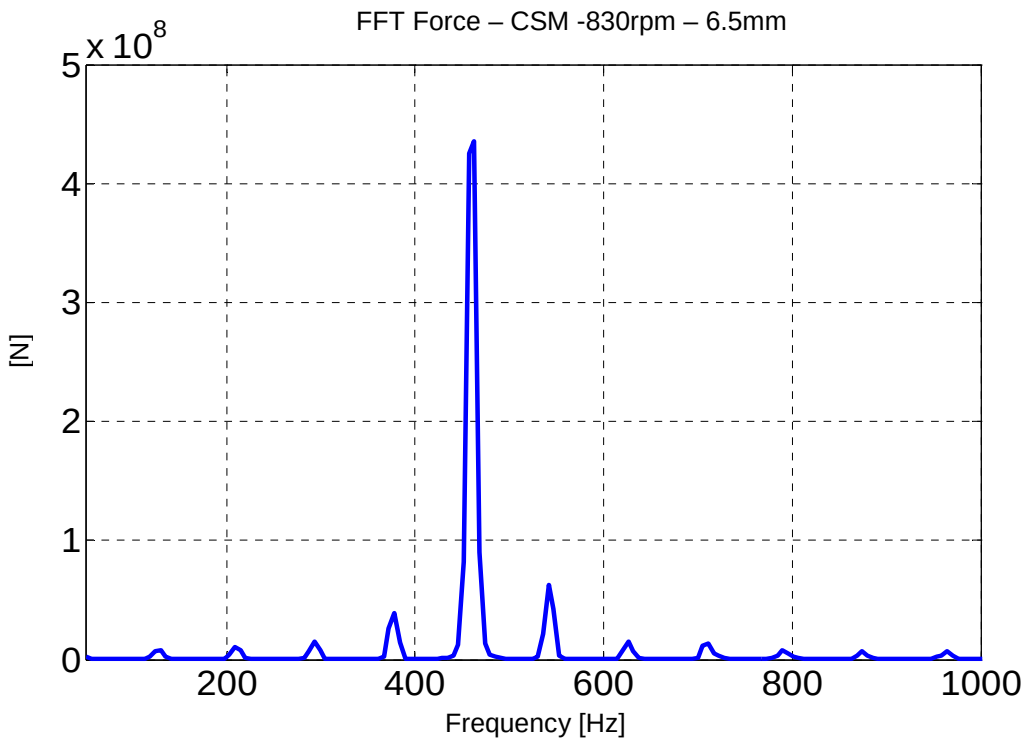


Fig. 4.59: Cutting force spectrum - CSM

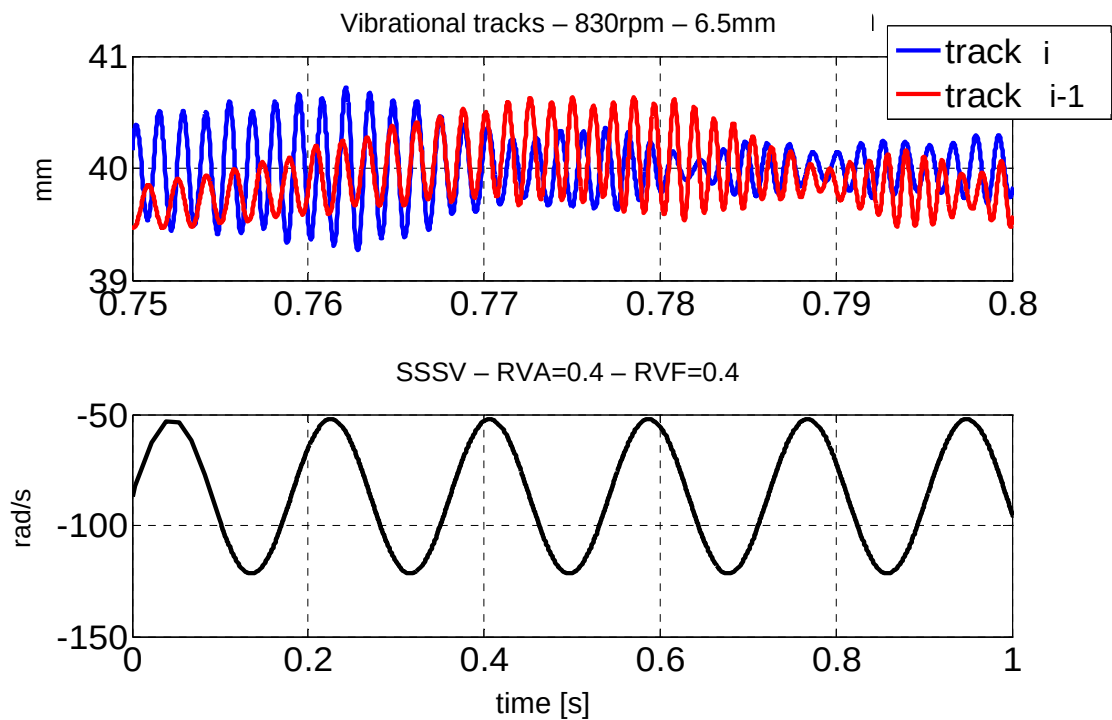


Fig. 4.60: Instantaneous chip thickness modulation – SSSV – RVA=0.4 – RVF=0.4

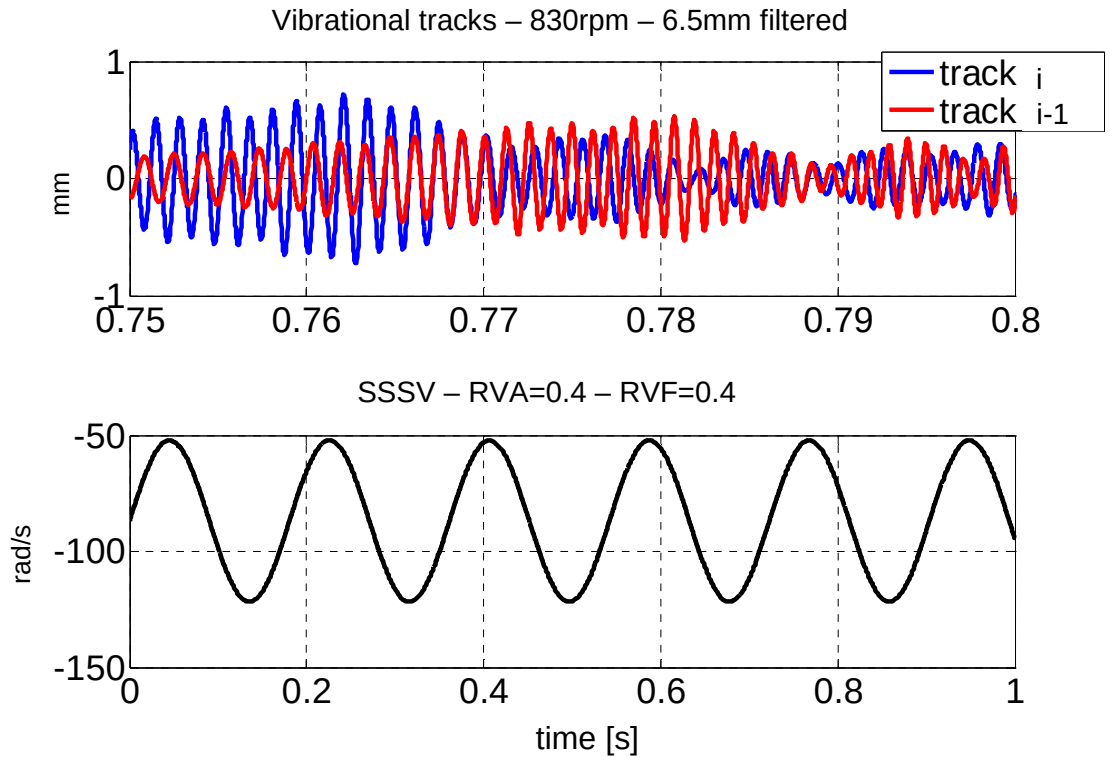


Fig. 4.61: Instantaneous chip thickness modulation – SSSV – RVA=0.4 – RVF=0.4 (filtered)

From the stability chart we can deduce that this milling operation originates a instable cutting if a CSM or a $RVA=RVF=0.4$ combination are selected. The other analyzed SSSV parameters combinations causes stable cuttings.

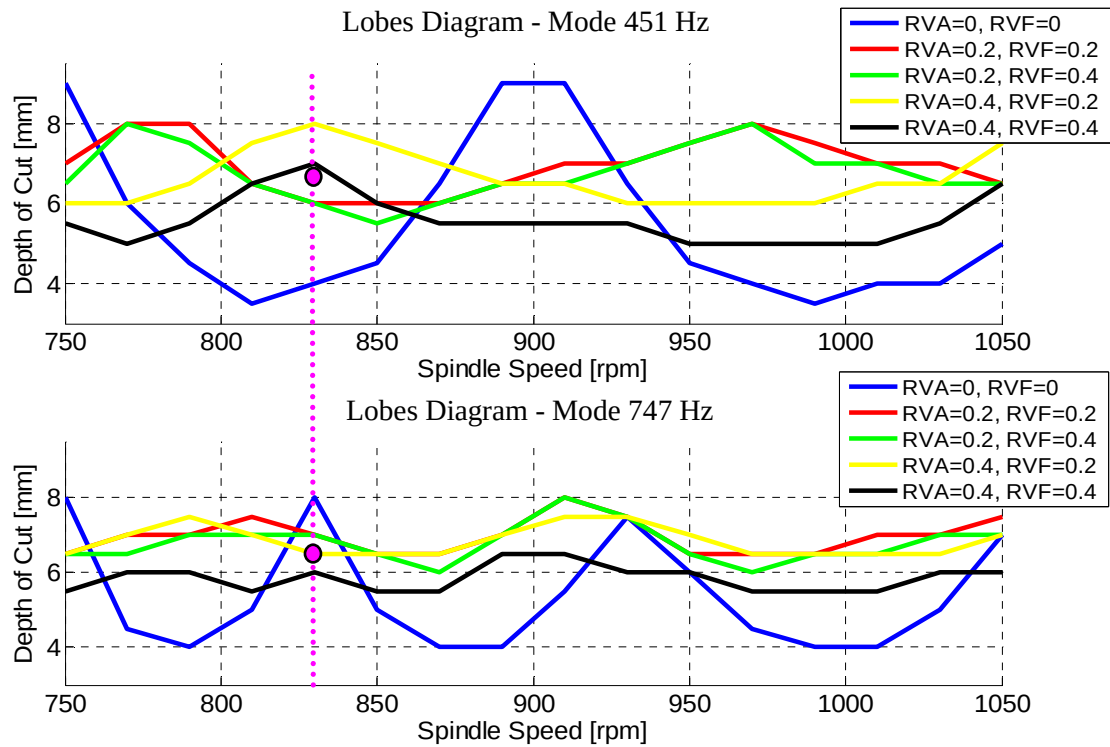


Fig. 4.62: Contribute of the two dominant modes – SSSV effects.

It is necessary to premise that stability chart reported in Fig. 4.57 has been lead basically by the behaviour of the mode at @450Hz and of the one at @750Hz, other two modes aren't so important.

The effects of the SSSV on each mode have been presented in Fig. 4.62.

In Fig. 4.58 the tracks related to the actual cutting edge and that linked to the previous one have been reported.

We can observe the chip thickness modulation. This milling involves instable cutting. The chip modulation corresponds to those linked to the worse cutting condition as formerly explained by Fig. 4.51.

This means that the responsible of the instability is the eigenmode @450 Hz because the nominal spindle speed corresponds to a minimum of the lobes diagram due to this mode.

This has been confirmed by a Fourier analysis of the cutting forces, Fig. 4.59.

Simulating the milling using a $RVA=RVF=0.4$ we have obtained Fig. 4.60 and Fig. 4.61.

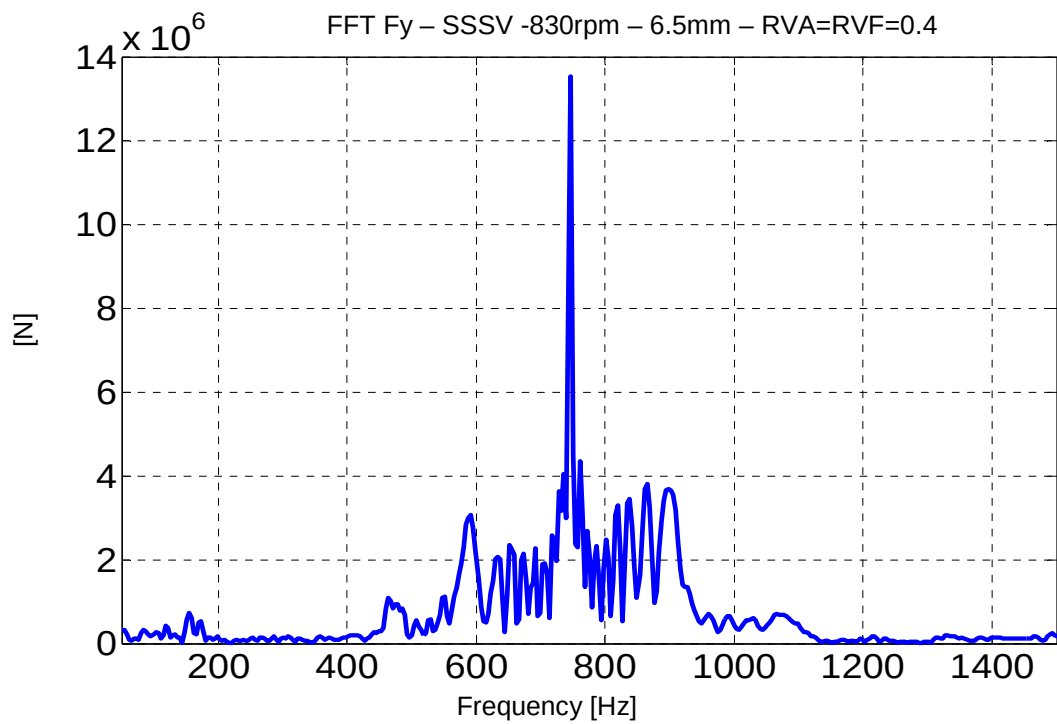


Fig. 4.63: Cutting force spectrum – SSSV – RVA=0.4, RVF=0.4

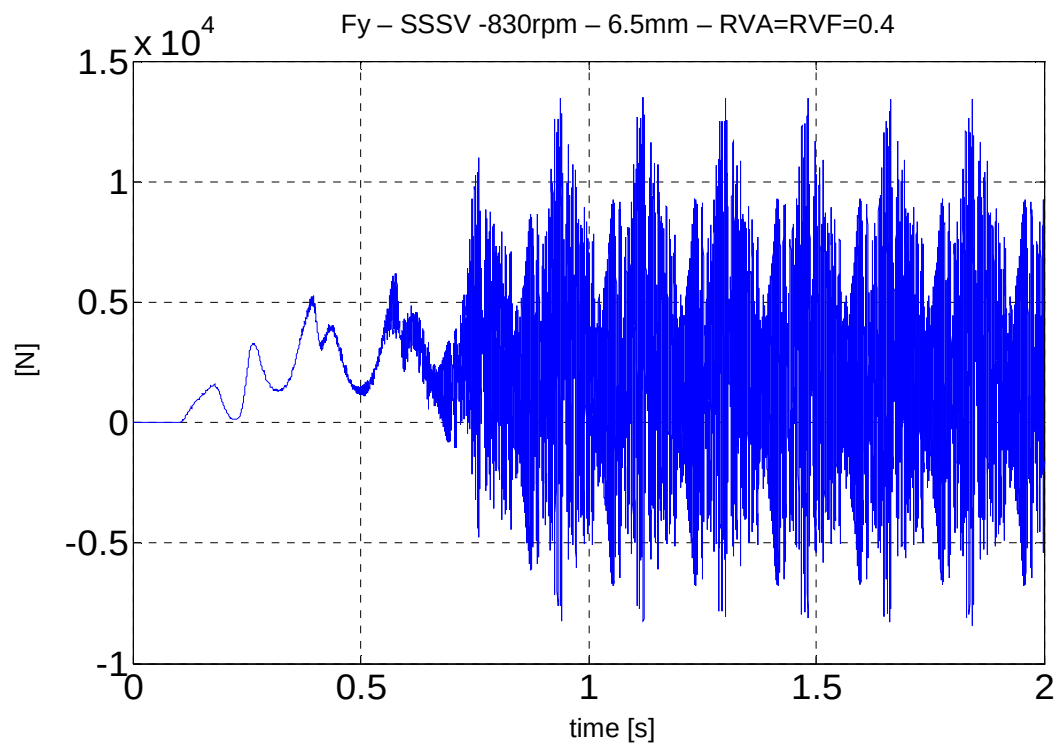


Fig. 4.64: Cutting forces– SSSV – RVA=0.4, RVF=0.4 – unstable cutting

We can note that the milling is instable, Fig. 4.64, but it is important to observe that the chatter frequency has changed value comparing to the CSM case so we can deduce that the SSSV can modify the contribute of each mode to the stability chart.

So we can conclude that it is possible to predict the behaviour of a system with a complex dynamics by considering the contributes of each single mode also using a VSM.

This topic hasn't been adequately analyzed in the specific literature.

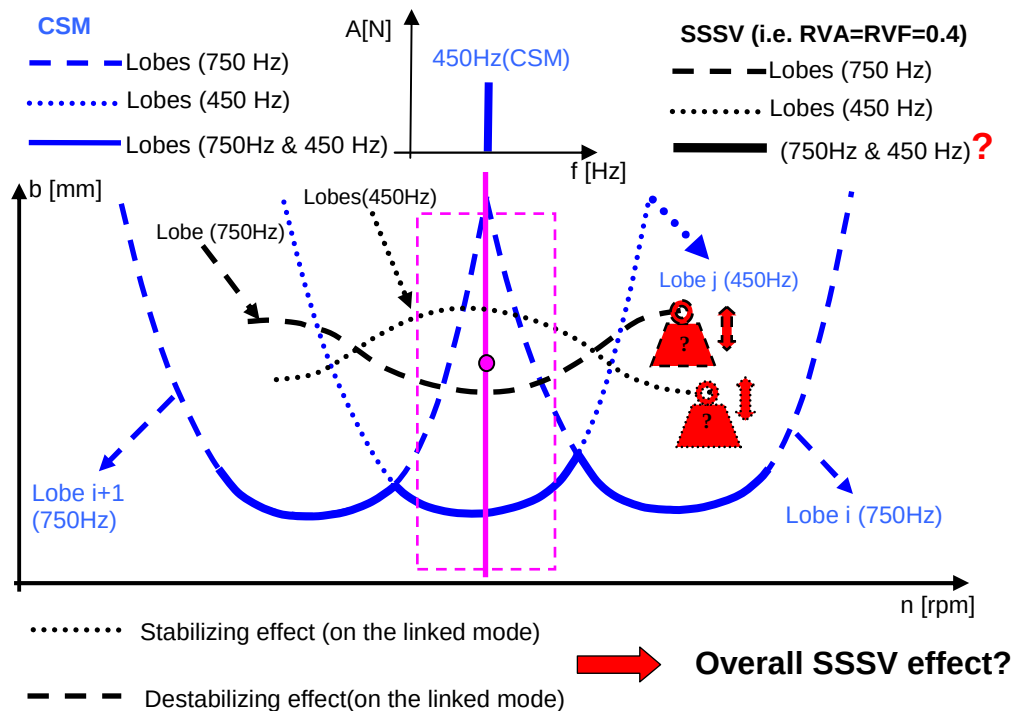


Fig. 4.65: SSSV effects on each single mode

The interaction due to the presence of different eigenmodes mechanism is graphically explained in Fig. 4.65 and Fig. 4.66.

As previously described in CSM the limiting contributes to the cutting stability is provided by the mode @450Hz.

In Fig. 4.65 the effects of the SSSV ($RVA=RVF=0.4$) on both modes have been presented. The VSM has a stabilizing effect for the mode @450 Hz (the nominal speed is in proximity of the minimum of the lobe) but destabilizing effect on the other dominant mode.

The global effect of the SSSV on the cutting process stability probably depends on the weight linked to each contribute.

For the analyzed case (RVA/RVF) the destabilizing effects has lead the global behaviour.

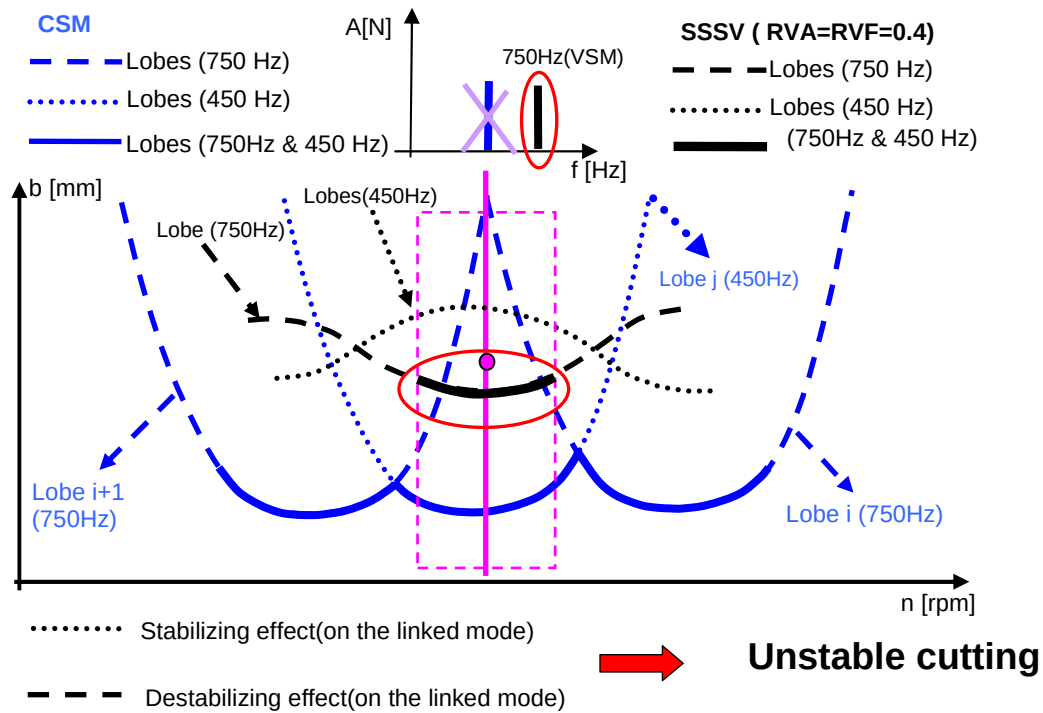


Fig. 4.66: SSSV global effect on the cutting process stability

In Fig. 4.67 and Fig. 4.68 the simulated results obtained using the $RVA=0.4/RVF=0.2$ SSSV are reported.

This parameters combinations has stabilized the cutting.

We can observe an important low frequency component but any component linked to the regenerative effects.

Referring to the considerations formerly reported in this case the stabilizing effect provided on the mode @450Hz by VSM predominates.

Probably for this axial depth of cut and for the used SSSV parameters any contribute linked to corresponding eigenmode doesn't become crucial and thus the cutting is stable.

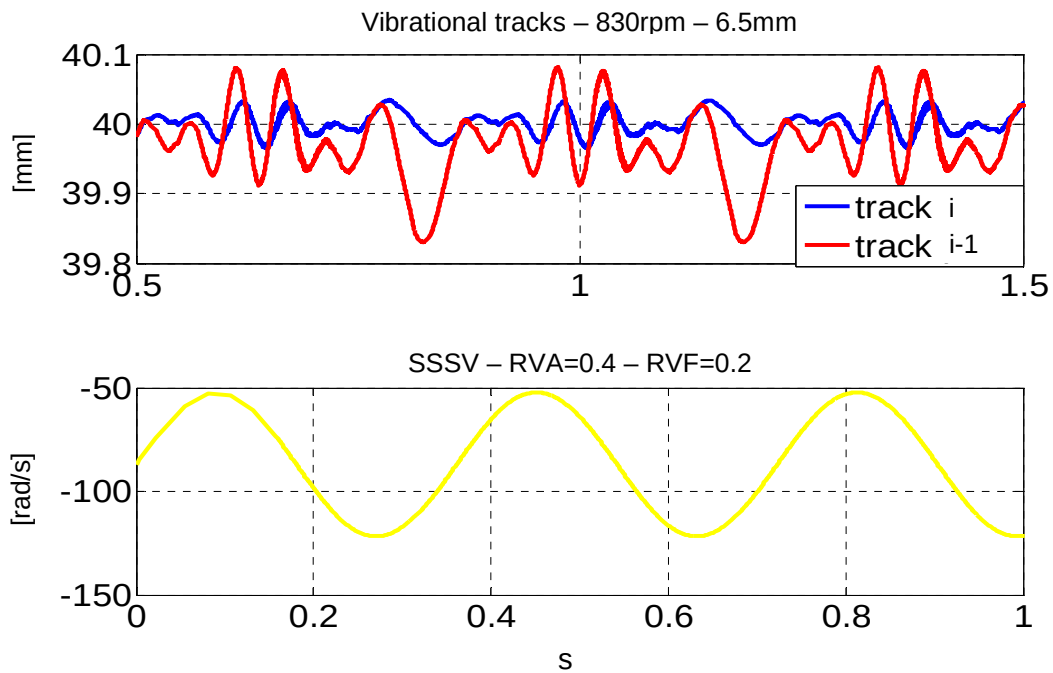


Fig. 4.67: Instantaneous chip thickness modulation – SSSV – RVA=0.4 – RVF=0.2

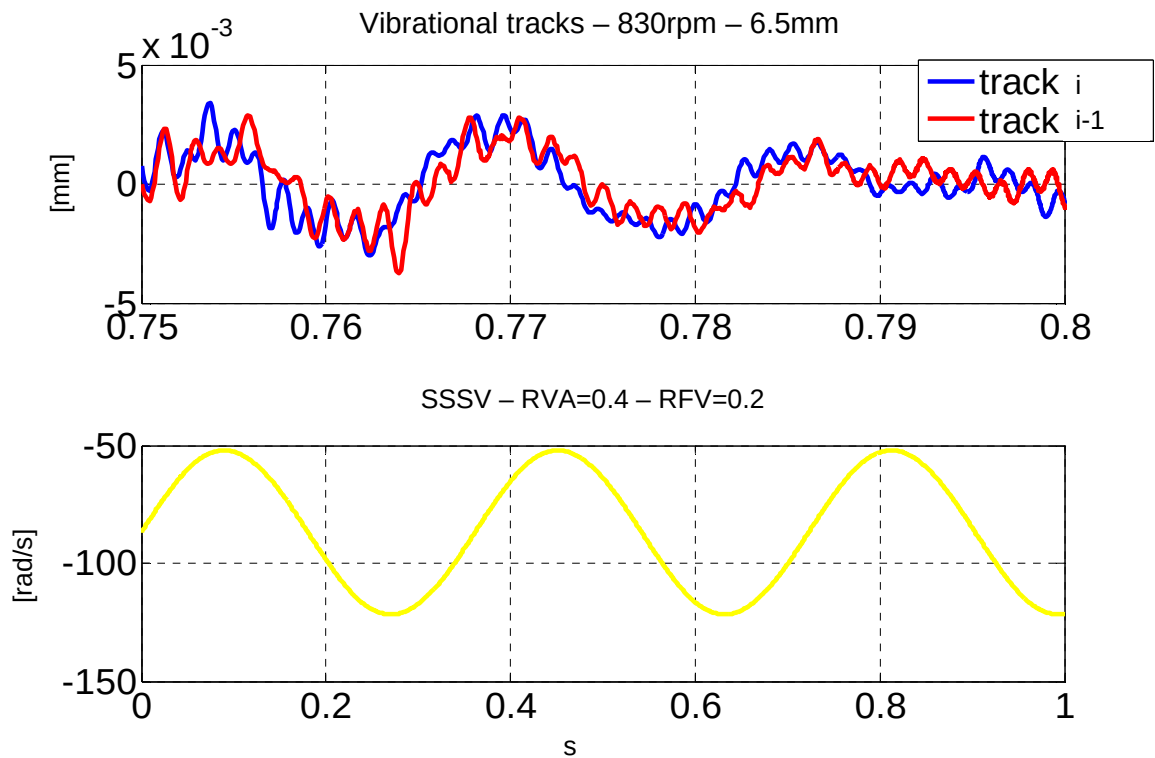


Fig. 4.68: Instantaneous chip thickness modulation – SSSV – RVA=0.4 – RVF=0.2 - zoom

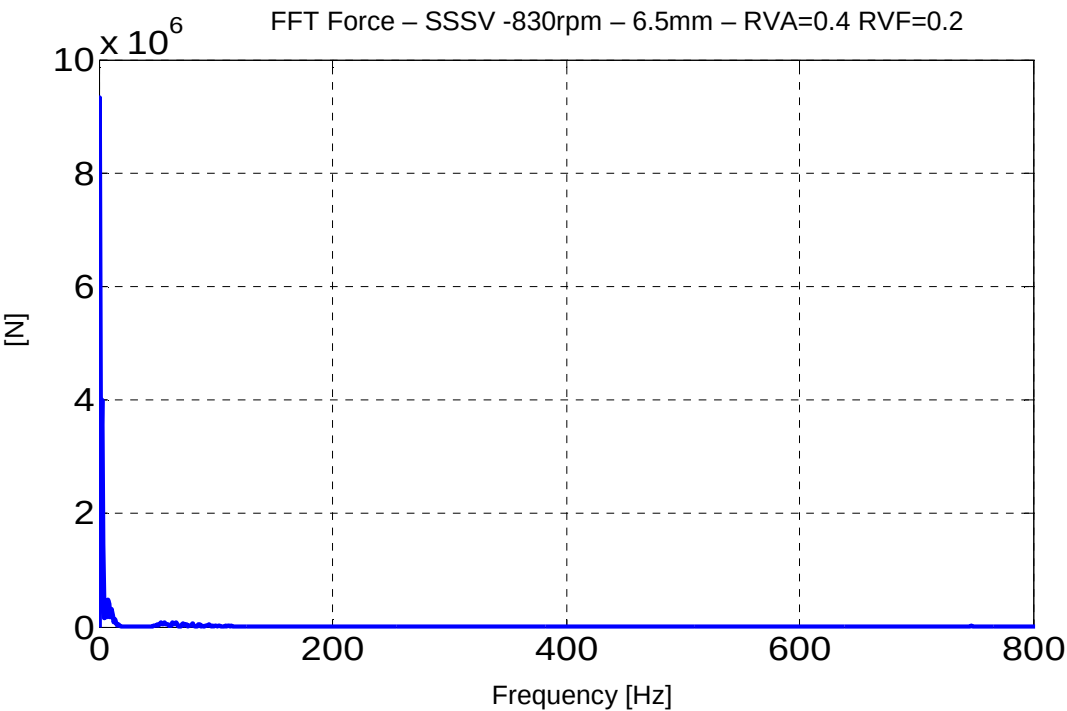


Fig. 4.69: FFT Cutting force – SSSV – RVA=0.4,RVF=0.2

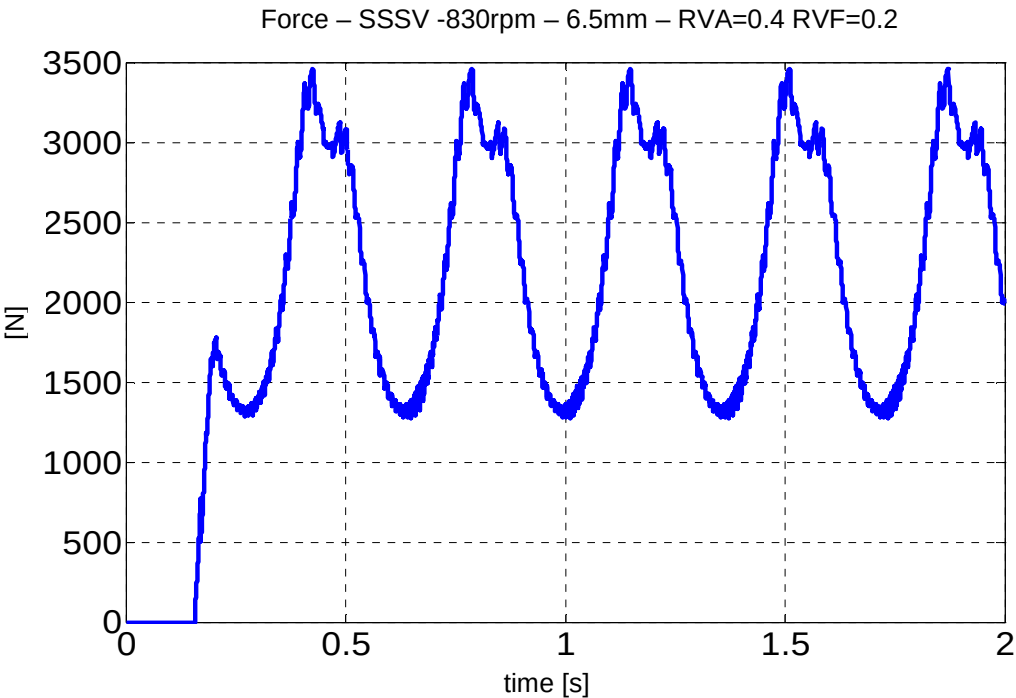


Fig. 4.70: Cutting forces – SSSV – RVA=0.4,RVF=0.2 – stable cutting

4.6.2 Vibrational Energy computation – SSSV effects

In this section a energetical interpretation of the cutting instability will be presented underlining the effects of the speed modulating technique.

This analysis strengthens the observations drawn from the 4.6.1.

We can state that there is a strong relationship between the energetical interpretation of the phenomenon and how the variable time delay affects the instantaneous chip thickness.

In this section a comparison between the analyzed SSSV profiles will be presented analyzing the results from an energetical point of view.

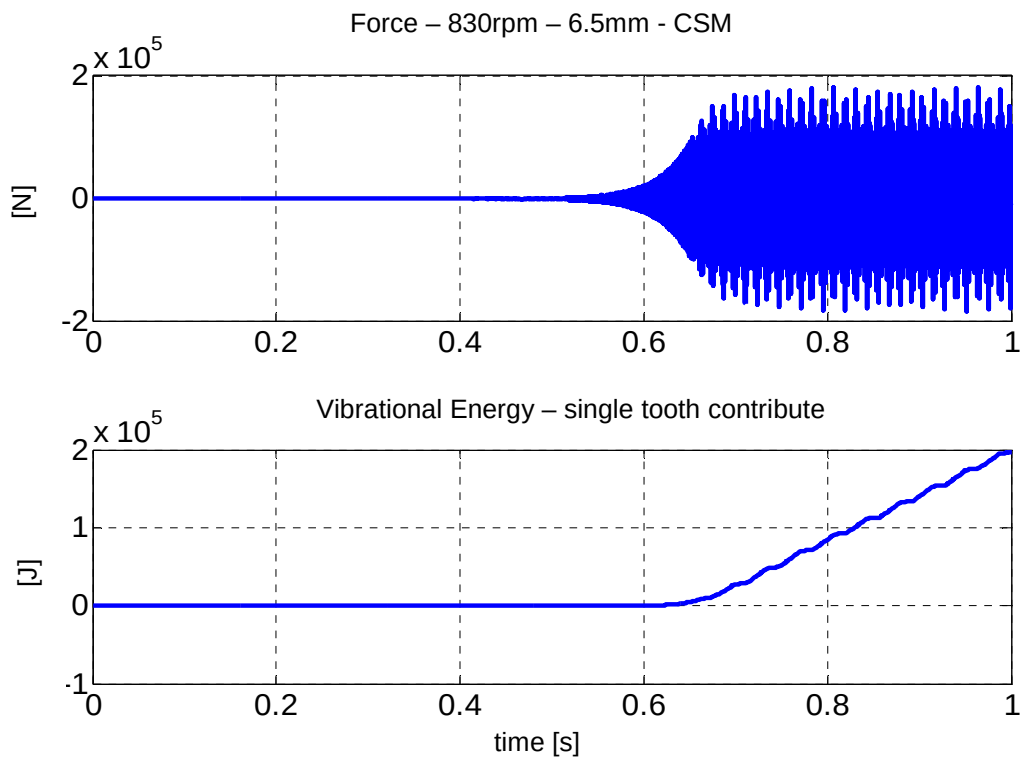


Fig. 4.71: Vibrational energy - single cutter contribute – CSM

Fig. 4.71 shows the vibrational energy that flows from the cutting process to the machine-tool during a unstable cutting. The computed energy is referred to a single cutting edge. In this cutting condition the regenerative effect introduces energy in the system that can't sufficiently dissipate it so the system is inevitably brought to instability.

In Fig. 4.72 the work introduced in the system by the cutting forces during a tooth passing has been underlined.

If we consider that more cutting edges are simultaneously working this effect will be obviously amplified.

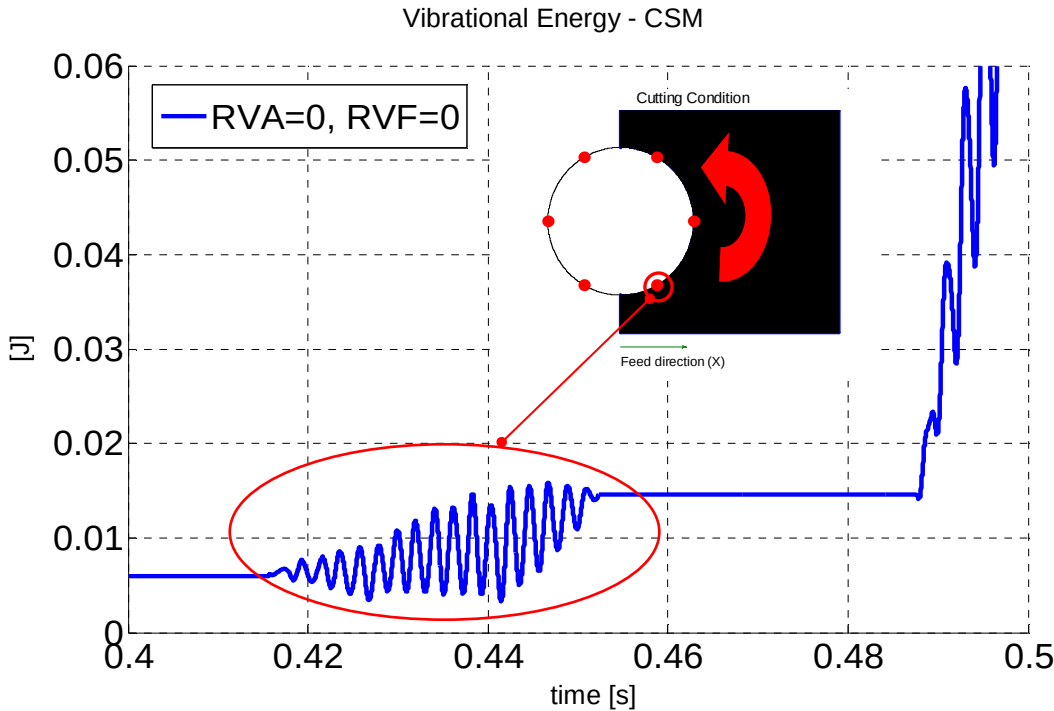


Fig. 4.72: Vibrational energy single cutter – CSM - zoom

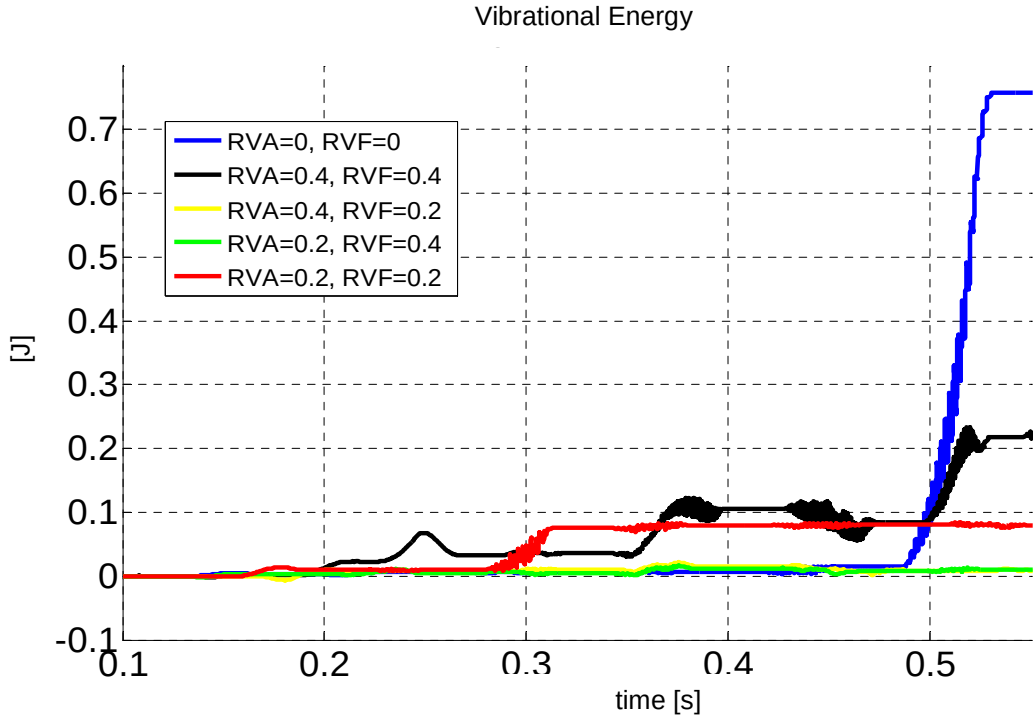


Fig. 4.73: Vibrational energy – SSSVs comparison

Analyzing both Fig. 4.73 and Fig. 4.74 it can be observed that during the unstable milling the energy is obviously growing up to a very high level (i.e. $RVA=RVF=0.4$ and CSM).

Moreover during stable cutting the vibrational energy introduced in the machine tool system is limited but we can observe different levels that have been occurred depending on the SSSV trajectories used.

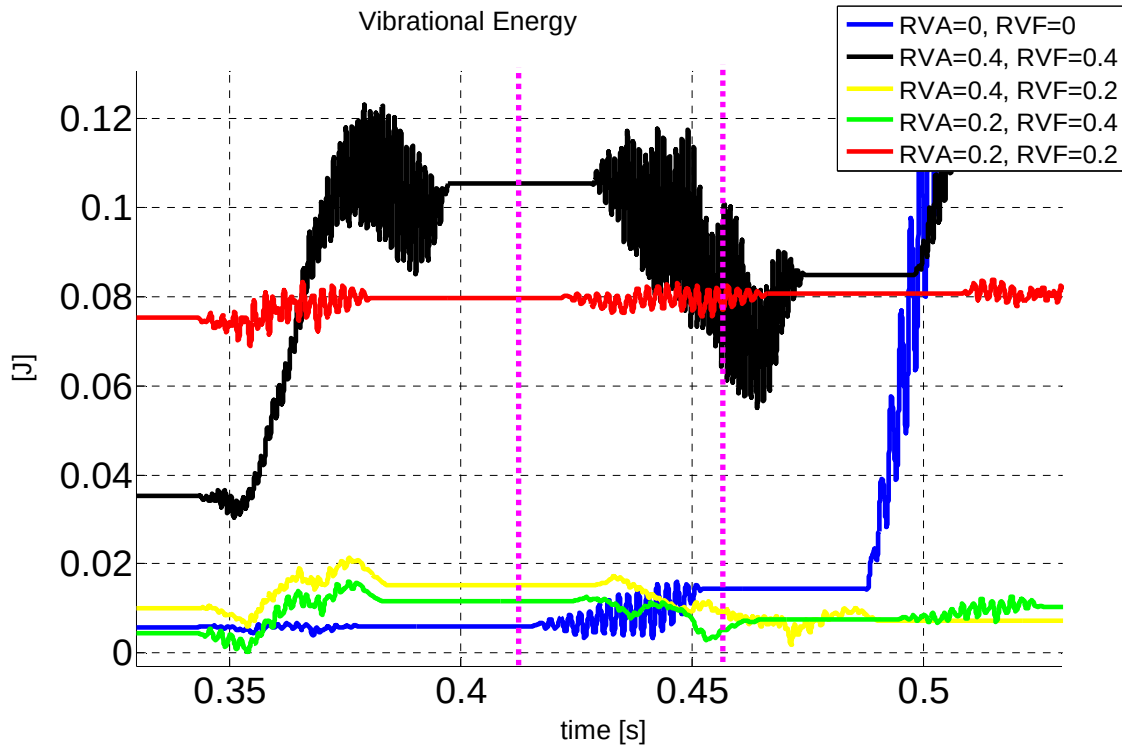


Fig. 4.74: Vibrational energy – SSSVs comparison - zoom

Looking at Fig. 4.57, if we consider the analyzed nominal spindle speed, we can conclude that the cases ($RVA=RVF=0.2$, $RVA=0.2/RVF=0.4$ and $RVA=0.4/RVF=0.2$) seem equivalent if we focus on the stability. It is necessary to recall that the step we used as axial depth increment in the time simulations is equal to 0.5mm.

We can assert that if we consider basically an energetical point of view the stable cutting milling operations are not completely equivalent so this analysis would suggest to choose the SSSV profile that minimize the energy introduced in the system. In this case the better analyzed SSSVs are both the $RVA=0.2/RVF=0.4$ one and the $RVA=0.4/RVF=0.2$ one. Furthermore we can state that ($RVA=RVF=0.2$) SSSV stabilizes the cutting and minimizes the spindle speed modulation but it doesn't minimize the work done by the cutting forces.

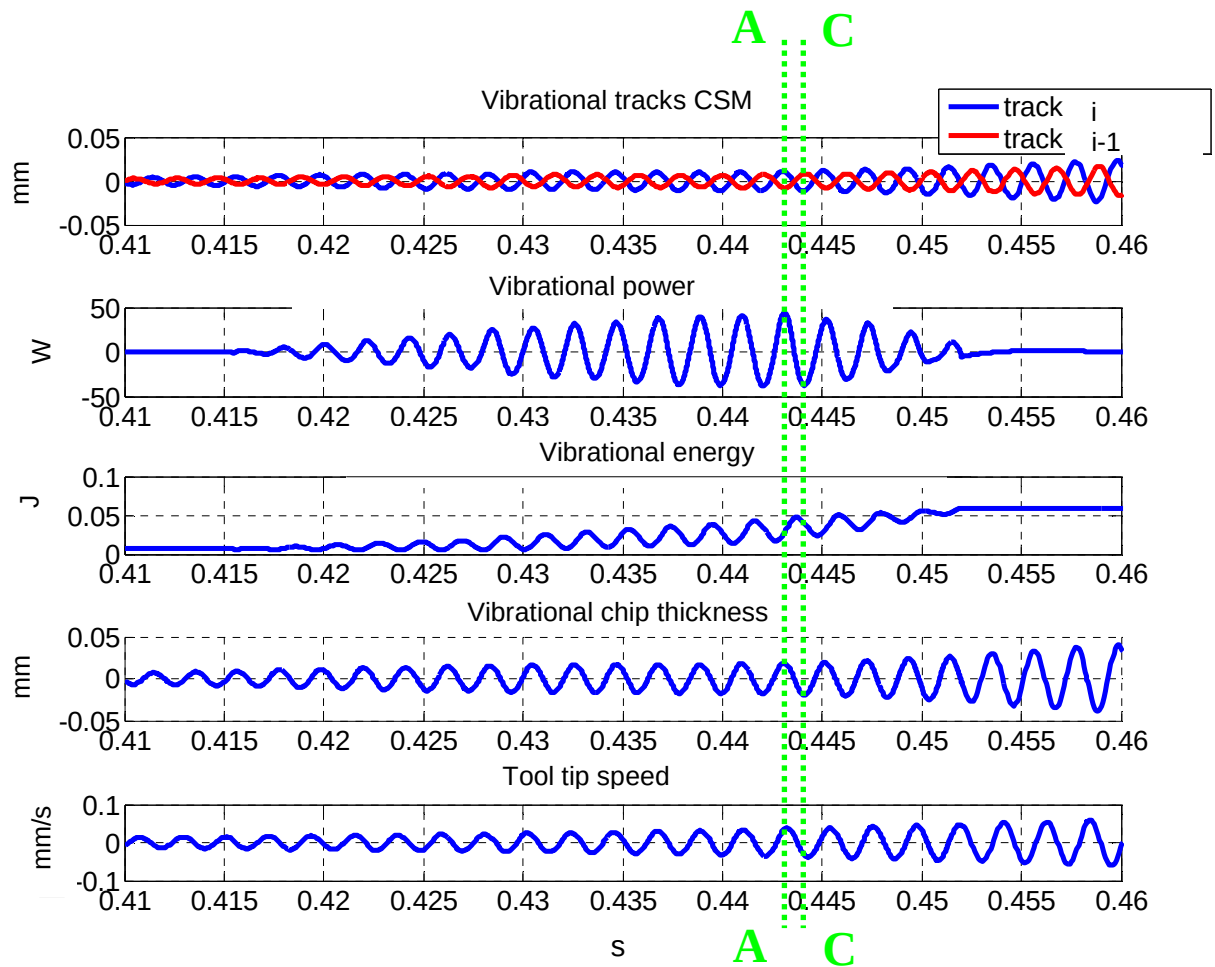


Fig. 4.75: Chip thickness modulation and vibrational energy - CSM

Nevertheless the results reported in Fig. 4.74 a general principle that can be used to select the most proper SSSV profile, choosing it from different stables one is the minimization of the speed modulation. Using this criterion a minimal excursion of the cutting forces, tool tip displacement and spindle power necessary for VSM have been guaranteed. We know that the tool tip displacement strictly affects the quality of the machined surfaces so it is necessary to limit the amplitude of the low frequency component that SSSV introduces.

As formerly mentioned we have found a dualism between the instantaneous chip thickness and the energy introduced in the dynamic system and consequently the stability of the cutting process.

Comparing following pictures it is possible to underline how the speed modulating instantaneously changes the relationship between the “track i ” and the “track $i-1$ ” and thus the mechanism explained in Fig. 4.51 and reviewed in Fig. 4.75.

Despite the ($RVA=RVF=0.4$ – maximum modulation) SSSV alters the worse chip thickness modulation pattern we can individuate the critical phase shift in some portions of the sub-graph, Fig. 4.76.

The alteration of the worse chip modulation pattern mechanism has been got more effective by the other analyzed RVA/RVF combinations.

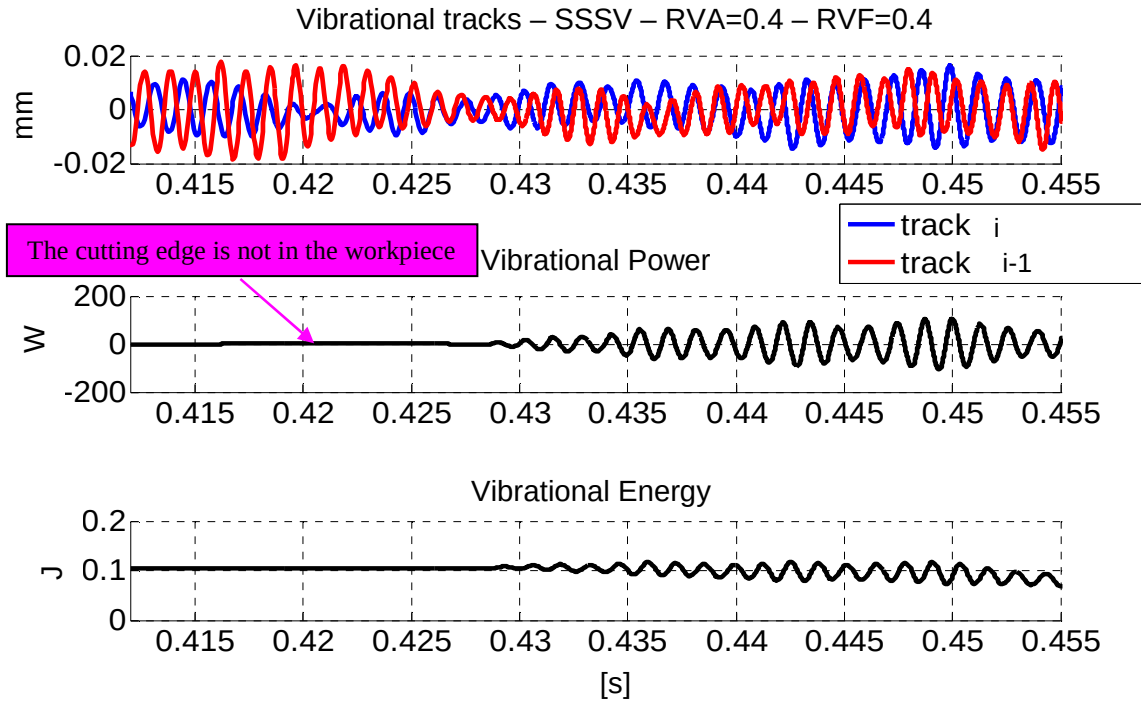


Fig. 4.76: Chip thickness modulation and Vibrational energy – ($RVA=RVF=0.4$)

In Fig. 4.77, Fig. 4.78, Fig. 4.79 we can't clearly recognize the critical phase shift. In some points of the tracks diagram we can detect that the instantaneous phase shift is equal to π but this condition doesn't endure for long time and moreover the vibration amplitude is very low so the vibrational work done by the cutting forces is not critical for the stability.

The idea to optimize the selection of the SSSV parameters could be based on how the speed modulation changes the critical phase shift pattern. The best parameters selection criterion could be the minimization of the time interval in which the worse phase shift occurs during the speed modulation.

For the analyzed full engagement face milling operation the worse instantaneous phase shift is equal to π but if we want to generalize the optimization procedure a detailed analysis of the critical phase shift is necessary. It will be proposed in the following paragraph.

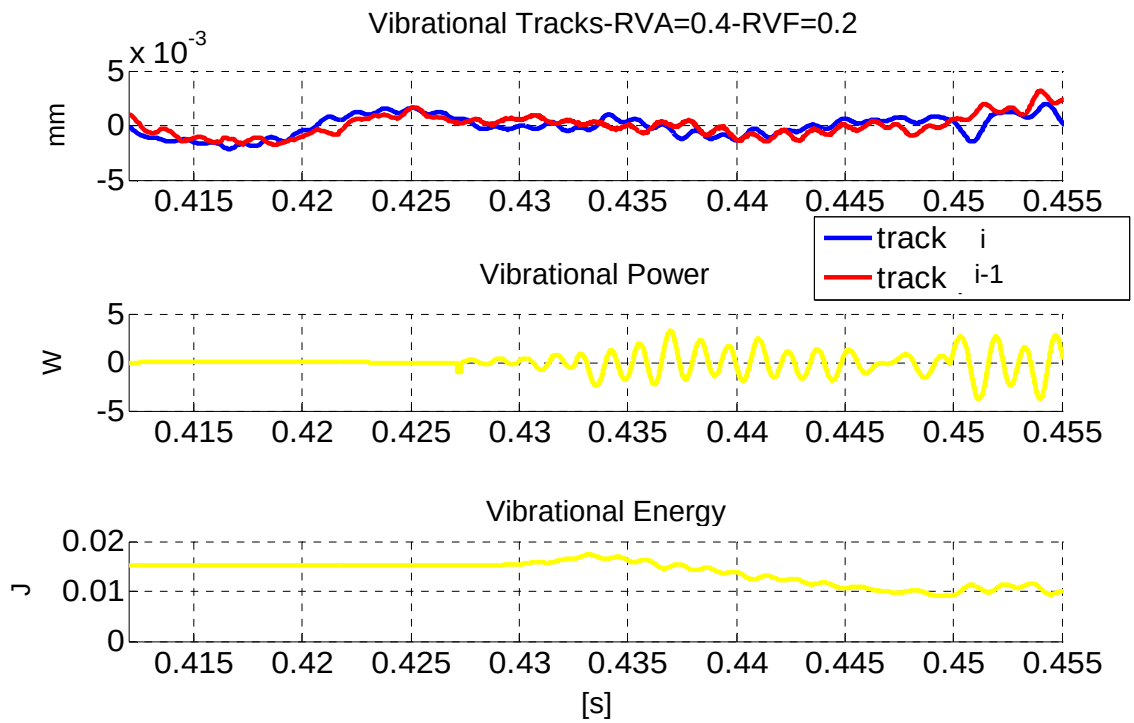


Fig. 4.77: Chip thickness modulation and Vibrational energy – (RVA=0.4,RVF=0.2)

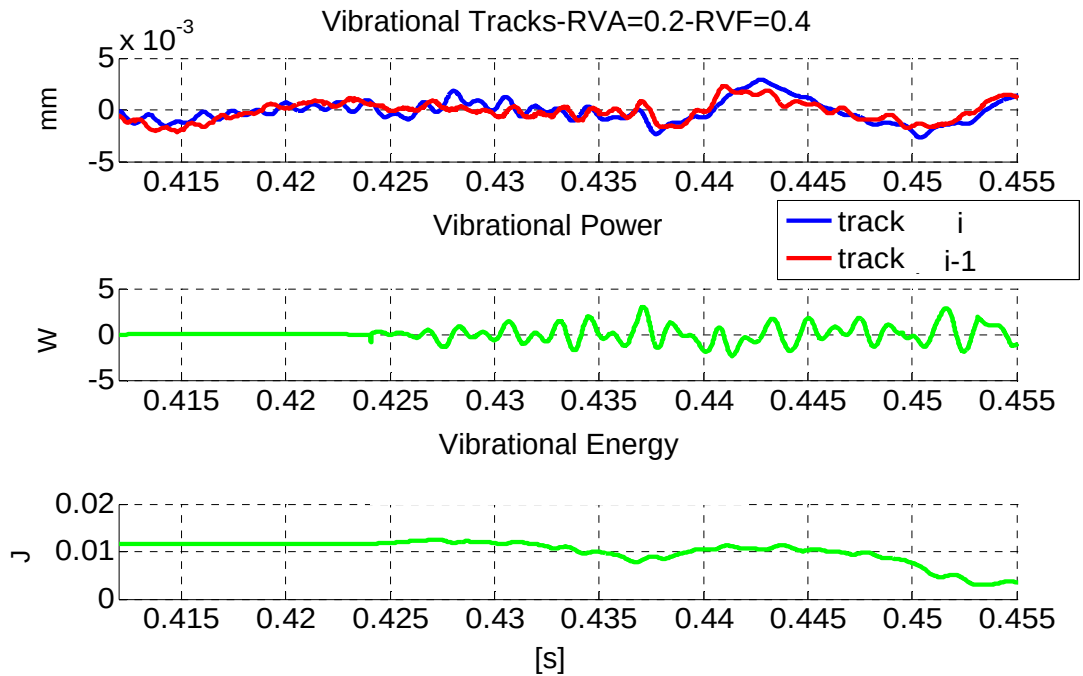


Fig. 4.78: Chip thickness modulation and Vibrational energy – (RVA=0.2,RVF=0.4)

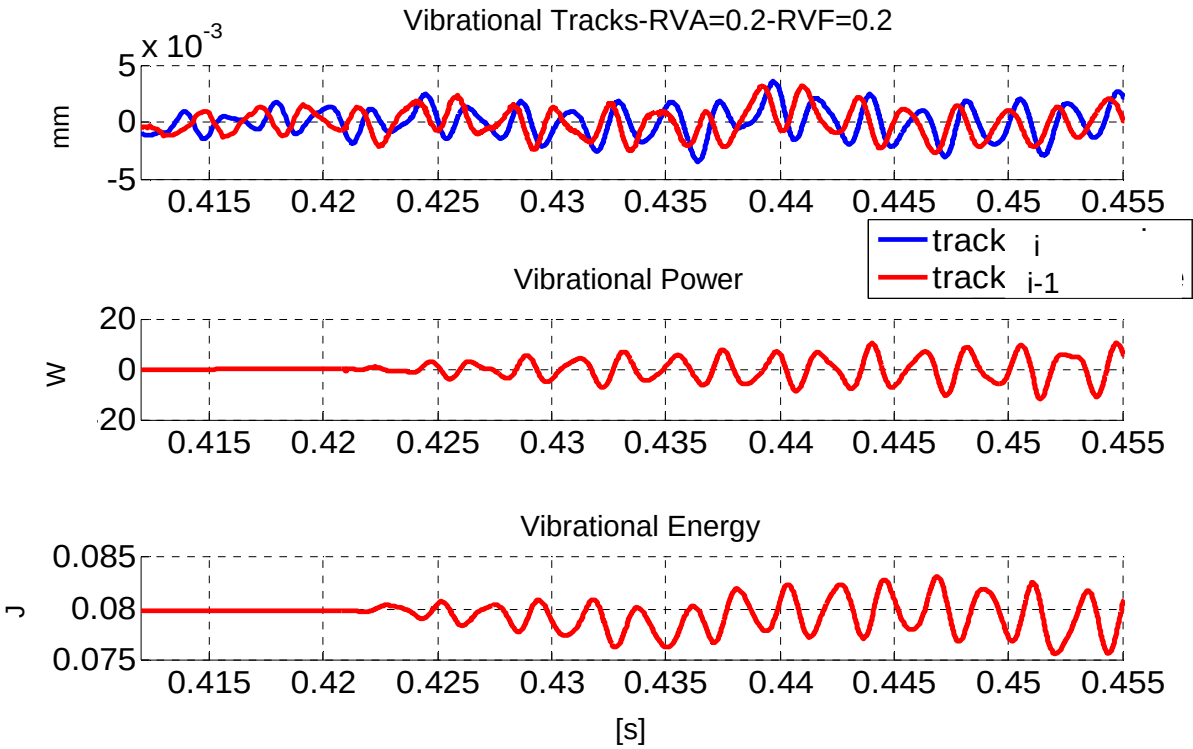


Fig. 4.79: Chip thickness modulation and vibrational energy – (RVA=0.2,RVF=0.2)

4.6.3 Inner and outer modulation at critical depth of cut

A detailed analysis of the critical chip thickness modulation due to the relationship between the actual position of the “cutter i ” and the track left on the workpiece by the “cutting edge $i-1$ ” will be proposed. The goal of this analysis is to understand which cutting parameters mostly affect the worse inner-outer modulation condition in order to define guidelines to select the best SSSV parameters.

So far this analysis hasn't been reported in the specific literature.

The reference case used for the analysis is the Constant Speed Machining.

This analytical analysis has been based on the concepts reported in Altintas Altintas and Budak [10]

It is necessary to recall them in order to explain the analysis that will be further reported.

Eq. 4.36 describes the relationship between the forces on the tool tip (F_x and F_y) and the tool tip displacement (Δ_x and Δ_y) as shown in Fig. 4.80.

Obviously the $A(t)$ matrix introduces the information about the cutting geometry: number of teeth, radial immersion, and cutting coefficients.

$$\text{Eq. 4.36:} \quad \{F(t)\} = \frac{1}{2} a \cdot K_t \cdot [A(t)] \cdot \{\Delta(t)\}$$

But it is possible to consider only the average component as explained in 1.1:

$$\text{Eq. 4.37:} \quad [A_0] = \frac{1}{T} \int_0^T [A(t)] \cdot dt = A_0 \left(\phi_{ex}, \phi_{sx}, \frac{K_r}{K_t} \right) = \frac{N}{2\pi} \begin{bmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{bmatrix}$$

So the dynamic equilibrium can be written, Eq. 4.38:

$$\text{Eq. 4.38:} \quad \{F\} e^{i\omega\tau} = \frac{1}{2} a \cdot K_t [1 - e^{-i\omega T}] \cdot [A_0] \cdot [G(i\omega)] \cdot \{F\} e^{i\omega\tau}$$

And $G(i\omega)$ is the transfer function matrix

So that the Differential Delayed Equation that describes the milling chatter phenomenon has non trivial solutions the determinant needs to be zero,

$$\text{Eq. 4.39:} \quad \det(\mathbf{I} + \Lambda \cdot [A_0] \cdot \mathbf{G}(\omega)) = 0$$

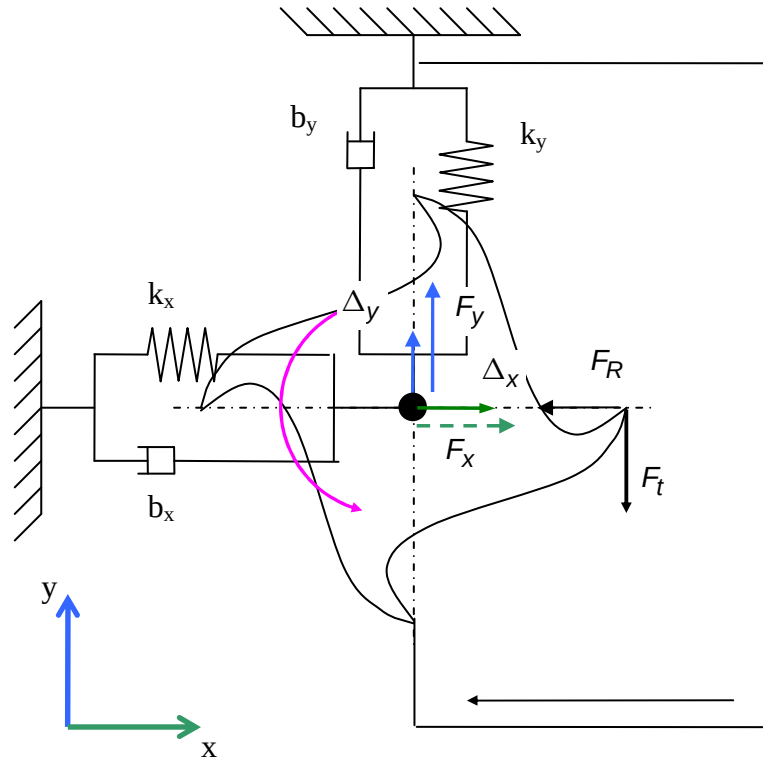


Fig. 4.80: Cutting geometry

Assuming:

$$\text{Eq. 4.40:} \quad \Lambda = -\frac{N}{4\pi} b K_t (1 - e^{-i\omega T (= \varepsilon)})$$

Where $\varepsilon = \varphi$ is the phase shift between inner (track “i-1”) and outer modulation (track “i”).

Λ that is the opposite of the inverse of the traditional eigenvalues is generally a complex one. So we can write:

$$\text{Eq. 4.41:} \quad \Lambda = \Lambda_R + i\Lambda_I$$

But we are interested in the axial depth of cut b so it is necessary to impose a real value for b .

In order to obtain the axial depth of cut we can write the following relationship

$$\text{Eq. 4.42:} \quad \Im(\Lambda \cdot (1 - e^{-i\varepsilon})^{-1}) = 0$$

But using the Euler's relationship for complex number:

$$\begin{aligned} \Im\left(\frac{\Lambda_R + i\Lambda_I}{1 - (\cos(\varepsilon) - i\sin(\varepsilon))}\right) &= 0 \Rightarrow \\ \Rightarrow \Im\left(\frac{(\Lambda_R + i\Lambda_I)(1 - \cos(\varepsilon) + i\sin(\varepsilon))}{(1 - \cos(\varepsilon))^2 + \sin^2(\varepsilon)}\right) &= 0 \Rightarrow \end{aligned}$$

Eq. 4.43: $\Rightarrow \Im((\Lambda_R + i\Lambda_I)(1 - \cos(\varepsilon) + i\sin(\varepsilon))) = 0 \Rightarrow$

$$\Rightarrow \Lambda_I(1 - \cos(\varepsilon)) + \Lambda_R \sin(\varepsilon) = 0 \Rightarrow$$

$$\Rightarrow \frac{\Lambda_I}{\Lambda_R} = \frac{\sin(\varepsilon)}{1 - \cos(\varepsilon)} = \frac{\cos\left(\frac{\varepsilon}{2}\right)}{\sin\left(\frac{\varepsilon}{2}\right)}$$

But if we know $\left(\frac{\Lambda_R}{\Lambda_I}\right)$ the phase shift $\varepsilon = \Phi$ can be obtained:

Eq. 4.44:
$$\varepsilon = \pi - 2 \tan^{-1}\left(\frac{\Lambda_R}{\Lambda_I}\right)$$

This approach is slightly different from the one used to study the regenerative phenomenon in turning where the characteristic equation is solved directly, having fixed a real dept of cut, by imposing that both the real and imaginary part are equal to zero.

After this briefly review of the traditional approach used to study the cutting process instability in milling operations it is necessary to analyze the relationship between the minimum depth of cut and the corresponding critical phase shift.

The following expression can be written:

Eq. 4.45:
$$b_{\min} = \min_{\omega} \left(\frac{-2\pi\Lambda_R(\omega)}{NK_t} \left(1 + \frac{\Lambda_I^2(\omega)}{\Lambda_R^2(\omega)} \right) \right)$$

And we can obtain:

$$\begin{aligned}
 \text{Eq. 4.46.} \quad b_{\min} &= \min_{\omega} \left(\frac{-2\pi\Lambda_R(\omega)}{NK_t} \left(\frac{\Lambda_R^2(\omega) + \Lambda_I^2(\omega)}{\Lambda_R^2(\omega)} \right) \right) = \\
 &= \min_{\omega} \left(\frac{-2\pi}{NK_t} \left(\frac{\Lambda_R^2(\omega) + \Lambda_I^2(\omega)}{\Lambda_R(\omega)} \right) \right)
 \end{aligned}$$

But $b_{\min} > 0$ so we can rewrite the Eq. 4.46:

$$\text{Eq. 4.47:} \quad \max_{\omega} \left(\frac{\Lambda_R^2(\omega) + \Lambda_I^2(\omega)}{\Lambda_R(\omega)} \right) \text{ con } \Lambda_R < 0$$

Considering the more comprehensive case with 2 degrees of freedom (2dof) used for milling operations:

The eigenvalues can be analytically computed because the related equation is a second order one:

In order to simplify the solution we consider the case in which:

$$\text{Eq. 4.48:} \quad G_{xy} = G_{yx} = 0$$

This simplification doesn't affect the generality of the results.

Using the 4.49 we can evaluate the eigenvalues:

Eq.

4.49:

$$\Lambda_{1,2} = - \frac{(\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy}) \pm \sqrt{(\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy})^2 - 4(G_{xx} G_{yy})(\alpha_{xx} \alpha_{yy} - \alpha_{xy} \alpha_{yx})}}{2(G_{xx} G_{yy})(\alpha_{xx} \alpha_{yy} - \alpha_{xy} \alpha_{yx})}$$

Where the transfer function matrix $G(i\omega)$ is:

$$\text{Eq. 4.50:} \quad G(i\omega) = \begin{bmatrix} G_{xx} & G_{xy} \\ G_{yx} & G_{yy} \end{bmatrix}$$

And:

$$\begin{aligned}
 \text{Eq. 4.51:} \quad G_{xx} &= G_{xx}(\omega) \\
 G_{yy} &= G_{yy}(\omega) \\
 G_{xy} &= G_{xy}(\omega) \\
 G_{yx} &= G_{yx}(\omega)
 \end{aligned}$$

And defining $[A_0]$ as done in Altintas we can underline the $\det(A_0)$:

$$\text{Eq. 4.52: } \Lambda_{1,2} = - \frac{(\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy}) \pm \sqrt{(\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy})^2 - 4(G_{xx} G_{yy}) \det(A_0)}}{2(G_{xx} G_{yy}) \det(A_0)}$$

When the $\det(A_0)=0$, two solutions can be computed:

$$\Lambda_1 = \infty (+)$$

$$\Lambda_2 \text{ indeterminate } (-)$$

The second solution can be found by solving the following limit:

Eq. 4.53:

$$\begin{aligned} \lim_{\det(A_0) \rightarrow 0} \Lambda_2 &= \lim_{\det(A_0) \rightarrow 0} - \frac{(\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy})^2 - (\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy})^2 - 4(G_{xx} G_{yy}) \det(A_0)}{2(G_{xx} G_{yy}) \det(A_0) \cdot \left((\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy}) + \sqrt{(\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy})^2 - 4(G_{xx} G_{yy}) \det(A_0)} \right)} = \\ &= - \frac{-2}{\left((\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy}) + \sqrt{(\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy})^2} \right)} = \frac{1}{(\alpha_{xx} G_{xx} + \alpha_{yy} G_{yy})} \end{aligned}$$

And now we are going to give a physical interpretation to the obtained results. If $\det(A_0)=0$ it means that the matrix A_0 have two columns that aren't independent and thus a force in the plane generates displacement only along one direction depending on the x and y compliances. The direction at right angle shows an infinite stiffness.

We can make the following assumption (in order to get a symmetric behaviour): This simplification doesn't affect the generality of the results.

$$\text{Eq. 4.54: } \tilde{G}_{xx} = \alpha_{xx} G_{xx}; \tilde{G}_{yy} = \alpha_{yy} G_{yy};$$

Hence the eigenvalues are:

$$\text{Eq. 4.55: } \Lambda_{1,2} = - \frac{(\tilde{G}_{xx} + \tilde{G}_{yy}) \pm \sqrt{(\tilde{G}_{xx} + \tilde{G}_{yy})^2 - 4(\tilde{G}_{xx} \tilde{G}_{yy}) \cdot \frac{\det(A_0)}{\alpha_{xx} \alpha_{yy}}}}{2(\tilde{G}_{xx} \tilde{G}_{yy}) \cdot \frac{\det(A_0)}{\alpha_{xx} \alpha_{yy}}}$$

When $\alpha_{xx} \alpha_{yy}=0$ the two solutions can be computed solving a limit:

We can obtain:

$$\text{Eq. 4.56:} \quad \Lambda_{1,2} = \pm \frac{i}{\sqrt{G_{xx} G_{yy}}}$$

It is important to observe that in this case the solutions correspond to two eigenmodes at the same frequency. This is the typical behaviour of the mode coupling mechanism formerly described.

It is possible to compute the phase shift linked to the analyzed situations:

First and foremost we can consider the case in which the modal coupling is not present.

Introducing a further assumption:

$$\text{Eq. 4.57:} \quad G_{xx} = G_{yy}$$

So we can obtain:

$$\text{Eq. 4.58:} \quad \bar{\Lambda} = \frac{1}{(\alpha_{xx} + \alpha_{yy}) \cdot G_{xx}(\omega)} \stackrel{\text{def}}{=} \frac{1}{\hat{G}_{xx}(\omega)}$$

If we consider a single pole transfer function $\bar{\Lambda}$:

$$\text{Eq. 4.59:} \quad \bar{\Lambda} = \frac{k((\omega_n^2 - \omega^2)^2 + (2\xi\omega_n\omega)^2)}{\omega_n^2((\omega_n^2 - \omega^2) - 2i\xi\omega_n\omega)}$$

where ω_n is the resonance frequency, k is the modal stiffness and ξ the damping ratio.

We can write the Eq. 4.60:

$$\text{Eq. 4.60:} \quad B_{1 \text{ pole}}(\omega) \stackrel{\text{def}}{=} \frac{\Lambda_R^2(\omega) + \Lambda_I^2(\omega)}{\Lambda_R(\omega)}, \quad \Lambda_R < 0$$

It is possible to verify that the frequency linked to the minimal depth of cut corresponds to the minimum of the real part of the dynamic compliance as reported in [3].

$$\text{Eq. 4.61:} \quad \omega_{\min \Re(G(\omega))} = \omega_{\max B_{1 \text{ pole}}(\omega)} = \omega_n \sqrt{1 + 2\xi}$$

So if $\omega_{\max B_{1\text{pole}}(\omega)}$ we can write:

$$\text{Eq. 4.62: } \bar{\Lambda}_{R \max B_{1\text{pole}}(\omega)} = -2k\xi \quad ; \quad \bar{\Lambda}_{I \max B_{1\text{pole}}(\omega)} = 2k\xi\sqrt{1+2\xi}$$

In Fig. 4.81 the real and imaginary part of $\bar{\Lambda}(\omega)$ have been diagrammed

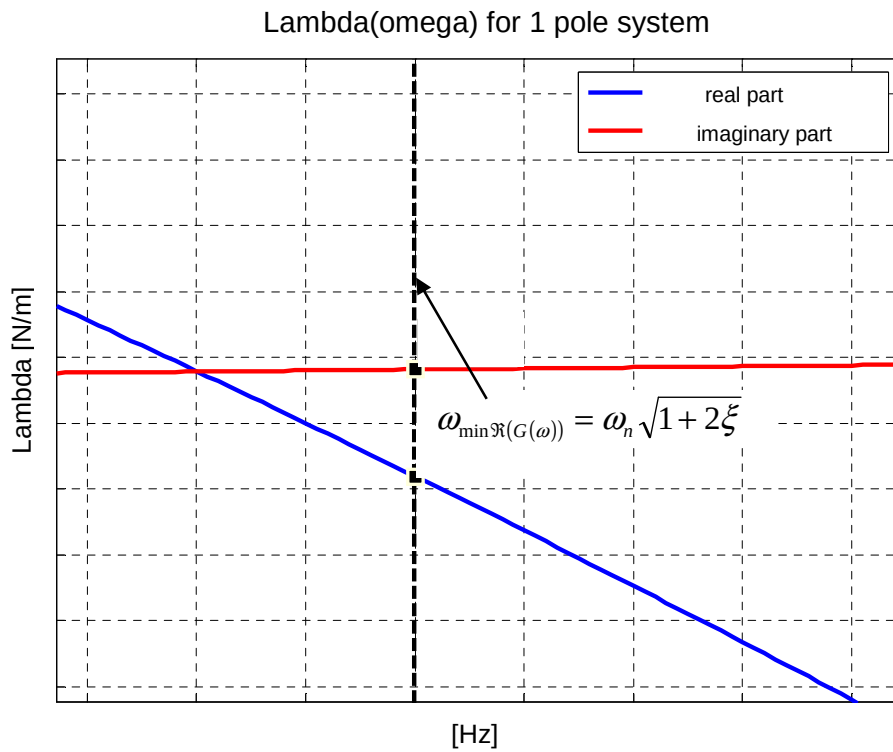


Fig. 4.81: Real part and imaginary part of Lambda

In order to compute the minimum b it is necessary to consider the $B_{1\text{pole}}(\omega)$ function. It is depicted Fig. 4.82 and its maximum (for the frequency $\omega_{\max B_{1\text{pole}}(\omega)}$) has been underlined.

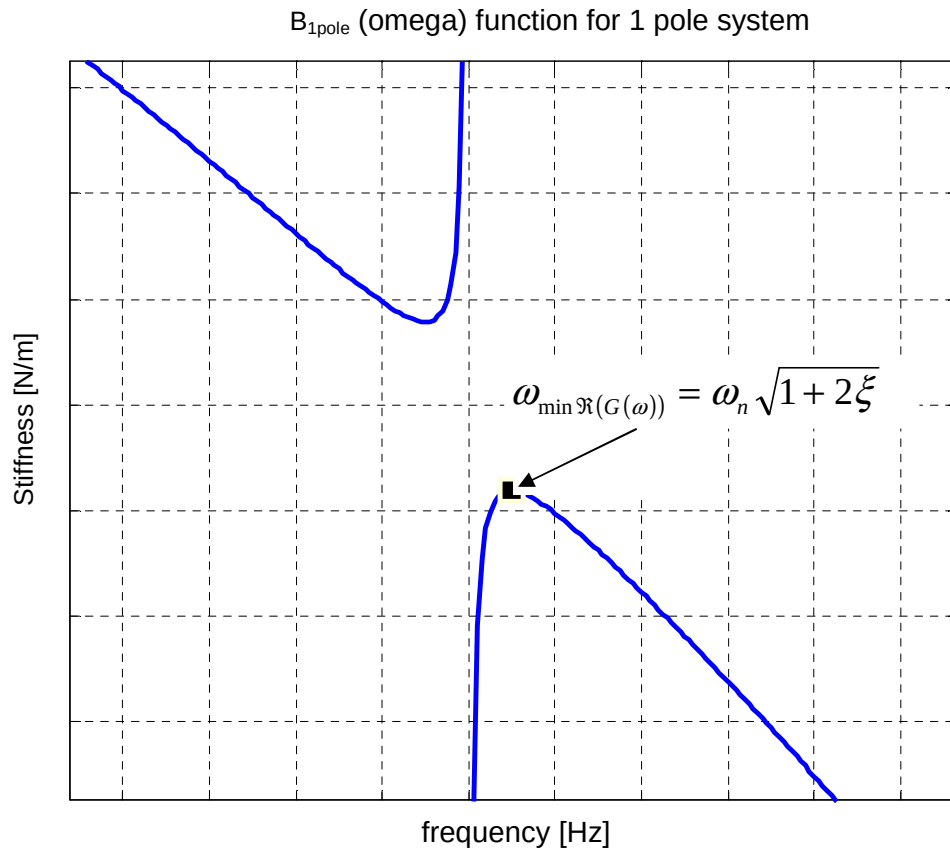


Fig. 4.82: Function B_{1pole}

In this ideal case the phase shift can be computed using the following equation
Eq. 4.44:

Thus:

Eq. 4.63:

$$\begin{aligned} \varepsilon_{b_{\min}} = \Phi_{b_{\min}} &= \pi - 2 \tan^{-1} \left(\frac{\bar{\Lambda}_R}{\bar{\Lambda}_I} \right) = \pi - 2 \tan^{-1} \left(-\sqrt{1 + 2\xi} \right) \cong .. \\ &\cong \pi - 2 \tan^{-1}(-1) \cong \frac{3}{2} \pi \end{aligned}$$

But it necessary to recall that the phase shift depends on the damping ratio as shown in Fig. 4.83. Generally typical values of damping ratio for mechanical system are very low (2-4%) so the phase can be considered approximately equal to $3\pi/2$.

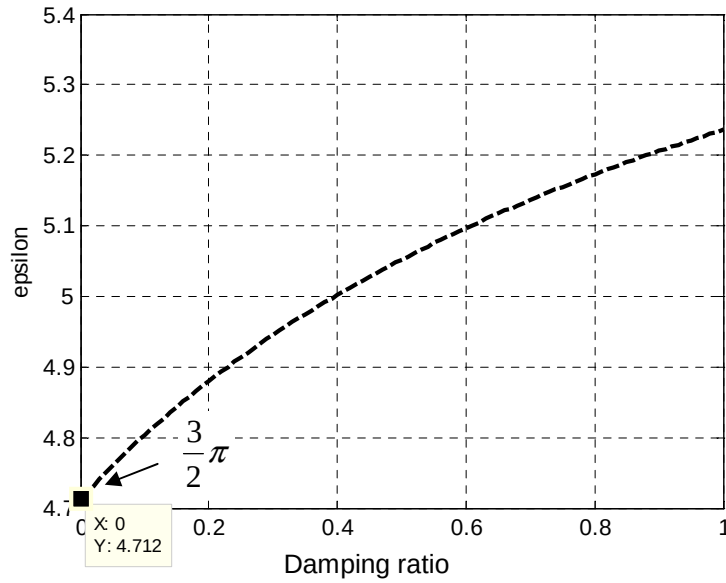


Fig. 4.83: Phase shift and damping ratio

The analyzed case is typical in turning operations.

If we consider the case in which the modal coupling is maximum, that is $\alpha_{xx}\alpha_{yy}=0$

We can use the following expression obtained from the eigenvalues solution:

Eq. 4.64:
$$\tilde{\Lambda}_{1,2} = \pm \frac{i}{G_{xx}(\omega)}$$

But if we consider the $\tilde{\Lambda}_1$; it would be equal to the eigenvalues reported in Eq. 4.58 if a phase shift, introduced by the imaginary unit, was neglected so

Eq. 4.65:
$$i = e^{i\frac{\pi}{2}}$$

And the phase shift:

Eq. 4.66:
$$\begin{aligned} \epsilon_{b_{\min}} = \Phi_{b_{\min}} &= \pi - 2 \tan^{-1} \left(\frac{\Re \left(\bar{\Lambda} e^{i\frac{\pi}{2}} \right)}{\Im \left(\bar{\Lambda} e^{i\frac{\pi}{2}} \right)} \right) = \pi - 2 \tan^{-1} \left(\frac{\bar{\Lambda}_R}{\bar{\Lambda}_I} \right) - \frac{\pi}{2} = \\ &= \frac{1}{2} \pi - 2 \tan^{-1} \left(-\sqrt{1+2\xi} \right) \cong \pi \end{aligned}$$

This is the case we have formerly analyzed: full engagement face milling.

So we can conclude that the coefficient that can be used to describe different cutting conditions and thus how vary the most critical phase shift is reported in Eq. 4.67:

$$\text{Eq. 4.67: } L = \frac{\overset{\text{def}}{\det}(A_0)}{\alpha_{xx}\alpha_{yy}}$$

When:

$$\begin{aligned} \text{Eq. 4.68: } L=0 &\rightarrow \varepsilon_{b \min}=3/2\pi \\ L=\infty &\rightarrow \varepsilon_{b \min}=\pi \end{aligned}$$

These analyzed cases correspond to the two extreme situations: respectively no modal coupling and maximum modal coupling.

Obviously there are infinite intermediate cutting conditions. For these intermediate conditions is not possible to compute analytically the critical phase shift but is necessary to solve numerically the characteristic equation.

When $\alpha_{xx}\alpha_{yy}=0$:

It means, Eq. 4.69, that a generic displacement for example along x doesn't produce a force in the same direction.

$$\text{Eq. 4.69: } \begin{Bmatrix} F_x \\ F_y \end{Bmatrix} \propto [A_0] \cdot \begin{Bmatrix} \Delta_x([G]) \\ \Delta_y([G]) \end{Bmatrix} = \begin{bmatrix} \alpha_{xx} & \alpha_{xy} \\ \alpha_{yx} & \alpha_{yy} \end{bmatrix} \cdot \begin{Bmatrix} \Delta_x([G]) \\ \Delta_y([G]) \end{Bmatrix}$$

Face milling operation with a full engagement of the tool in the workpiece and an high ratio between the tangential cutting coefficient and the radial one can be considered a case in which the modal coupling is maximum.

If we consider the case in which $L=0$ the mechanism that the process uses to introduce energy in the system is different and it is explained in Fig. 4.84.

In this case the cutting forces introduce energy in the system acting on the same direction of the tool vibration that origins the chip thickness modulation. This a very interesting result because a detailed and general analysis of the critical phase shift hasn't already proposed in the specific literature. Only some studies referred to the turning case can be found, [88].

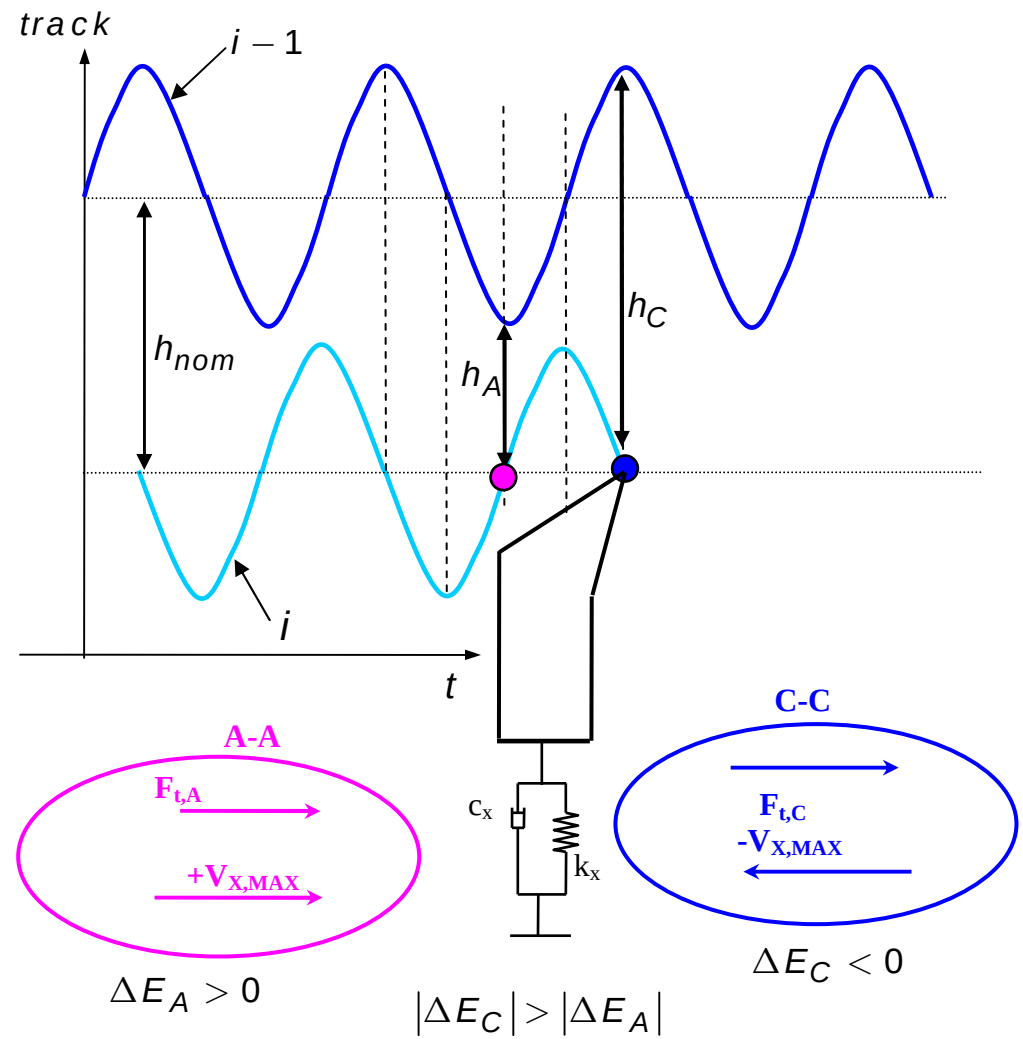


Fig. 4.84: Regenerative effect – energetic explanation – $L=0$

4.7 An analytical approach to optimize SSSV in Milling

A relationship between the inner-outer tracks modulation and the energy introduced in the dynamic system during the cutting has been analyzed. A critical chip thickness modulation for Constant Speed Machining has been figured out and moreover a detailed analysis of how this condition is strictly connected to the characteristic of the milling such as radial immersion, number of cutting edges and cutting coefficients has been presented.

If this critical chip thickness modulation linked to a well known nominal spindle speeds endures the system is brought to instability.

The Sinusoidal Spindle Speed Variation allows to continuously vary the inner-outer tracks modulation and consequently it breaks the mechanism that introduces energy into the system.

The effectiveness of the SSSV in the chatter suppression depends strongly on the RVA/RVF parameters so the selection of the proper spindle profile is an important issue.

For the planar displacements dynamics, we have proposed a function to measures an heuristic distance between the phase shift between inner and outer modulation that would be occurred if the instantaneous value of cutting speed was kept constant and the worst phase shift condition previously identified.

The functional depends on the speed variation law hence, the optimum parameters (RVA and RVF) can be derived solving the following classical optimization problem, Eq. 4.70.

$$\text{Eq. 4.70: } \begin{cases} \max_{RVA, RVF} \left(J = \int_t^{t+T} (|\Phi(t | RVA, RVF) - \Phi_{\text{worst}}|) dt \right) \\ \text{s.t.: } \underline{a} < RVA < \bar{a}, \underline{b} < RVF < \bar{b} \\ \forall \Omega_0 \in [\Omega_{\text{inf}}; \Omega_{\text{sup}}] \end{cases}$$

And n_r is a number of period considered for the integration

$$\text{Eq. 4.71: } T = \int_0^{n_r \cdot \frac{2\pi}{RVF}} \Omega(t) dt$$

And the “instantaneous phase shift” or better the phase shift that would be occurred if the instantaneous value of cutting speed was kept constant has been computed using the approximate relationship Eq. 4.5.

Obviously the dependence of the chatter frequency to the instantaneous spindle speed has been considered $\omega_c = \omega_c(\Omega(t))$.

It is necessary to assert that this optimization can be performed only using the data from a traditional stability analysis in the frequency domain.

Maximizing this functional for each nominal spindle speed is equivalent to select the Spindle Speed Variation trajectory that introduces less energy in the machine tool system because we have attested that there is dualism between the instantaneous chip thickness modulation and the work done by the cutting forces.

A graphic interpretation has been provided in Fig. 4.85 considering a defined nominal spindle speed and a specific RVA/RVF parameters combination

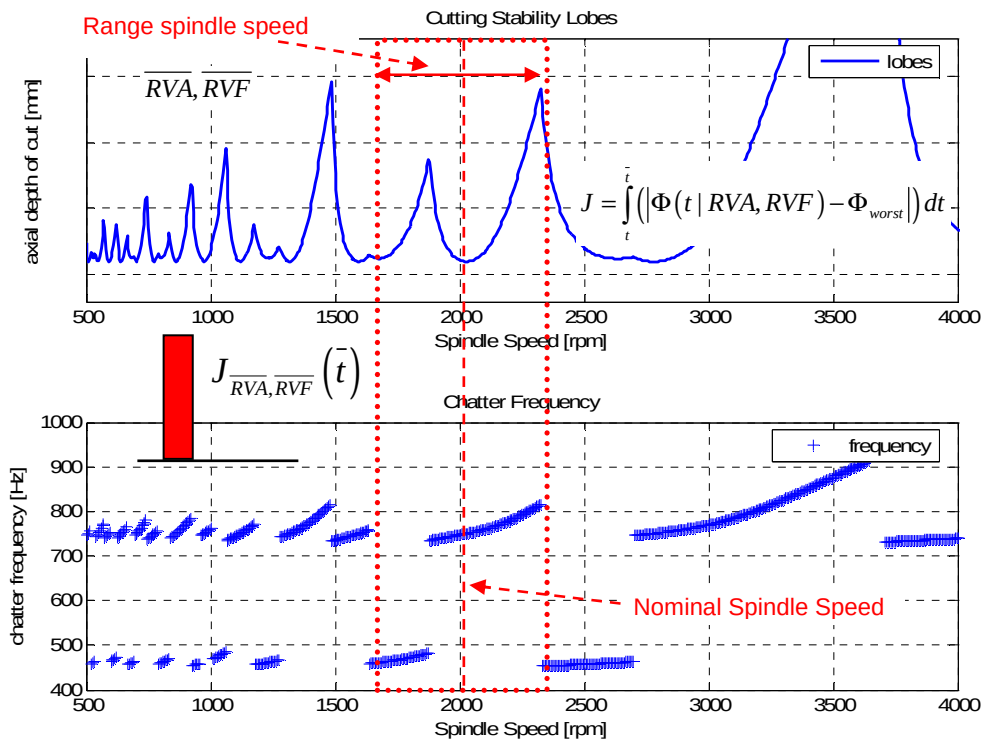


Fig. 4.85: Optimization procedure – graphical explanation

This approach has been applied to the test-case formerly analyzed. The milling operation is described in Fig. 4.6 hence the critical condition is $\Phi = \pi$ as explained in 4.6.3.

The simplified case with only one pole has been analyzed firstly.

Fig. 4.85 presents the obtained results.

A discrete matching between the results obtained with the approach that considers the worst modulation condition and the time simulations results has been obtained.

It is therefore necessary to state some further considerations.

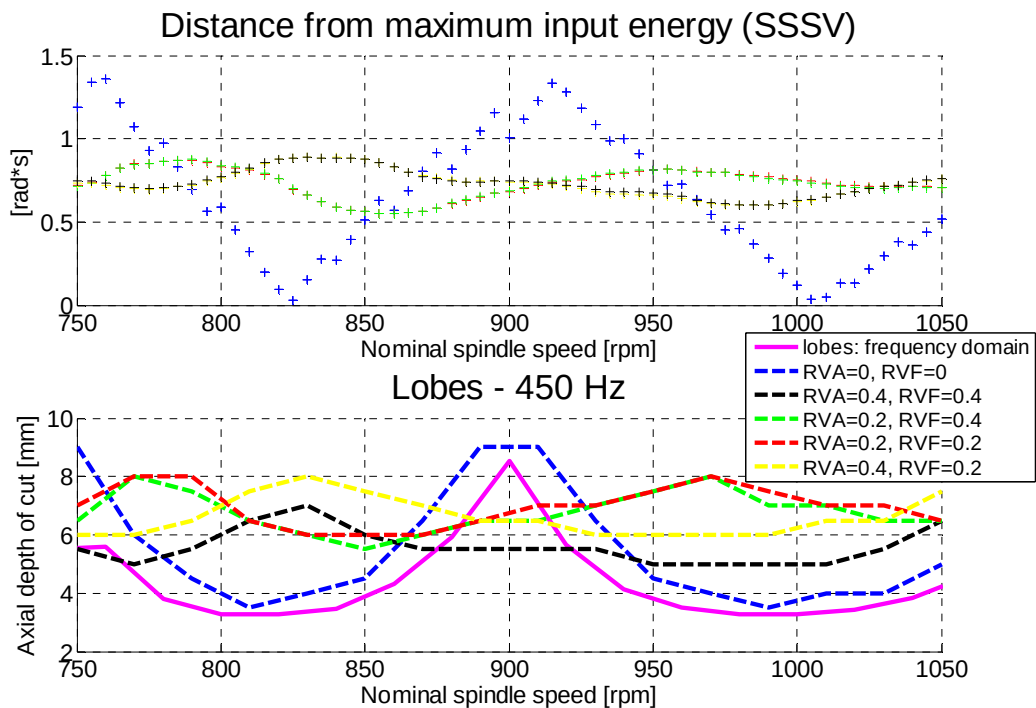


Fig. 4.86: SSSV trajectory optimization – time domain comparison (@450Hz)

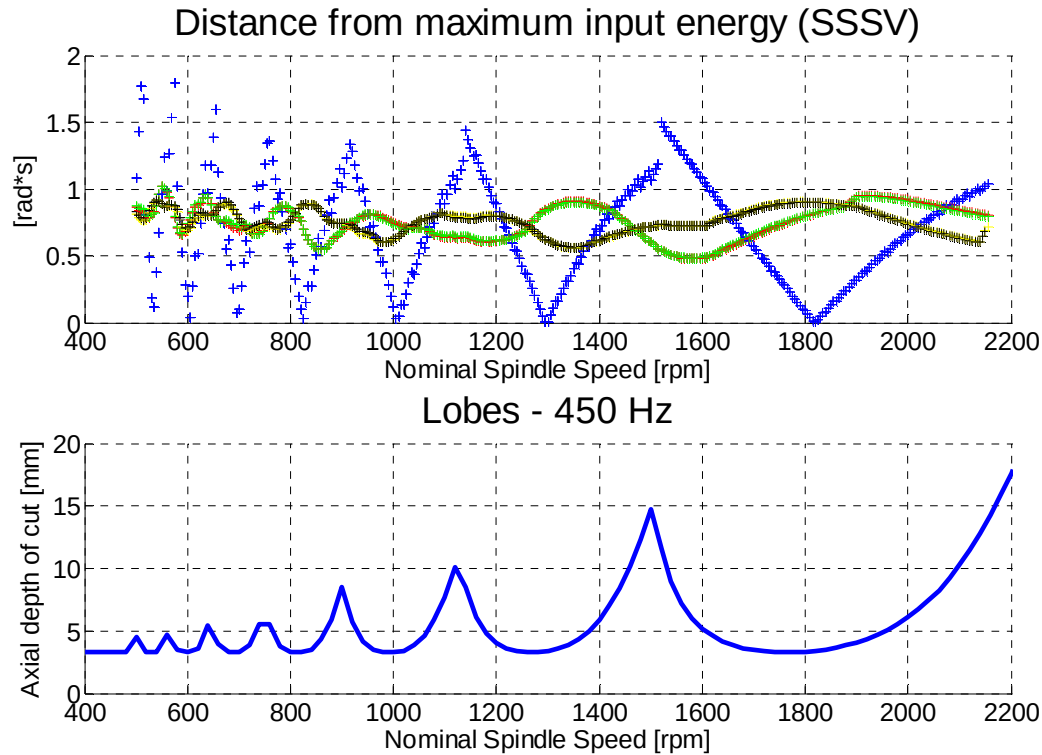


Fig. 4.87: SSSV trajectory optimization – mode @450Hz

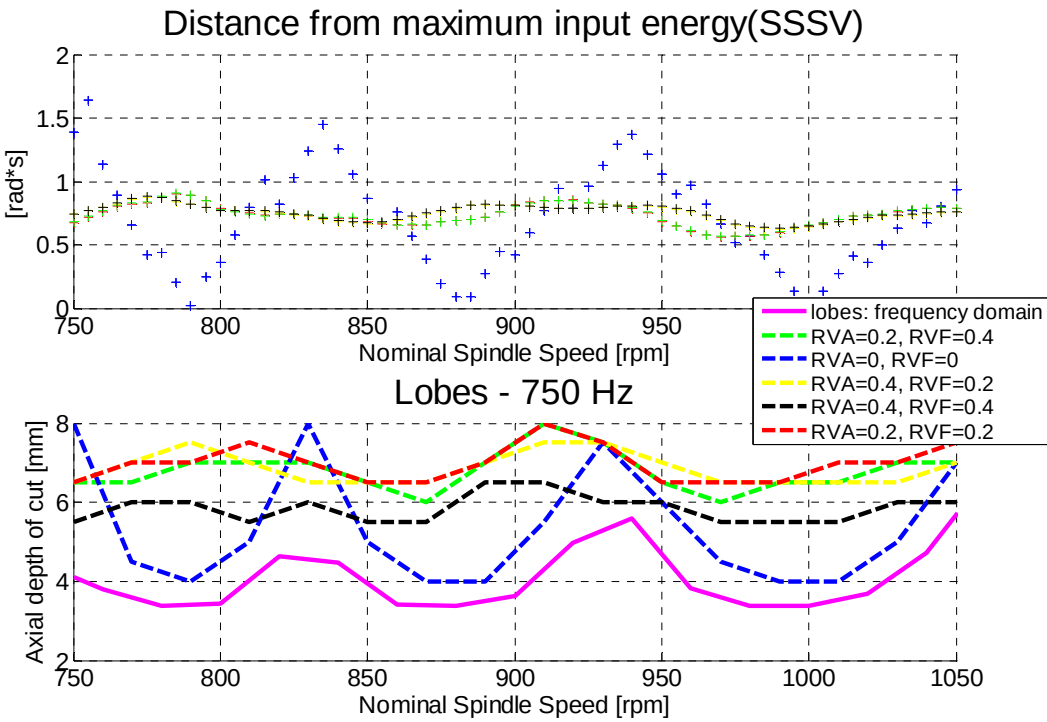


Fig. 4.88: SSSV trajectory optimization – time domain comparison (@750Hz)

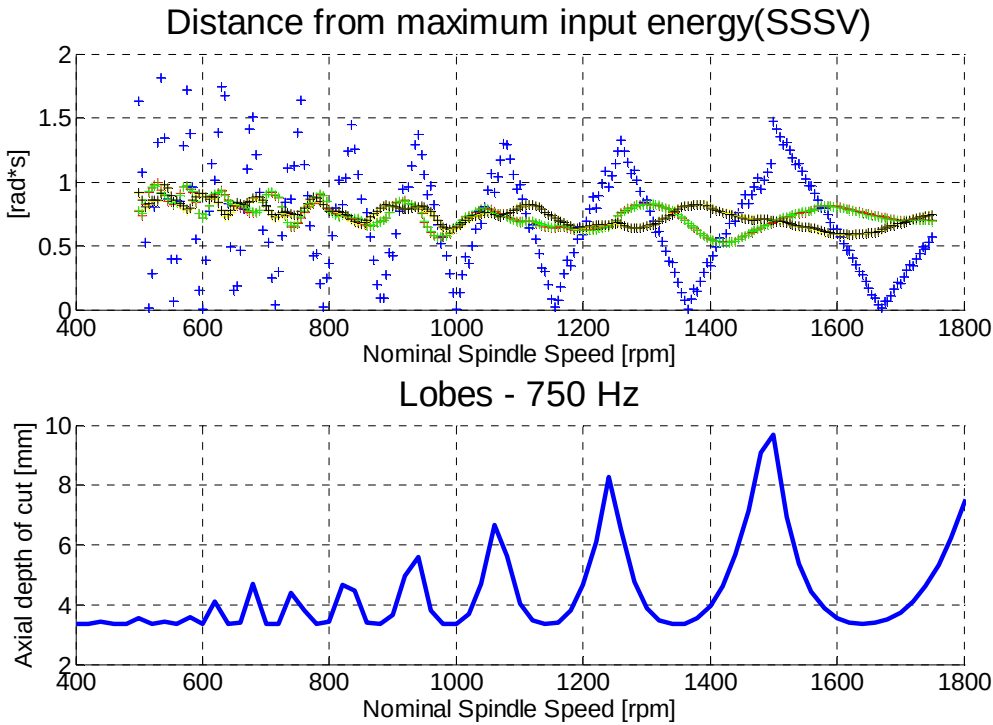


Fig. 4.89: SSSV trajectory optimization – mode @750Hz

The described approach allows catching the zones of the stability chart in which the SSSV is more convenient than CSM.

Moreover the method well reproduces the trends of the different analyzed parameters combinations.

An aspect that that hasn't been reproduced is the different stability limits obtained using the $RVA=RVA=0.4$ one and the $RVA=0.4/RVF=0.2$ one.

From the time domain simulation we have deduced that the RVF parameter is less influent than the RVA one. Using the analytical approach this aspect has been stressed.

This can be pointed out observing the trends of the analyzed SSSV trajectories especially focusing on higher nominal spindle speeds.

Moreover we can conclude that the VSM technique seems convenient also in the right part of the stability chart even if in these regions would be easy to select the right constant spindle speed.

Analogous conclusions can be drawn from the analysis of the results reported in Fig. 4.88.

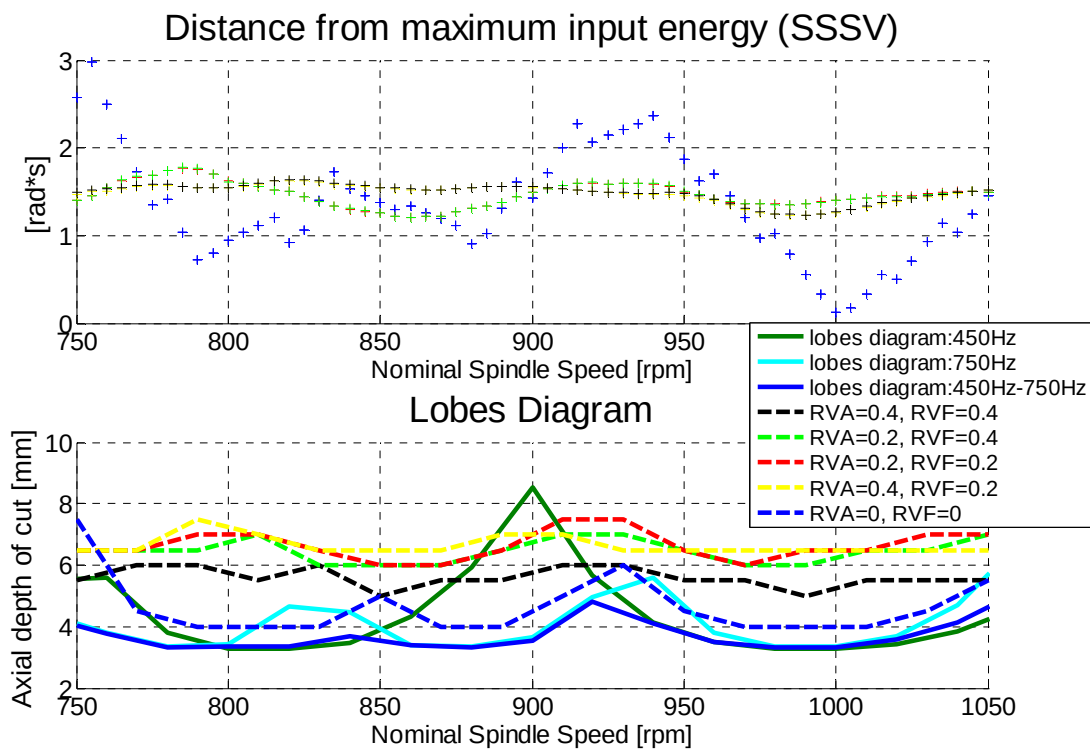


Fig. 4.90: SSSV trajectory optimization – time domain comparison (450Hz & 750Hz)

In this latter case the different sinusoidal trajectories don't manifest evident different performances: all the RVA/RVF combinations have given comparable stability limits also from time domain simulations.

Another aspect that can be observed from the presented results is the shapes of the CSM stability lobes obtained with the proposed method: they look different from the ones obtained by the traditional frequency approach or by time domain simulations.

We guess that this effect is due to the fact that in the optimization procedure only the “critical phase shift” has been considered but we haven’t taken into account that each inner-outer modulation is linked to a corresponding chip thickness so the work done by the cutting forces depends both on these two contributes.

The results obtained considering two poles have been reported in Fig. 4.90: also in this more complex case the method seems to be quite reliable even if it is less effective and precise than the cases in which only one pole has been considered.

4.8 Discussion and future works - SSSV

The goals of this study were to investigate the capabilities of the Sinusoidal Spindle Speed Variation to suppress the regenerative chatter and to define some basic guidelines to select the more effective speed trajectory.

A first analysis of the problem was made by time domain simulations; an industrial case with a complex machine tool dynamics has been considered.

The SSSV technique is particularly effective for low nominal spindle speeds. In this region of the stability chart it is difficult to select the more convenient spindle speeds linked to the stability pockets.

We have also observed that the SSSV flattens the lobes diagram so the choice of the nominal spindle speed is not crucial.

We have noted that the VSM is convenient especially when the corresponding nominal spindle speed is close to the minima of the lobes because the speed modulation more effectively alters the regenerative effect that reaches maximum levels close to these regions.

It is necessary to remember that the SSSV needs power requirement from the spindle drive and moreover the speed modulation impairs the surface finishing and the tool wear.

Moreover a more detailed energetic interpretation of the mechanism that brings the system to the instability has been proposed.

A dualism between the inner-outer cutting edge tracks modulation and the energy introduced into the machine-tool by the cutting system has been studied.

A critical analysis of the worst inner-outer tracks modulation has been proposed, studying the influence of the geometrical properties of the milling operation.

The effects of the spindle speed modulation on the work done by the cutting forces and thus on the stability of the cutting process have been analyzed.

A fast method to select the more effective spindle speed trajectory based on an analytical approach has been proposed.

The method seems to give correct tips comparing to the ones from the time domains simulations.

Nevertheless some interesting results an experimental campaign would be necessary to verify the effectiveness of the proposed approach and to verify the effect of the SSSV on the quality of the machined surfaces.

Moreover it would be necessary to apply the proposed method to different milling operations.

A more general optimization approach could be proposed in order to consider also the power consumption, the quality of the surface finishing and the tool wear.

5 Active chatter suppression

In this chapter an active damping control strategy will be presented as a solution to increase the spindle performance during cutting especially focusing on the suppression of the regenerative chatter phenomenon.

An exhaustive review of the previous literature works has been reported first. The effectiveness of the designed control strategy will be pointed out basically using simulated results. Moreover, a first implementation stage of the active controller on the physical prototype has been presented.

5.1 Vibration control in spindle systems: state of the art

The presence of vibrations in machining processes has a negative effect on the quality of the machined surfaces. As formerly described these vibrations are sources of machine fatigue, tool damage, high cutting edge wear, machine tool damage, wear of machine tool components and annoying high noise levels.

Spindle is considered one of the most crucial machine-tool components especially in high speed machining and its performance is often strictly related to the quality and the surface finishing of the machined workpieces.

Vibrations can be reduced via an active or passive approach or a semi-active one as comprehensively explained by Preumont in [99].

In Fig. 2.14 and Fig. 2.15 a passive and an active damping solution for a generic flexible structure are respectively depicted.

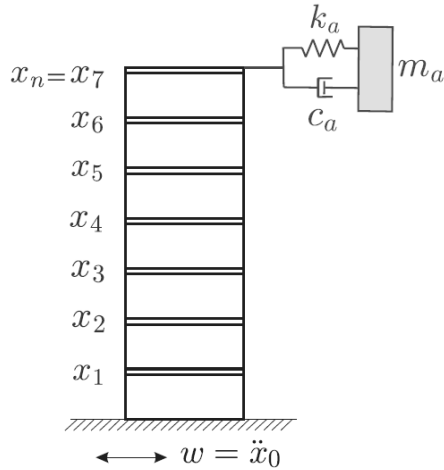


Fig. 5.1 : Dynamic vibration absorber DVA

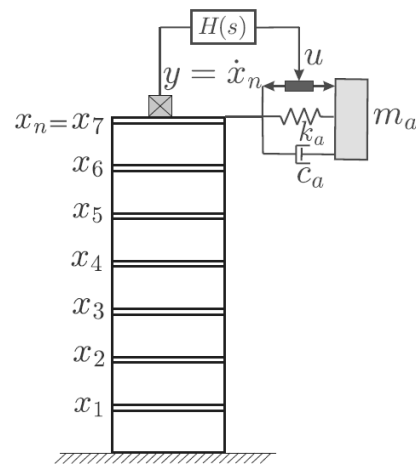


Fig. 5.2: Active mass damper AMD

The DVA is purely passive but operates only on the target mode on which it is tuned leaving the other modes unaffected.

On the other hand AMD is fully active and operates on all controllable modes but requires an adequate sensor and actuator.

There is also the possibility to combine these two approach in what is called “hybrid control” in order to achieve the advantages of both the techniques.

A lot of vibration control applications in machining process have been proposed and analyzed in the specific literature.

Generally active vibration reduction methods are preferred to passive ones (i.e. Benning [123]) because allow do damp different frequency components of the system.

Tarng et. al. [108] implemented a tuned vibration absorber on a lathe for chatter suppression and reported a meaningful increase in cutting stability.

Knospe et. al. [126], Knospe and Tamer [125] developed an adaptive open loop control algorithm for active magnetic bearings (AMB) to suppress rotor unbalance response. The algorithm takes advanced of the fact that the unbalance disturbance can be predicted and the control action can counteract the disturbance.

The unbalance disturbance is very important in wood machining because its contribute, if the spindle speed is very high, it is the most important one.

In these cases Iterative Learning Control (ILC) techniques can be proposed to suppress the vibration.

Some works from different fields can be suggested, Lee et. al. [145] and Lee et. al. [144].

Nonami et. al. [128] developed a controller for flexible rotor. The controller was designed whit a reduced order model [99], [115], [116], however the spillover phenomenon, which usually occurs due to un-modelled dynamics, was significantly reduced due to controller robustness. Nonami et. al. [127] implemented a controller for milling spindle supported by active magnetic bearings. Experimental tests showed good performance of the controller.

Chen in [93] presented an active control strategy to improve the performances of a wood-cutting circular blade. In this work magnetic actuators and a Linear Quadratic Gaussian control were used. Meaningful results were caught in surface quality.

Different types of actuators are particularly suitable for machine tool control applications: the Active Magnetic Bearings (AMB) and Piezo Actuators.

Especially the dynamic capabilities of piezoelectric actuators make them suitable for active vibration suppression including active chatter suppression and for active error compensation.

A number of researchers have investigated the application of piezoelectric technology to turning, and boring machine processes. The piezoelectric actuators are used basically to actively control vibration and to compensate for machine tool inaccuracies, obviously if these can be measured.

Piezo actuators have been used in some milling and grinding applications. The amount of research works done in this area is considerably lower than the amount of work focused on the enhancement of boring and turning machines.

The reason for this is that AMB are usually the preferred choice for active vibration control of rotating machinery even if they are very expensive.

Nevertheless some researchers explored the use of piezoelectric actuator for active control of rotating systems, pioneering work was done by Kascak et.al. [129].

Andren et. al. [130] designed an active tool holder for lathe. The holder has an integrated piezoelectric actuator and accelerometer. The goal of the work was to reduce the vibrations. A meaningful enhancement of the surface finishing due to the vibration reduction system was obtained.

Kim and Nan [132] and Kim and Kim [133] have developed a similar system for ultra-precision lathe. They use a force feed back to control the depth of cut. The feedback signal is obtained from the piezoelectric actuator that controls the cutting tool. Feed table error is measured prior to the machining. This information is then used to control the position of the cutting tool so that the feed table position error is compensated for.

Xu and Han [134] developed another machining tool with error compensation for turning. The tool uses two piezoelectric actuators, one for active error compensation and the other for ultrasonic vibration cutting. Experimental cutting tests shows that roundness is improved with no enhancements in roughness when the error compensation is used without the aid of ultrasonic vibration.

Ganguli in [98],[100] dealt with the regenerative chatter in turning and in milling via numerical simulations. The author proposed some applications based on active control. In order to avoid a lot of cutting tests to tune the controller and to study the technique effectiveness a chatter demonstrator “hardware in the loop” for turning operation was introduced.

In [97] Ganguli presented a comprehensive analysis of the regenerative chatter phenomenon using the root loci. It was found that the instability of the cutting process is not always due to a structural pole but sometimes to a poles linked to the delay effects. An interesting interpretation of the various regions of the stability chart was given.

Min et. al.[135], developed a smart boring tool for line boring in the automotive industry. The boring bar is fixed to the rotating spindle at one end with a roughing cutter and finishing cutter attached to the free end. A piezo-electrical actuator controls the finishing cutter. A non contact power transformer provides power. The non-contact power transformer also transfers data between the smart tool and machine tool itself. The smart tool is able to measure the cutting forces and allows lower error positioning.

Chiu et. al. [136] presented an overhung boring bar servo system for on-line correction of machine errors. The control of the cutting tip is obtained via level structure with a piezoelectric actuator. A 40% improvement in roundness was observed.

In Tewani [109] the use of an active dynamic absorber to suppress machine tool chatter in a boring bar was studied. The vibrations of the system are reduced by moving an absorber mass using an active device such as a piezoelectric actuator,

to generate an inertial force that counteracts the disturbance acting on the main system.

Another work on the control of a boring bar was proposed in [111] by Browning: this industry-ready system consists of three principle subsystems: active clamp, instrumented bar, and control electronics. For the described system, cutting performance has extended existing chatter thresholds (cutting parameter combinations) for nickel alloys by as much as 400% while maintaining precision surface finish on the machined part.

Jang and Tarn in [112] the use of a piezoelectric actuator to act as an active vibration damper on a cutting tool is reported. The piezoelectric actuator with an inertial mass is attached to the controlled cutting tool. The actuator resonance can be well tuned over a wide frequency range by adjusting the size of the inertial mass, so that the actuator can provide an extremely large damping force to suppress undesired vibration of the cutting tool at the resonance frequency of the actuator.

Pratt in [106] and [107] designed both a control system based on a simple linear feedback and a non-linear control strategy to improve the performances of a boring bar. The non-linearity of the cutting process has been analyzed.

The use of piezoelectric actuator for spindle system was proposed by different researchers [95], [96], [137], [138]. The basic principle proposed is an hybrid system where rotor is supported by piezoelectric actuators via bearings. The research efforts were focused on the use of the system for vibration control in aircraft engines and industrial machinery.

Nagaya et. al. [139] proposed an active vibration control system for controlling micro-vibrations of machine spindle. Four piezoelectric actuators, two for each orthogonal axis, support the front spindle bearing. They are arranged in a push-pull configuration. The authors performed numerical simulations and concluded that vibration can be reduced via the proposed system. Further research work was done by the same author in [140] where an auto-tuning vibration absorber is presented in which the absorber creates an anti-resonance state.

In Sims and Zhang [103], [104] an active piezo layer damping system was design in order to increase the MRR during the machining of thin-walled and flexible structure for aerospace industry. This techniques was study in details by Stanway [110] that presented a state of the art on active constrained layer damping.

Kwan et.al. [124] and [90] reported on a development of a controller for chatter suppression in octahedral hexapod machine, Fig. 5.3. It is clear from the simulations that the cutting stability increased dramatically by introducing closed loop control due to the spindle dynamics enhancement. In this work it was done only simulating work. A complete model of the spindle was proposed considering the FEM spindle model, the piezo-actuators model, the cutting process model and the model of the controller considering moreover the effects of the amplifiers. In this work a H^∞ control was proposed.

Laufer et. al. [141], Shankar et. al. [142], Regelbrugge et. al. [143] and Jeffrey et. al. [91] described developments of a smart spindle unit for active chatter suppression of milling machine. The goal of the work was to develop a chatter suppression system for a horizontal hexapod milling machine in order to increase material removal rate (MRR) which is limited by the onset of chatter. The spindle system was controlled by four actuators made of PMN-based electrostrictive ceramic.

The control strategy was the Linear Quadratic Gaussian (LQG) and it is based on the model of the system. A state observer was also designed to predict the states of the system [116], [118], [119], [120]. This control technique is generally used to increase the damping of the system.

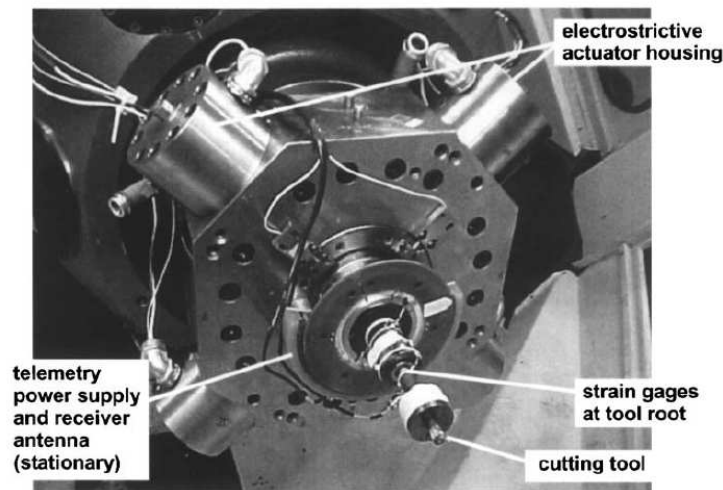


Fig. 5.3: Smart spindle mounted on hexapod

Lead Magnesium Niobate (PMN) electrostrictive material is similar to piezoelectric material in the sense that both exhibit mechanical displacements as results of applied voltage. There are some differences: PNM displacement is proportional to the square of the applied voltage, it exhibits less hysteresis and it has a much higher capacitance.

The experimental cutting tests have shown an important enhancement of the cutting stability limit especially for partial immersion of the cutting tool.

This application is the meaningful work done on chatter suppression in milling operations.

Kern in [146] designed an AMB spindle, a robust control strategy was implemented and meaningful MRR increment was found.

Ries in [147] used ceramic piezo-actuators in order to actively suppress the regenerative chatter vibration, very promising results were obtained and the MRR was increased up to 50%.

Hynek [105] presented a wood cutting controlled spindle. The spindle has four piezoelectric actuators in a push-pull configuration. Two eddy current probes measure the displacement of the spindle shaft in proximity to the actuators. A

control strategy based on a pulse was developed in order to improve the surface finishing and to reduce the high of the peaks left on the machined surfaces and caused by the kinematic of the tangential cutting.

Another proposed application by Simoes et. al. [92] dealt with the control of a rotor system using piezoelectric stack actuators, in this work an optimal modern control techniques was proposed (Linear Quadratic Regulator LQR) [117] and [120]. Simulations and experimental test revealed an important increment of the damping of the system.

In this work we have investigated, like some authors did, the capabilities of active vibration control technique to increase the performance of an active spindle. The goal of work has been focused on the milling regenerative chatter suppression. The reference case is an active spindle that has four piezoelectric actuators, two for each axis in push pull configuration. The spindle was designed for wood cutting enhancements by Hynek [105].

This work has come from the collaboration with the Loughborough University, Wolfon School of Mechanical Engineering and Manufacturing and more precisely the Mechatronics Research Group.

The proposed strategy was suggested by Ho et. al. in [94] that theoretically analyzes the performance of a mixed approach that combines LQG control and an input estimation methodology. Generally the controllers based on LQG are particularly effective to damp transient response of the system and they are designed without considering the disturbance effects.

This mixed strategy uses a linear observer based on Kalman Filter to estimate the disturbances that obviously cannot be measured. An additional control law obtained from the disturbance prediction and inverting the system dynamics was added to the LQG control action. The application of this combined strategy has revealed important enhancements on the system response.

In the spindle system the cutting forces can be considered as disturbances on the system. The aim of the work is to apply this combined strategy to the spindle.

Even if the spindle was designed for wood cutting a study of the behaviour of the spindle and the closed loop strategy effectiveness has been proposed via simulations for metal cutting.

The problem of the cutting force prediction via indirect measurements has been addressed with a modified Kalman filter. This method for the indirect cutting force measurement-estimation was proposed in Park et. al. [101] and Albrecht et. al. [102]. Both authors used a spindle shaft/spindle tool displacement to predict the cutting forces using a linear and optimal Kalman filter [119].

Obviously the enhancement of the spindle dynamic behaviour can improve the spindle performances also during wood-cutting. A first implementation of the designed control strategy on the real time application has been attempted in order to verify the damping properties of the controllers. Some differences between the model and the real system especially caused by the effects of the signals discretization in the real time application have been underlined. Probably a new tuning of the controller is necessary.

5.2 Introduction

As formerly mentioned an active control technique based on optimal control (LQG) has been proposed to increase the dynamic performances of the active spindle system. Generally the goal of this kind of control is to increase the structural damping of the system as described in Fig. 5.4 and explained in Preumont [99].

The concept of the optimal control is very close to that of the pole placement one even if a different technique to tune the controller is used. This technique doesn't consider only the performance of the system but a trade off between performances and the control effort.

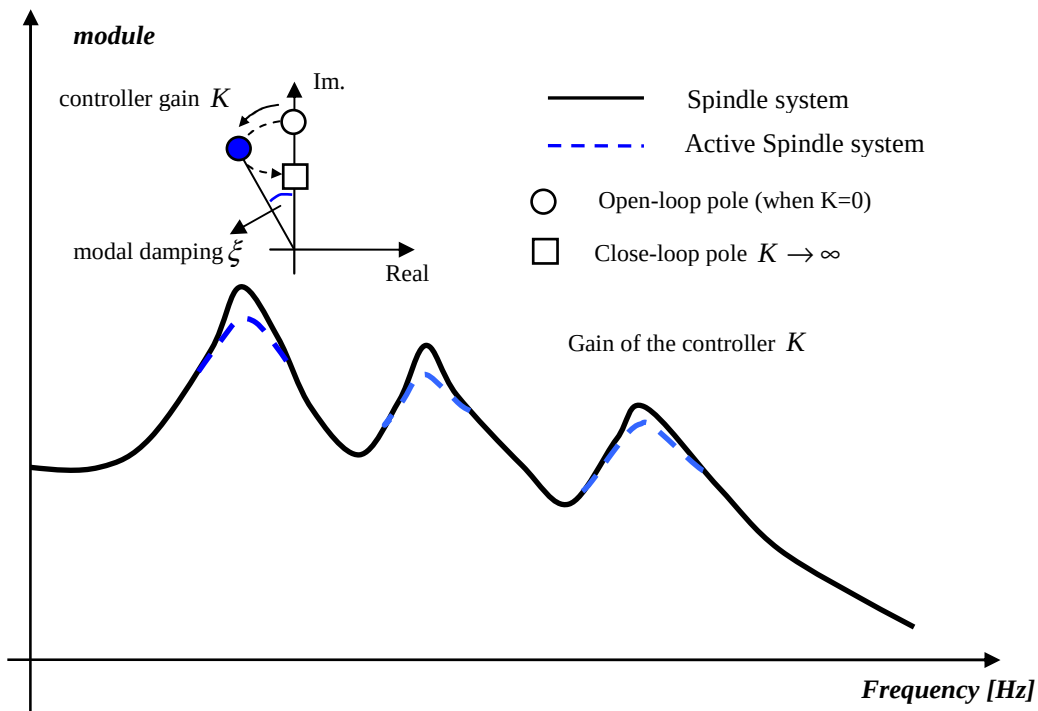


Fig. 5.4: Structural damping increment

From the chatter theory, see Chapter 1.1, we know that the unconditioned stability limit is roughly affected by the minimum of the real part of the tool tip-workpiece dynamics. This statement is not always true, there are some exceptions especially in milling operations when the considered dynamics are very complex.

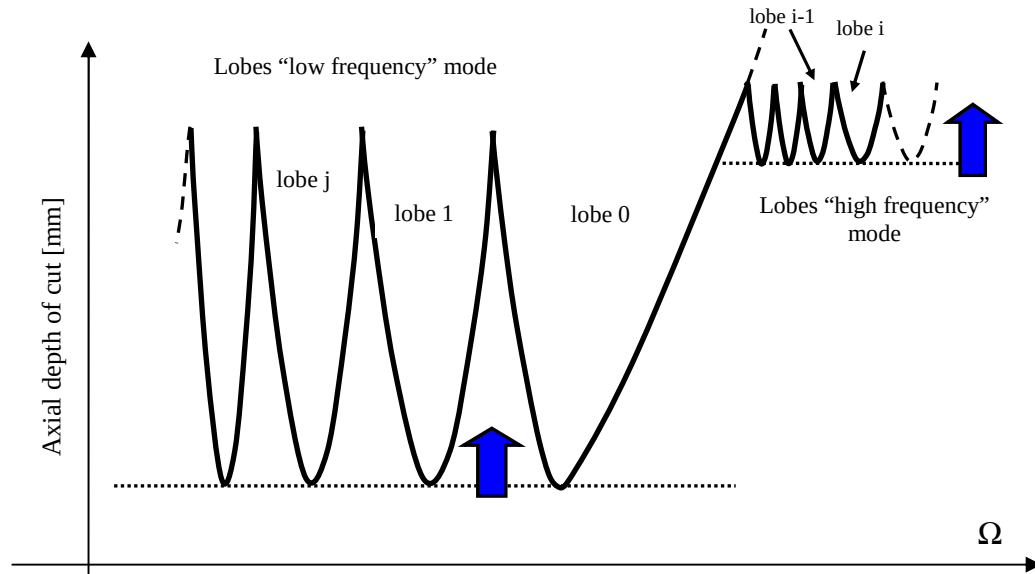


Fig. 5.5: Role of the spindle in high speed machining

An increasing of the system damping provokes a reduction of the dynamic compliances of the most resonance peaks; the minimum of the real part of the FRF becomes less negative thus enhancing the stability limit as explained in Fig. 5.5.

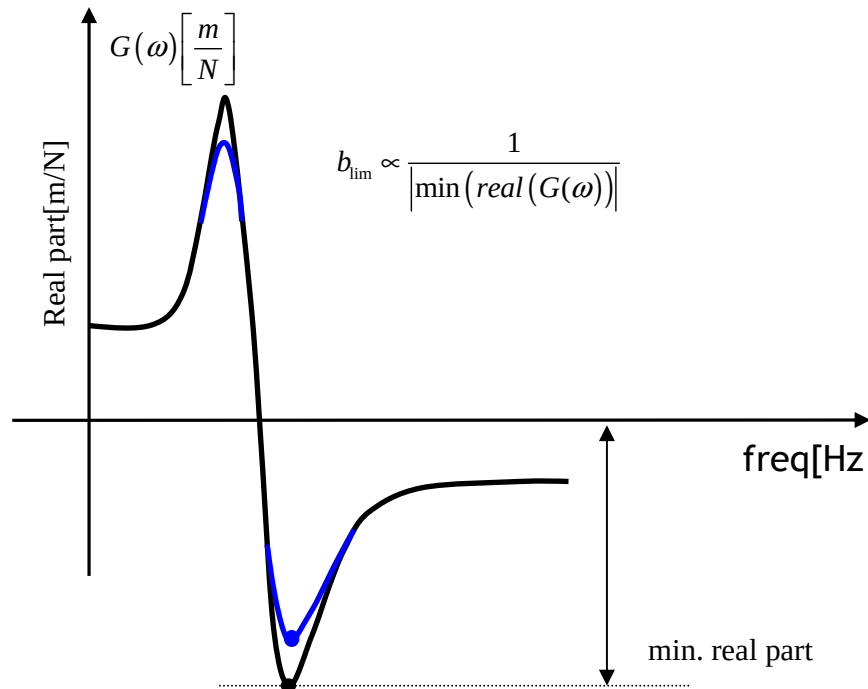


Fig. 5.6: One pole dynamic compliance – real part

5.3 Goals of the work

As said in the previous section the goal of this work is to increase the dynamic performances of a spindle, shown in Fig. 5.7.

The spindle is a prototype built by the Mechatronics Department of Loughborough University (U.K.).

The spindle was originally designed for wood cutting applications, in fact the external motor has a very low maximum available power and torque.

Despite this limitation that doesn't allow us to physically use the prototype for metal cutting (for example aluminium cutting) we would evaluate, basically by numerical simulations, the hypothetical enhancement involved by the implementation of an active damping control technique.

The small scale prototype had been instrumented with an incremental encoder, two eddy current sensors (with a phase shift of 45° respect to the axis of the actuators), signal conditioning circuits, four driving amplifiers (one amplifier for each piezo electric actuator) and a control computer.

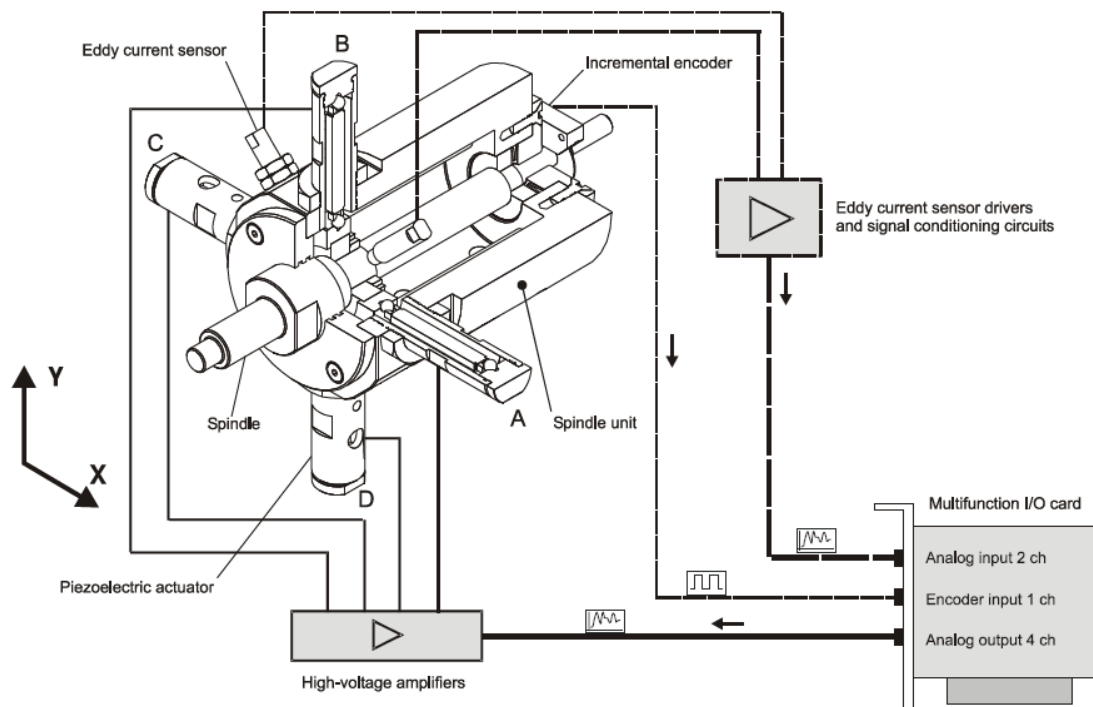


Fig. 5.7: Experimental set up

These amplifiers send the corresponding voltage level to the piezo electric actuators which apply a force against the spindle and cause a controlled displacement.

The small scale planer's instrumentation includes control computers, which are used to acquire the sensor readings in order to adjust the input voltage for the piezo electric actuators in real time.

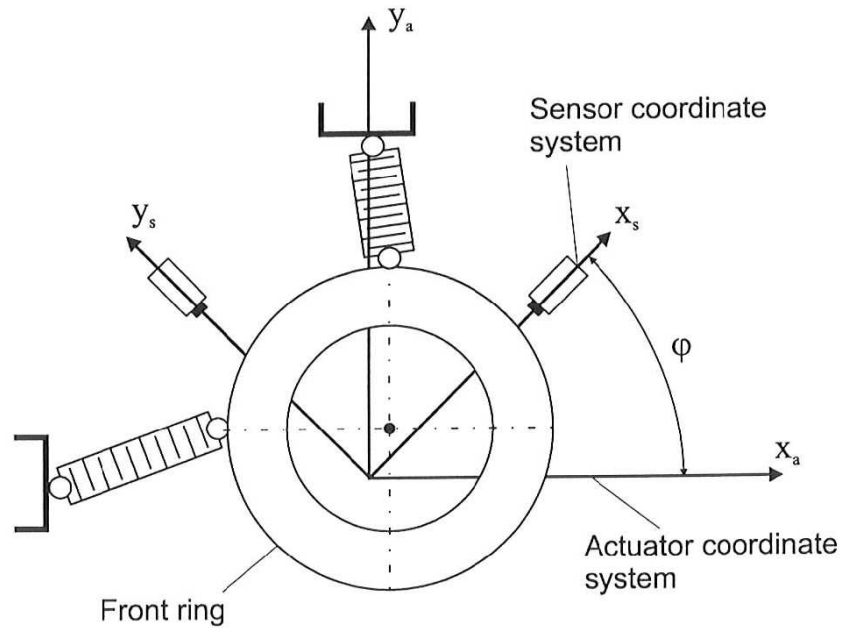


Fig. 5.8: Actuator mounting

The Matlab xPC Target prototyping environment is used to carry out the real-time control applications. The Matlab xPC-Target prototyping environment consists of a host computer and a target computer, which are connected via Ethernet link with capacity 10 Mb/sec as shown in Fig. 5.9, Hynek [105].

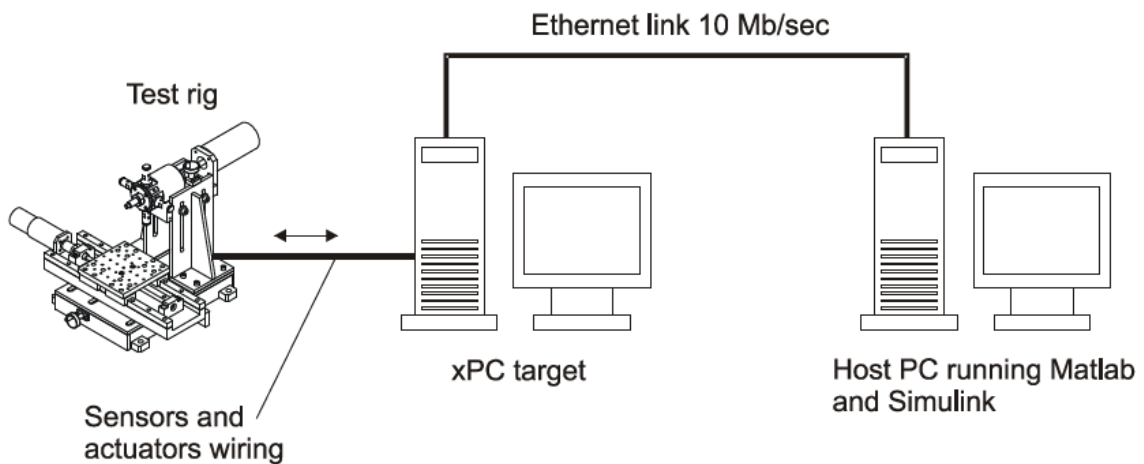


Fig. 5.9: Real time application

In order to test the effectiveness of the proposed strategy a complete model of the system is necessary. The model of the spindle, the piezo-actuator model, the cutting process and the model of the controller will be developed. The complex modelling phase will be described in the following sections.

The Fig. 5.25 pointed out the designed control strategy; it is similar to the scheme typically used for state-space control and optimal control.

An observer allows estimating the states of the system using only the displacement measurements from the eddy current probes. The estimates of the states are used to compute the proper control strategy ($\propto -K \cdot \hat{x}$) where $\hat{x}$ are the state estimations. The designed observer is also used to estimate the cutting forces during the milling operations. This feature can be useful to monitoring the cutting process as shown by Albrecht [101] and by Park [102] but moreover to design a compensation control action that tries to reduce the tool tip deflection due to the cutting forces.

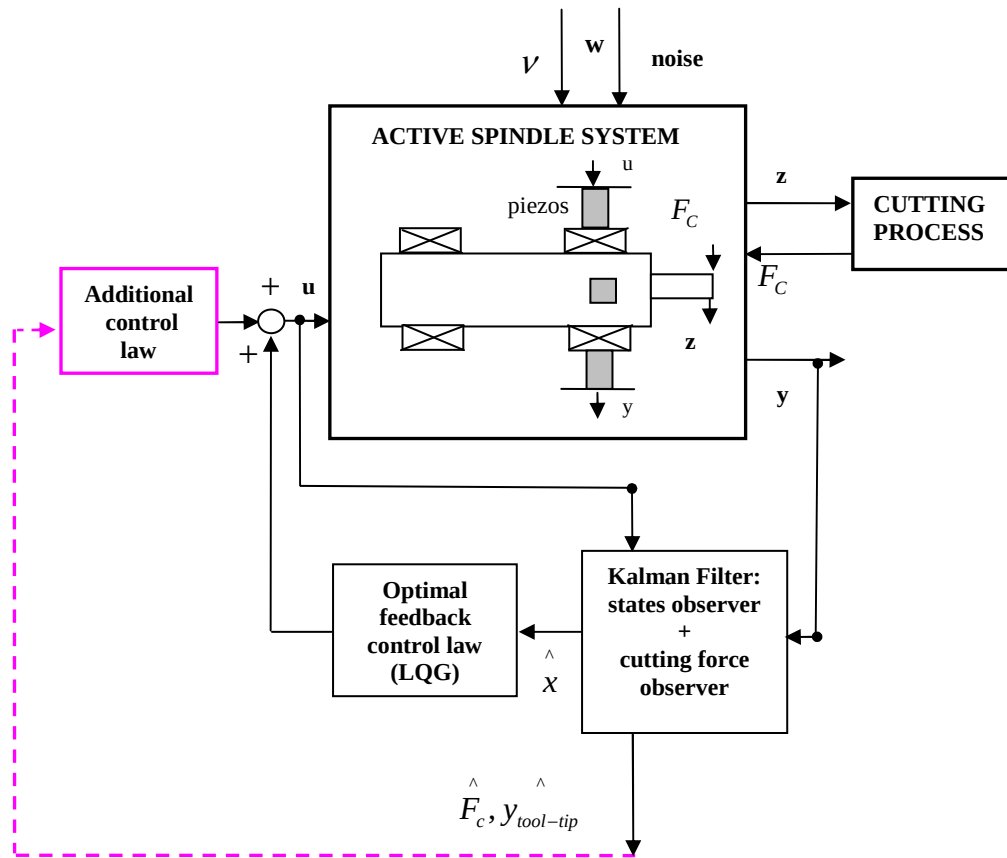


Fig. 5.10: Control strategy description

Where:

- $\hat{x}$: state estimation
- y : eddy current output measurements
- $y+v$: noisy measurements
- u : piezoelectric actuators input voltage
- w : white noise noise disturbance on the states

- ν : white noise on the measurements
- z . performance metric (tool tip displacement)
- $\hat{F}_c$: cutting force prediction
- F_c : cutting force (system disturbance or unknown system input)

5.4 Spindle Modelling

The overall smart spindle model is obtained by joining the FEM model of the mechanical part of the spindle to the piezo-actuators model as pointed out in Fig. 5.32.

5.4.1 FEM Spindle Modelling

The modelled spindle prototype is shown in Fig. 5.11, it was built by Hynek [105]. The spindle cutter head is not depicted in this picture. The spindle is mounted on a flexible frame.

A beam based FEM software is used to model the spindle system; this approach is generally suitable for “rotor system” modelling as described in Chapter 2. The FEM software was properly developed by Loughborough University and it is briefly described in Appendix A.2

The flexibility of the shaft, the bearing stiffness, the stiffness of the support, the inertial properties of the shaft and of the tool have been considered. In this simplified model the flexibility of the spindle housing hasn't been taken into account in order to get an easy FEM model.

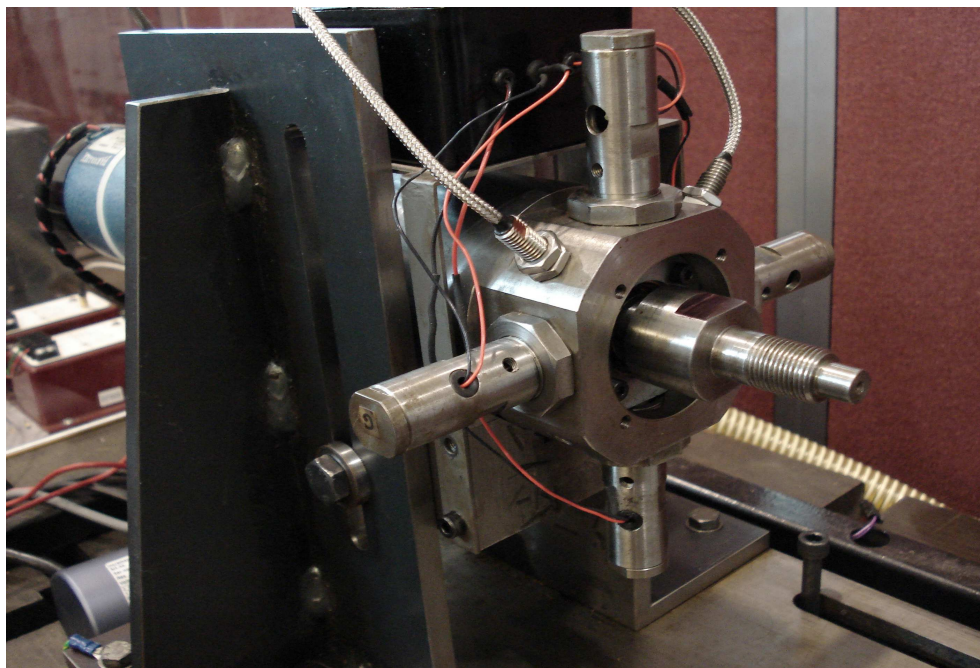


Fig. 5.11: Experimental test rig – active spindle (without tool)

The simplified spindle geometry is reported in Fig. 5.12; obviously some approximations are introduced in order to allow the implementation of the beam modelling approach.

The model also considers the inertial properties of the ring on which the piezo-actuators act.

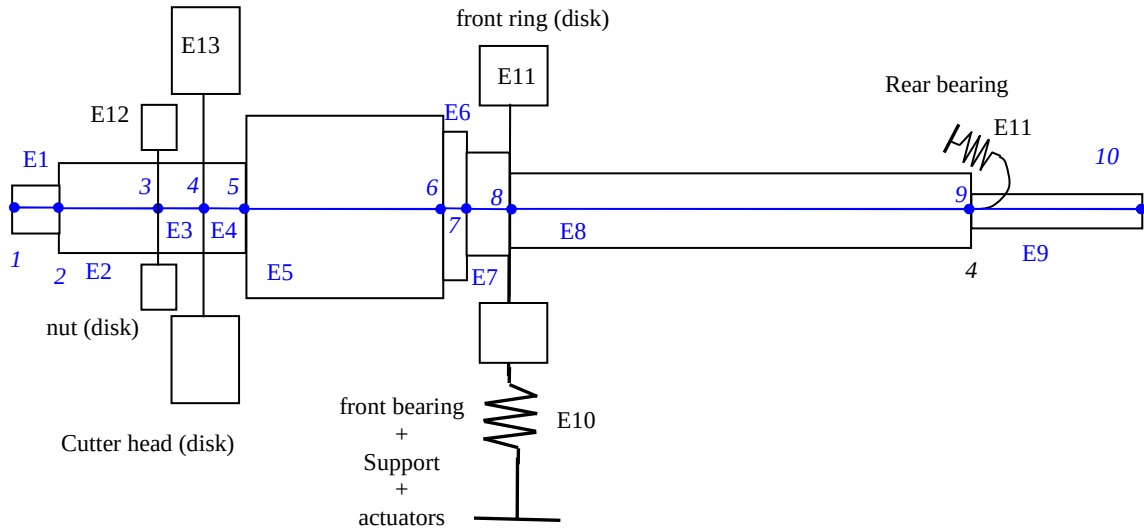


Fig. 5.12: Mechanical model of the spindle

The input file of the FEM model is reported in the Appendix A.3

The resulting FEM model is presented in Fig. 5.13. An infinite radial stiffness for the back bearing is assumed. The FEM package originally considers 6 dof for each node. The proposed model considers only 4 dof for each node because they are sufficient to describe the bending behaviour of the spindle.

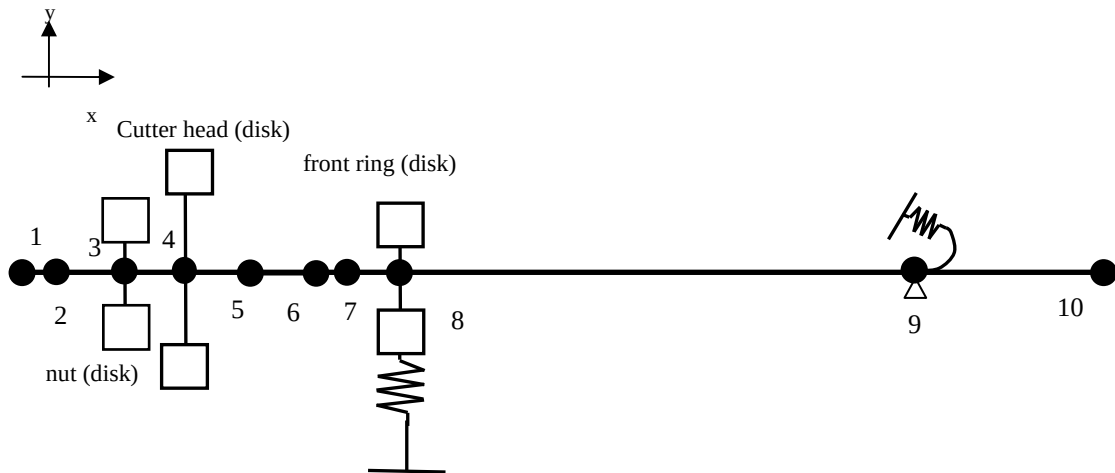


Fig. 5.13: FEM model of the spindle , Y or Z axis

The total number of dof is 38 dof (10x4-2 constraints), a 76th order model is therefore obtained.

Obviously the spindle model has a symmetrical behaviour in the orthogonal planes X-Y and Z-X.

The damping can be introduced into the system using the following relationship but only the stiffness matrix proportional term β has been used.

Eq. 5.1:
$$B = \alpha \cdot M + \beta \cdot K$$

A part of the damping is also introduced using the damping coefficient related to the model of the bearing (spring-damper component).

The damping was tuned in order to obtain the damping ratios of the first eigenmodes close to 2-3% that are typical values for analogous systems (from experimental data and modal identification procedure).

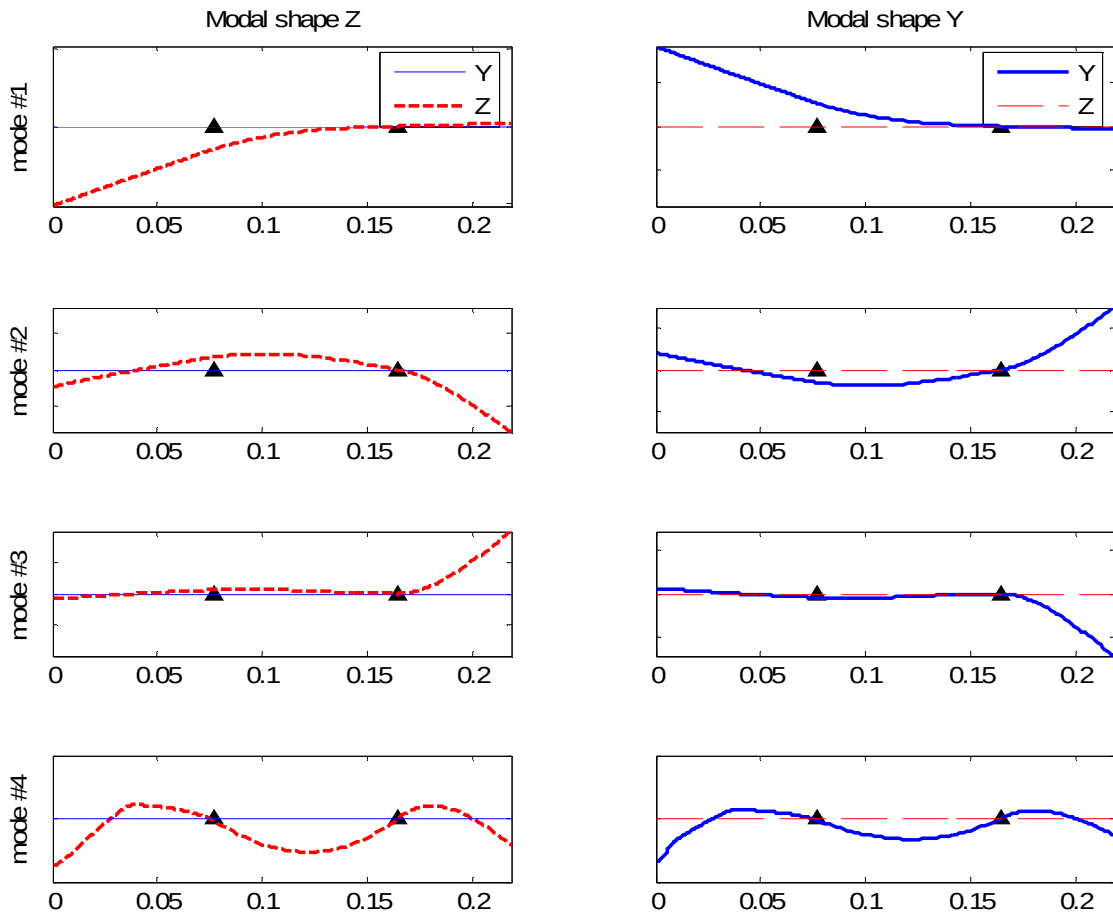


Fig. 5.14: Modal shapes - FEM model

It’s important to observe that the poles of the system and consequently all the modal parameters will change when the overall model of the active spindle is obtained.

The eigenvector analysis is reported Fig. 5.14, the corresponding eigenvalues analysis is shown in Tab. 5.1.

Only the first four eigenfrequencies are reported.

<i>mode</i>	<i>Frequency [HZ]</i>
<i>Mode #1</i>	502
<i>Mode #2</i>	1590
<i>Mode #3</i>	1998
<i>Mode #4</i>	6361

Tab. 5.1: Frequencies

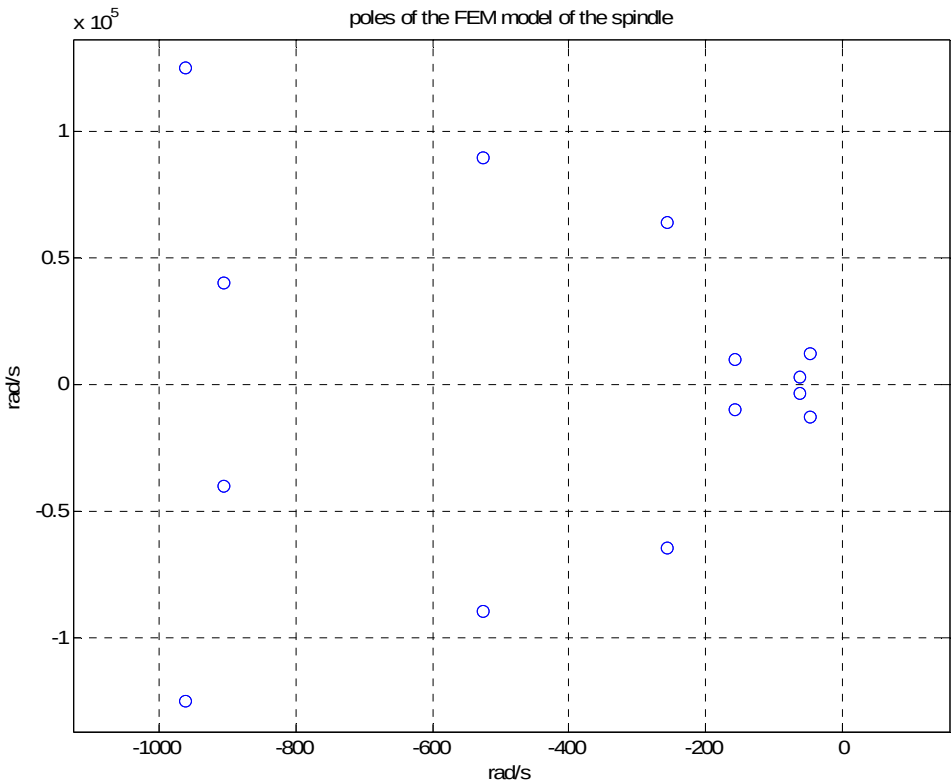


Fig. 5.15: Poles of the FEM spindle model

In order to comprehend the dynamic behaviour of the system the corresponding poles are plotted in Fig. 5.15. Moreover, the modal damping ratios are reported in Fig. 5.16 and it is clear as the damping introduced by the elastic elements affects the damping ratios of the first modes.

An equivalent representation of the real-imaginary plot is reported in Fig. 5.17.

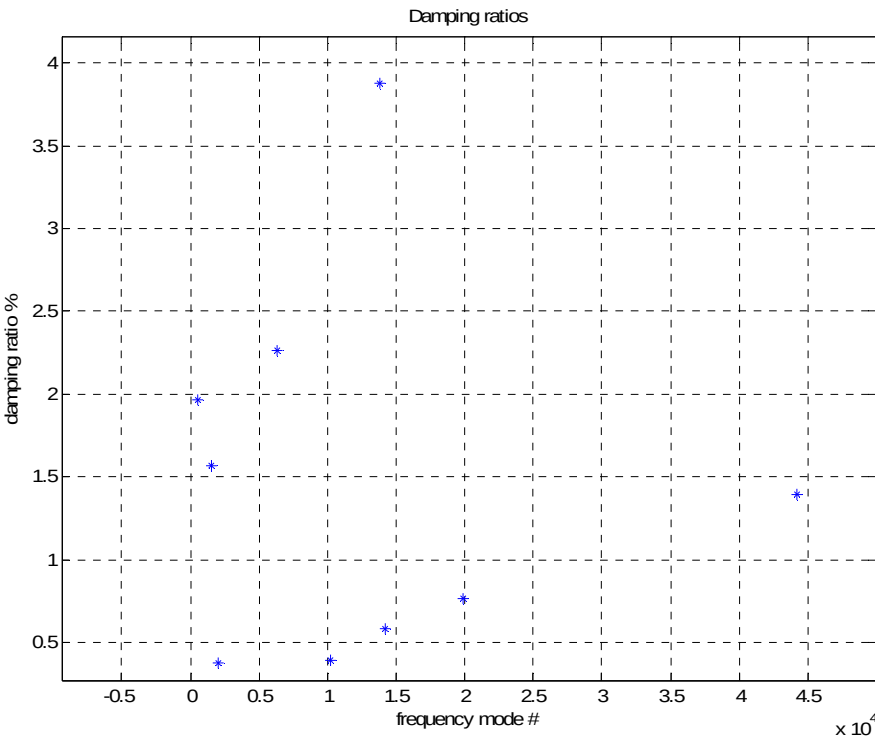


Fig. 5.16: Damping ratios FEM model

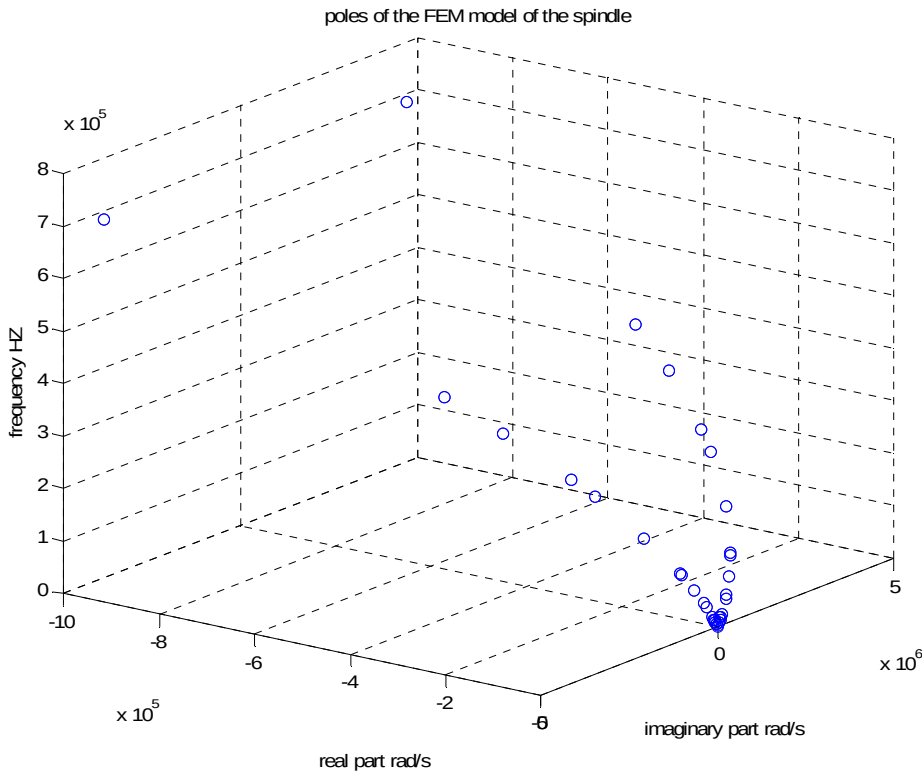


Fig. 5.17: Poles FEM spindle model - real-imaginary-frequency

5.5 Piezoelectric actuators

The Piezoelectric actuators are electromechanical transducers, which convert electrical energy into mechanical energy. They are widely used in different kinds of applications ranging from static and dynamic micro positioning, optics, smart structures, precision machining and active vibration control.

They are capable of providing large displacements in micrometer range and generating large forces with typical frequency bandwidth up to 30 KHz. The maximum force delivered by the piezoelectric actuator is usually in the range from 0.2 kN up to 10 kN depending on its stiffness. The maximum displacement of the piezoelectric actuators is in the range from 2 μm for single layer actuators up to 200 μm for multi layered large stacks.

Piezoelectric actuators are manufactured from piezoelectric ceramic which becomes electrically polarized if it underlay mechanical stress. This characteristic of the piezoelectric ceramic is termed as direct piezoelectric effect and it is widely used to manufacture sensors such as accelerometers and load cells. Conversely, if the electric field is applied to piezoelectric ceramic, it expands its shape in direction of the polarization. This is known as inverse piezoelectric effect on which the piezoelectric actuators are based on. There are several different types of piezoelectric actuators.

The tree main types are: flexure elements, tube actuators and stack actuators.

The flexure elements is produced from thin piezoelectric ceramic strips which are combined into a bimorph like pointed out in Fig. 2.15.

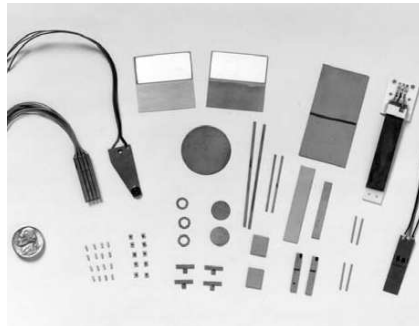


Fig. 5.18: Flexure elements

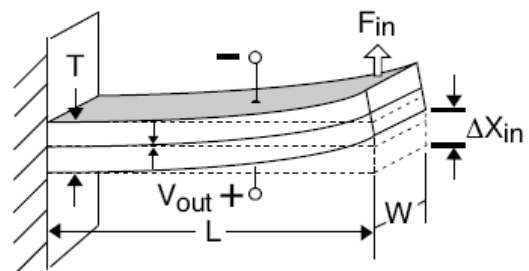


Fig. 5.19: Flexure element principle

The operative principle of this element is based on simultaneous contraction of one strip with the expansion of the other one thus it results in bending.

This type of actuator is capable of providing high displacements (i.e. up to few millimetres) with the drawback that the generated force is relatively low.

Piezoelectric ceramic tubes are monolithic actuators which contract laterally and longitudinally when a voltage is applied between the inner and outer electrodes. This type of actuator is widely used in scanning microscopes.



Fig. 5.20: Piezo stack actuator

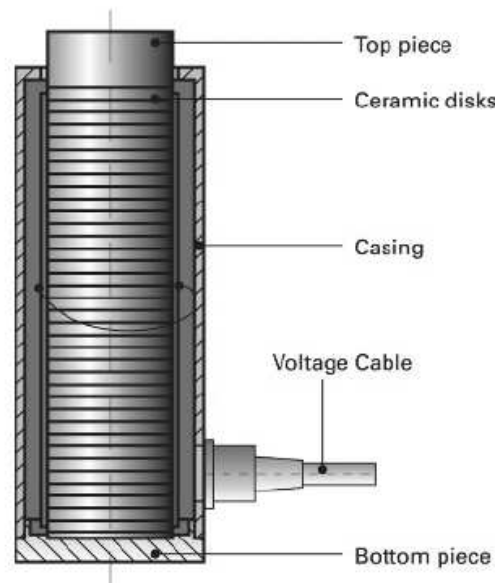


Fig. 5.21: Piezo stack actuator - scheme

Stack type actuator is constructed as a stack of thin piezoelectric ceramic layers as described in Fig. 5.20 and Fig. 5.21. The stack expands if voltage is applied to its electrodes.

This type of actuator is able to deliver relatively large displacements (i.e. $<200\ \mu\text{m}$) and very high force (i.e. $<30\ \text{kN}$).

Due to their characteristics (delivering relatively large displacement with high force) the stack type piezoelectric actuators are widely used for precision machining, vibration cancellation (chatter suppression) and dynamic positioning of structures and parts.

5.5.1 Piezoelectric actuator modelling

In Fig. 5.22 a simple scheme of stack type piezo actuator is shown. We can observe some geometrical properties of the stack actuator and how the layers are mechanically connected (series) and moreover the voltage system (V) used to polarize the layers (parallel connection).

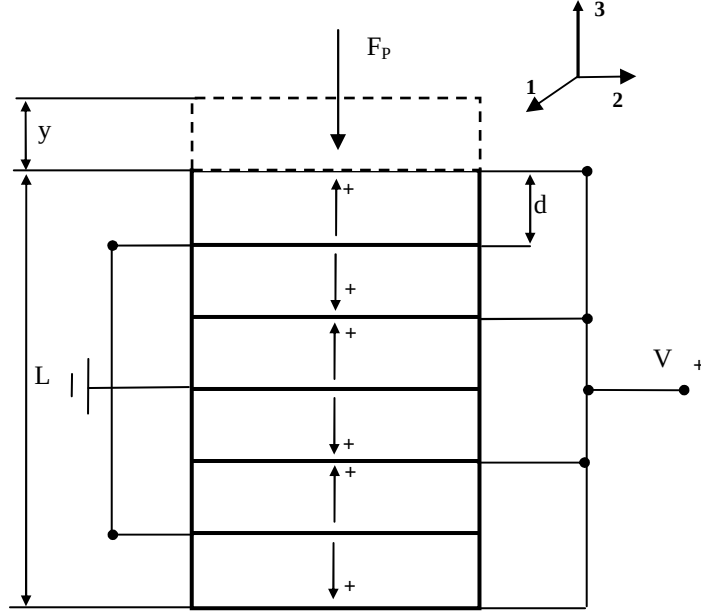


Fig. 5.22: Piezoelectric stack type actuator

Where d is the thickness of each layer and n is the number of layers of the actuator, consequently $L = n \cdot d$ is the overall length of the stack and “A” is the cross-sectional area. Generally the thickness of single layer is very small (0.1-0.3 mm) comparing with the cross-sectional area.

The ceramic piezo-actuators have anisotropic behaviour that depends on the analyzed direction, Fig. 5.22. Considering the “direction 3” as the main direction the following equations can be written, Hynek in [105] and Devos in [113]:

Eq. 5.2:

$$e_3 = S_{33}^E \cdot \sigma_3 + d_{33} \cdot E_3$$

$$D_3 = d_{33} \cdot \sigma_3 + K_3^\sigma \cdot \epsilon_0 \cdot E_3$$

Where:

- E_3 : electrical field
- D_3 : electrical displacement
- K_3^σ : relative dielectric constant at constant mechanical stress
- S_{33}^E : mechanical compliance at constant electric field
- d_{33} : strain constant relating to mechanical strain to applied electrical field

Moreover, actual piezoelectric actuators have non-linear characteristics. The non linear behaviour is caused by the fact that the piezoelectric constants depend on applied voltage and mechanical stress developed in the actuator.

As a result, the piezoelectric actuator shows hysteresis, look at Fig. 5.23. Typical values are 10-15% for “soft” piezoelectric (low stiffness) actuators and 1% for “hard” ones (hard stiffness).

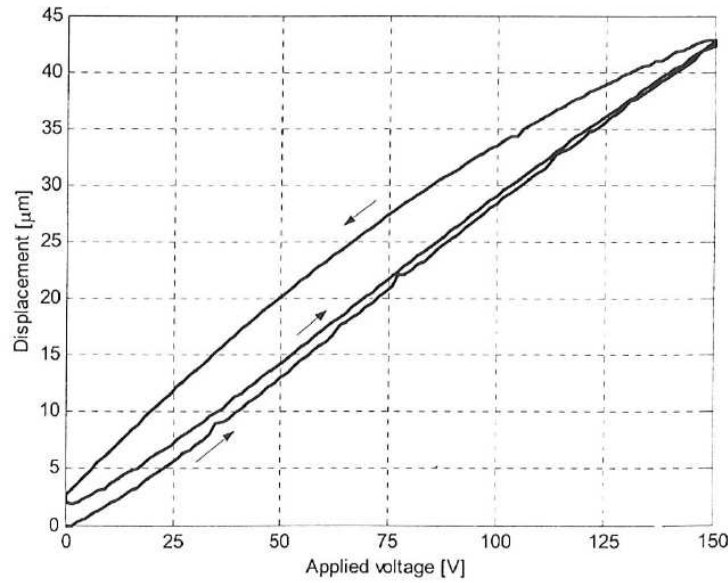


Fig. 5.23: Piezo-actuator non-linear behaviour

And σ_3 is the mechanical stress: $\sigma_3 = \frac{F_p}{A}$ and e_3 is the related strain $e_3 = \frac{y}{L}$.

The following Eq. 5.3 and Eq. 5.4 are used to evaluate the electrical displacement D_3 and the electrical field E_3 :

$$\text{Eq. 5.3} \quad D_3 = \frac{q}{n \cdot A}$$

$$\text{Eq. 5.4} \quad E_3 = \frac{V}{d}$$

Substituting the Eq. 5.3 and Eq. 5.4 in the Eq. 5.2:

$$\begin{aligned} \text{Eq. 5.5:} \quad y &= \frac{S_{33}^E}{A} \cdot L \cdot F_p + n \cdot d_{33} \cdot V \\ q &= n \cdot d_{33} \cdot F_p + n \cdot \frac{A \cdot K_3^\sigma \cdot \epsilon_0}{d} \cdot V \end{aligned}$$

And defining the quantities C_p and k_p , respectively the capacitance and the mechanical stiffness of the piezo-actuator by the Eq. 5.6:

$$\text{Eq. 5.6:} \quad C_p = n \cdot \frac{A \cdot K_3^\sigma \cdot \epsilon_0}{d}$$

$$k_p = \frac{A}{S_{33}^E \cdot L}$$

So the Eq. 5.7 can be obtained:

$$\text{Eq. 5.7:} \quad y = \frac{1}{K_p} \cdot F_p + n \cdot d_{33} \cdot V$$

$$q = n \cdot d_{33} \cdot F_p + n \cdot C_p \cdot V$$

That explains the mechanical-electrical behaviour of the piezo-actuator.

Considering a constant voltage V , Fig. 5.24 shows the force-displacement characteristic. The “blocked force” and the “no load displacement” can be defined (Eq. 5.8)

$$\text{Eq. 5.8:} \quad F_p = (y - n \cdot d_{33} \cdot V) \cdot k_p; y = 0 \rightarrow$$

$$\text{maximum force} \rightarrow F_{p0} = -k_p \cdot n \cdot d_{33} \cdot V$$

$$y = \frac{1}{K_p} \cdot F_p + n \cdot d_{33} \cdot V; F_p = 0 \rightarrow$$

$$\text{maximum displacement} \rightarrow y_0 = n \cdot d_{33} \cdot V$$

The piezoelectric material is manufactured from Lead Zirconate Titanate (PZT). It is necessary to notice that in Eq. 5.2 the hysteresis effects haven't taken into account in order to obtain a linear model of the active spindle.

In many analyzed works from the specific literature authors didn't consider the non-linearity of the piezoelectric actuators too.

Obviously this involves less accuracy in the model but allows us to use the theory of linear systems.

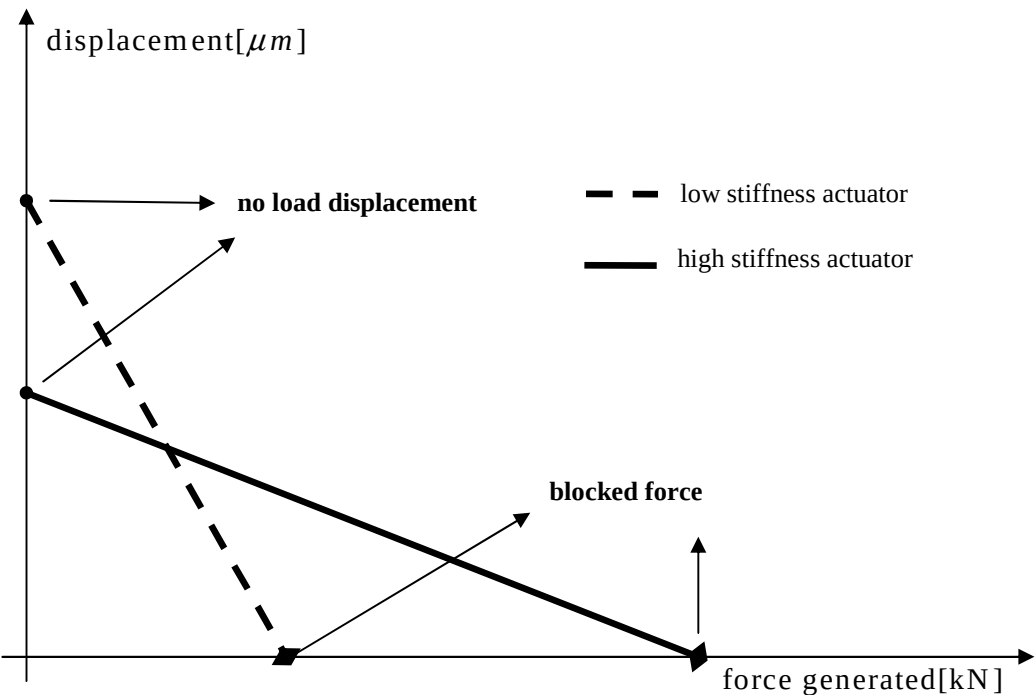


Fig. 5.24: Force-displacement characteristic ($V=cost$)

Generally the piezo data sheets report the “blocked force” and the “zero load displacement” values.

Property	Symbol	Value	Units
Strain constant	d_{33}	230 – 700	$10^{-12} \frac{m}{V}$
Relative dielectric constant	K	1300 – 2400 -	-
Compliance	S_{33}	13 – 20	$10^{-12} \frac{1}{Pa}$

Tab. 5.2: Typical piezo-material properties (PZT)

Quantity	Symbol	Value	Units
Max input voltage	V	150	V
Actuator stiffness	K_p	14	N/mm
Zero load displacement @ 150V	V_0	40	μm
Blocked force	F_{p0}	0.56	kN
Actuator capacitance	C_p	2.3	μF
Cross-sectional area	A	25	mm^2
Length	L	19.5	mm

Tab. 5.3: Actuators specification

5.5.2 Equivalent mechanical model

An equivalent mechanical model that describes this behaviour is an infinitely stiff pusher connected in series with a spring, where the displacement of the infinitely stiff pusher is proportional to the applied voltage (i.e. $n \cdot d_{33} \cdot V$ term in equation Eq. 5.7 and the spring stiffness is equal to the actuator stack stiffness k_p).

The mechanical model reported in Fig. 5.25 shows a typical push-pull configuration. This configuration is used to avoid tensile strength that can damage the actuators itself. Each piezo-actuator is able to act only in one direction, applying a positive voltage; the displacement in the opposite direction is allowed by the action of the other actuator.

Another design expedient used to protect the actuator against shear stress is shown in Fig. 5.8.

Generally commercial actuators are protected against tensile strength by a preload spring but this however affects the effective displacement of the actuator.

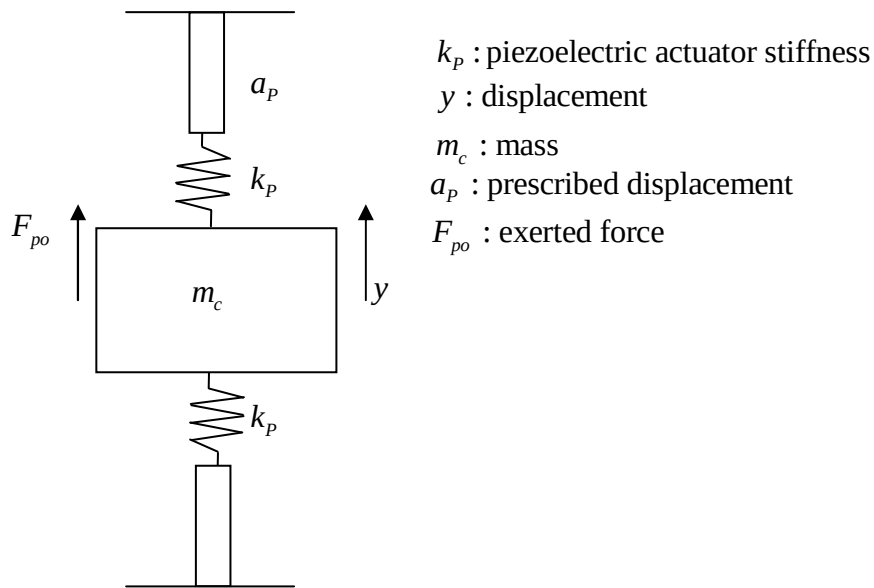


Fig. 5.25: Mechanical model of a piezo-actuator

Eq. 5.9:
$$F_{po} = k_p \cdot (a_p - y)$$

Eq. 5.10:
$$F_{po} \neq F_{p0}$$

Obviously the behaviour of the piezo-actuators depends on the environment in which it works.

Two different situations can be resumed in Fig. 5.26.

It is possible to graphically individuate the operative points of the two configurations as shown in Fig. 5.27 and Fig. 5.28.

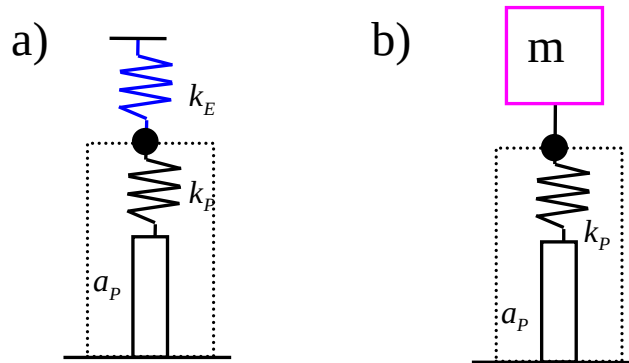


Fig. 5.26: Possible piezo environment – a) stiffness environment vs b) weight environment

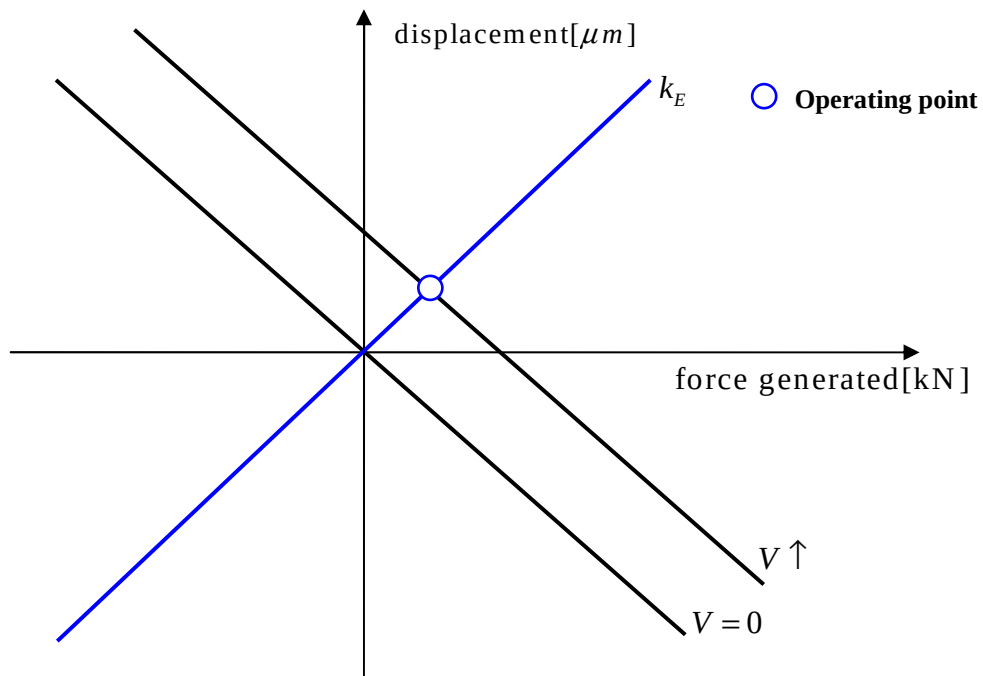


Fig. 5.27: Operating point – stiffness environment

The operative points can be found by imposing the equilibrium of the system that corresponds to the intersection of the curve that describes the piezo actuator and those that describes the external environment one.

The situation a) illustrated in Fig. 5.26 represents the situation of a push-pull configuration. This scheme can be used to model the active spindle depicted in Fig. 5.11.

The situation described in Fig. 5.26 b) can be illustrated in the following figure:

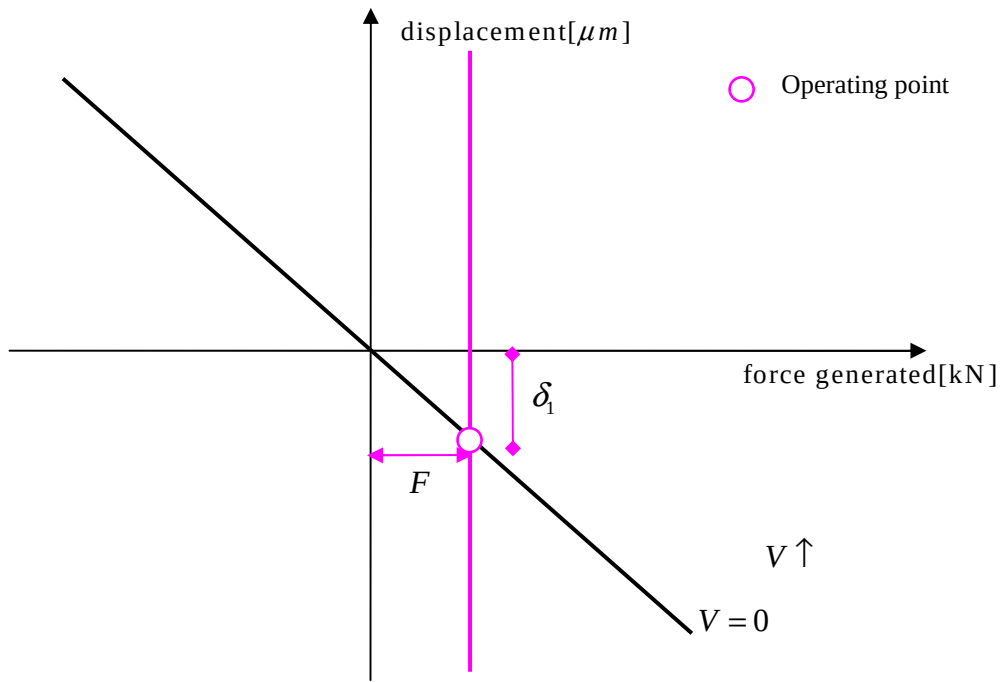


Fig. 5.28: Operating point – constant force environment

As previously described the push/pull configuration depicted in Fig. 5.25 allows a reduced displacement due to the opposite actuator but this configuration shows a stiffness that is double compared to the non push-pull one if the opposite actuator has the same stiffness.

This can be also graphically shown in Fig. 5.29:

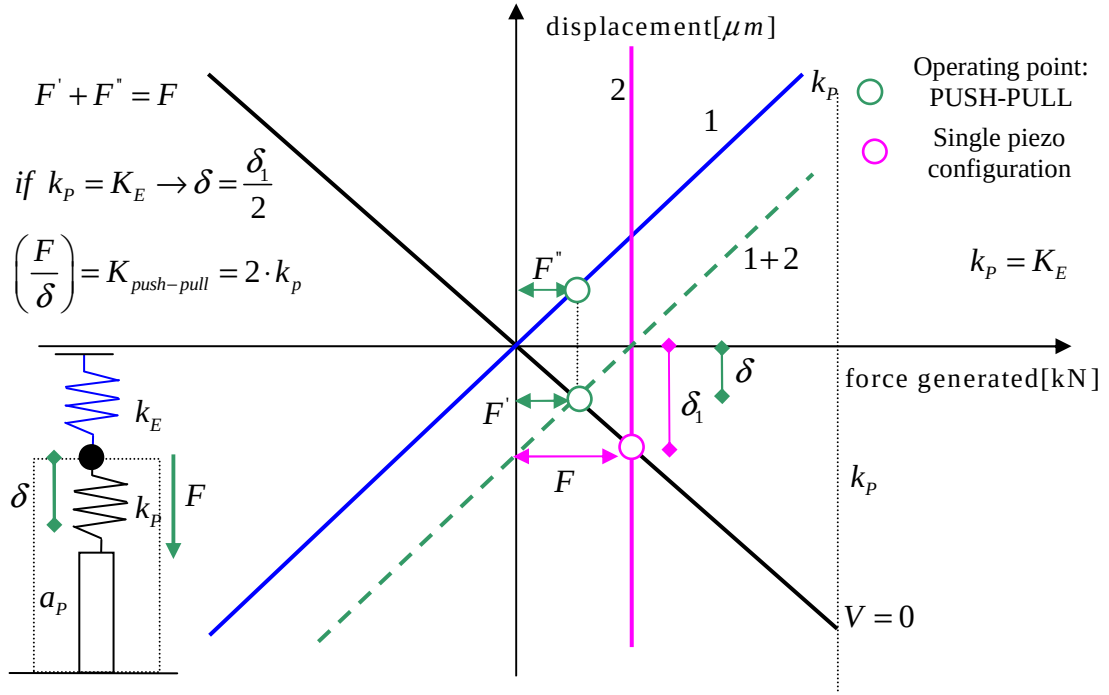


Fig. 5.29: Push-pull configuration – stiffness increment

5.5.3 Electrical model

The model presented in the previous subsection is related only to the piezo-actuator. The following figure consider also the driving amplifier that imposes the input voltage. In this case the piezo-actuator is considered as a capacitor and R_0 is the output resistance of the amplifier.

Writing the Kirchhoff's equations to the electrical system of Fig. 5.30:

$$\text{Eq. 5.11:} \quad V_i = R_0 \cdot i + V_0$$

Where:

V_i : amplifier input voltage

V_0 : voltage applied to the actuator

And $i = \frac{dq}{dt}$ is the driven current.

Eq. 5.12 is obtained from Eq. 5.7.

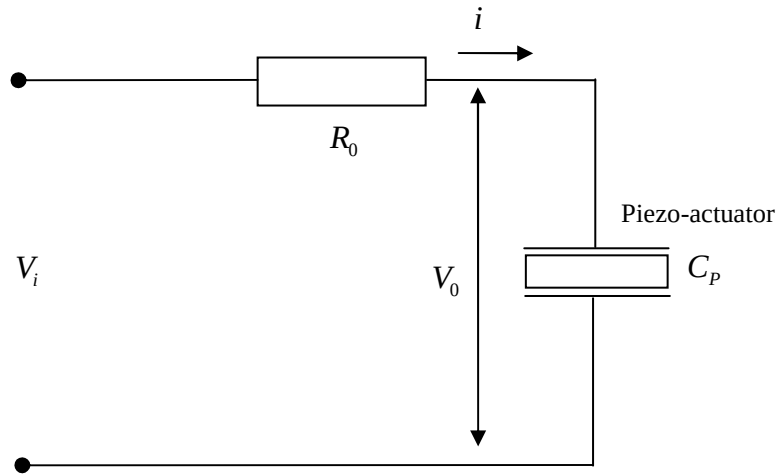


Fig. 5.30: Electrical model of the piezoelectric actuator

$$\text{Eq. 5.12:} \quad i = \frac{dq}{dt} = k_p \cdot n \cdot d_{33} \frac{dy}{dt} - k_p \cdot n^2 \cdot d_{33}^2 \cdot \frac{dV_0}{dt} + C_p \cdot \frac{dV_0}{dt}$$

$$\text{Eq. 5.13:} \quad \eta = n \cdot d_{33}$$

$$\text{Eq. 5.14:} \quad \frac{dV_0}{dt} = \frac{1}{R_0 \cdot (C_p - k_p \cdot \eta_p^2)} \cdot \left(V_i - V_0 - R_0 \cdot k_p \cdot \eta_p \cdot \frac{dy}{dt} \right)$$

$$\text{Eq. 5.15:} \quad F_{po} = -k_p (y - \eta_p \cdot V_0)$$

Is the force that acts on the structure (pay attention to the sign, it is opposite to the force depicted in Fig. 5.22).

Using Eq. 5.15 a State space Model can be obtained:

$$\left\{ \frac{dV_0}{dt} \right\} = \left[-\frac{1}{R_0 \cdot C_1} \right] \cdot \{V_0\} + \left[0 \quad -\frac{C_2}{C_1} \quad \frac{1}{R_0 \cdot C_1} \right] \cdot \left\{ \begin{matrix} y \\ \frac{dy}{dt} \\ V_i \end{matrix} \right\}$$

Eq. 5.16:

$$\left\{ \begin{matrix} F_{po} \\ V_0 \\ i \end{matrix} \right\} = \left[\begin{matrix} C_2 \\ 1 \\ -\frac{1}{R_0} \end{matrix} \right] \cdot \{V_0\} + \left[\begin{matrix} -k_p & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_0} \end{matrix} \right] \cdot \left\{ \begin{matrix} y \\ \frac{dy}{dt} \\ V_i \end{matrix} \right\}$$

$$C_1 = C_p - k_p \cdot \eta_p^2$$

$$C_2 = k_p \cdot \eta_p$$

Where

$$\left\{ \begin{matrix} y \\ \frac{dy}{dt} \\ V_i \end{matrix} \right\} = u; \{V_0\} = x; \left[\begin{matrix} F_{po} \\ V_0 \\ i \end{matrix} \right] = y$$

Eq. 5.17:

$$A = \left[-\frac{1}{R_0 \cdot C_1} \right]; B = \left[0 \quad -\frac{C_2}{C_1} \quad \frac{1}{R_0 \cdot C_1} \right]$$

$$C = \left[\begin{matrix} C_2 \\ 1 \\ -\frac{1}{R_0} \end{matrix} \right]; D = \left[\begin{matrix} -k_p & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_0} \end{matrix} \right]$$

And

Eq. 5.18:

$$\dot{x} = A \cdot x + B \cdot u$$

$$y = C \cdot x + D \cdot u$$

For the considered test rig:

$$\begin{aligned}
 k_p &= 14 \frac{N}{\mu F} \\
 \text{Eq. 5.19: } C_p &= 2.3 \mu F \\
 \eta_p &= 0.23 \frac{\mu m}{V}
 \end{aligned}$$

η_p can be determined from the following:

$y_0 = n \cdot d_{33} \cdot V$ in a zero load configuration.

So

$$\begin{aligned}
 \text{Eq. 5.20 } C_1 &= C_p - k_p \cdot \eta_p^2 = 2.3 \mu F - 14 \frac{N}{\mu m} \cdot \left(0.23 \frac{\mu m}{V} \right)^2 = 1.5594 \mu F \\
 C_2 &= k_p \cdot \eta_p = 14 \frac{N}{\mu m} \cdot 0.23 \frac{\mu m}{V} = 3.22 \frac{N}{V}
 \end{aligned}$$

$$\text{Eq. 5.21: } R_0 = 65 \Omega$$

So the matrix of the state space representation Eq. 5.18 are reported in Eq. 5.22.

$$\begin{aligned}
 \text{Eq. 5.22: } A &= [-9.8657]; B = [0 \quad -2.0649 \cdot 10^6 \quad 9.8657] \\
 C &= \begin{bmatrix} 3.22 \\ 1 \\ -0.0154 \end{bmatrix}; D = \begin{bmatrix} -14 \cdot 10^6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0154 \end{bmatrix}
 \end{aligned}$$

We can notice that the model of the piezo-actuator has only one state, the voltage V_0 .

Hynek [105] underlined the effects of R_0 that is the resistance of the amplifier on the piezo actuator characteristics: the blocked force and the external stiffness. Basically the value of R_0 affects the bandwidth of system, this is pointed out in Fig. 5.31.

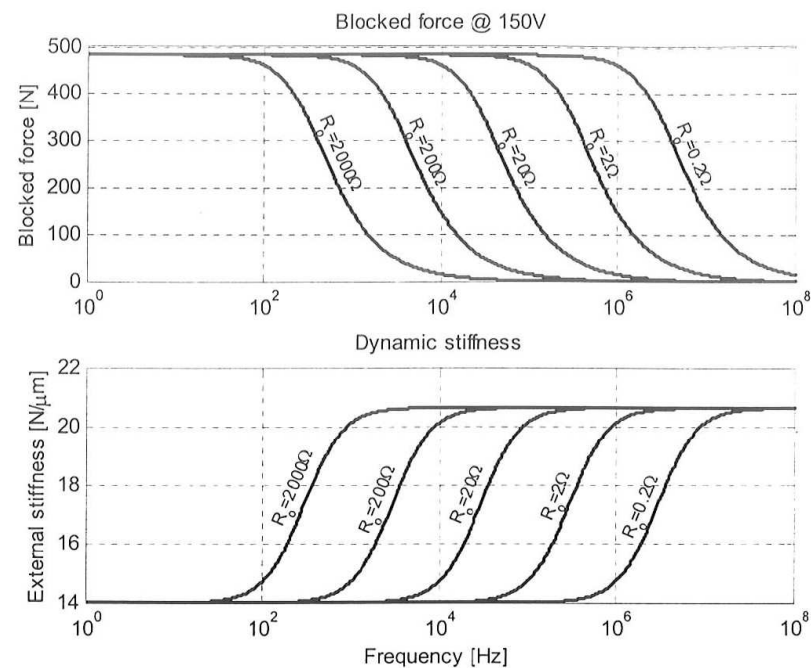


Fig. 5.31: Effects of the electrical resistance

5.6 Smart Spindle overall model

The model of the smart spindle is obtained as depicted in Fig. 5.32. The state space model of the piezo-actuators described in the previous section is connected to the FEM model of the mechanical part of the spindle. Both models are in the state space formulation.

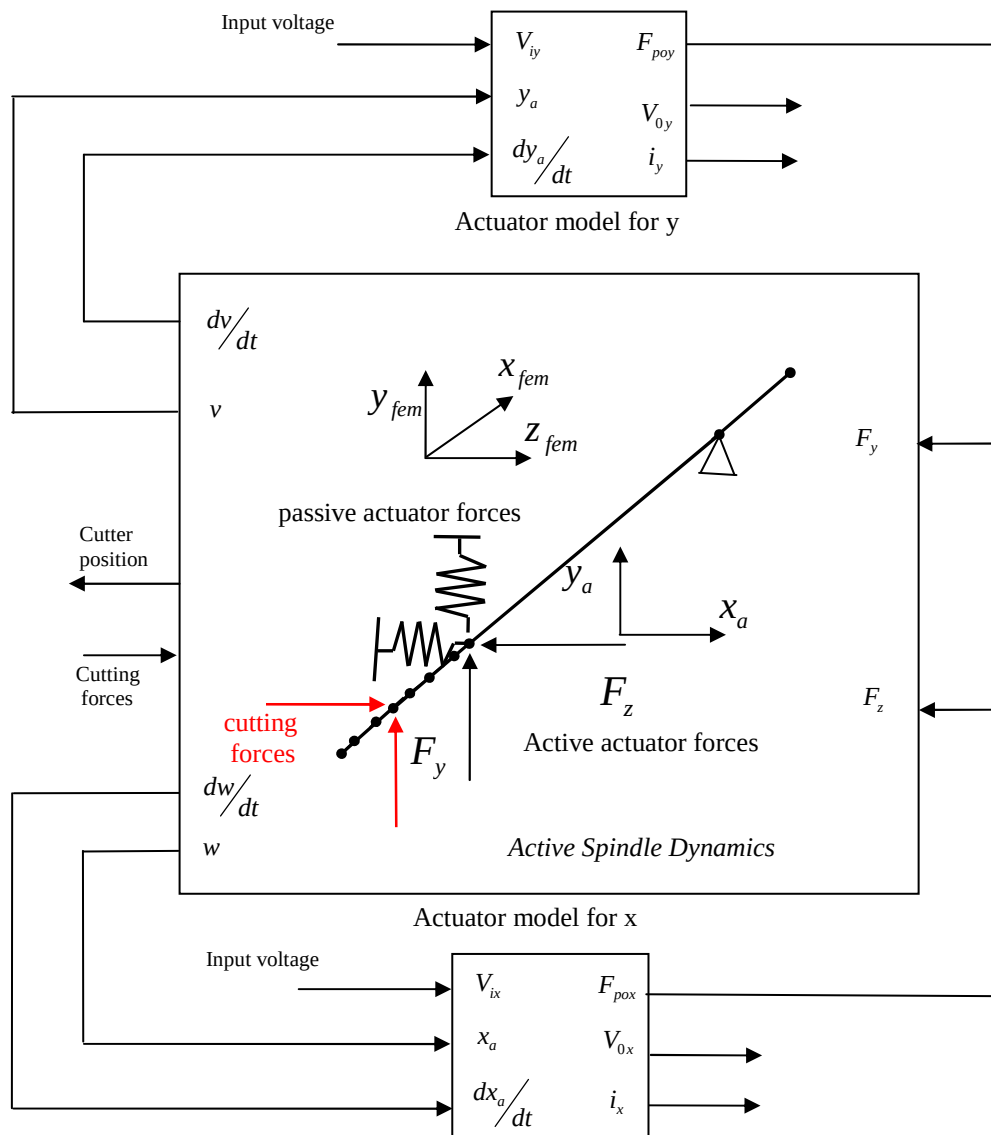


Fig. 5.32: Active spindle model

The piezo-actuator acts on the shaft of the spindle with a force, the feedbacks from the FEM spindle model are used by the piezo model to determine the states

of the system and obviously the linked outputs such as the voltage on the piezo device and the driving currents.

The overall model of the spindle has 78 states: 76 of these are linked to the FEM spindle model, as formerly described and two states, one for each actuator.

A schematic representation of the overall model is proposed also in Fig. 5.33.

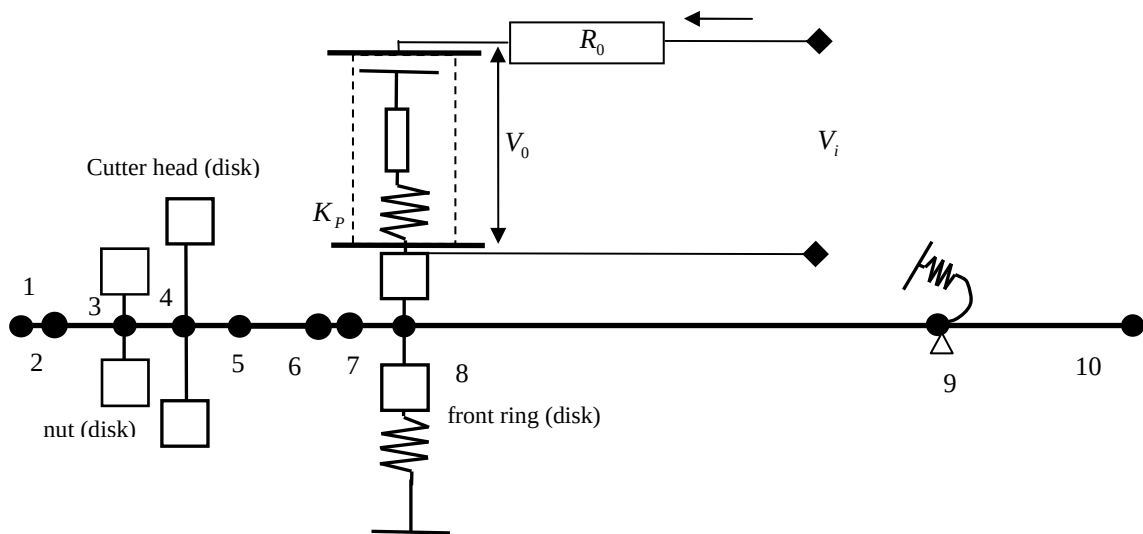


Fig. 5.33: FEM model of the spindle and piezo actuator model

After having obtained the model of the active spindle the eigenmode-eigenvalue properties of the system have been evaluated. The following figures point out the effects of the actuators on the mechanical behaviour of the spindle.

It is necessary to outline that the FEM model hasn't a proper real physical meaning if the actuators models aren't considered. The presence of the actuators stiffens the system, this effect, for example, can be observed looking at Fig. 5.34: there was a migration of the low frequency poles toward higher frequency values. Moreover the damping is affected by the piezo models.

The spindle becomes more rigid due to the push-pull effect, adding a piezo-actuator is equivalent to put a parallel stiffness. The explanation of this effect can be appreciated in Fig. 5.29.

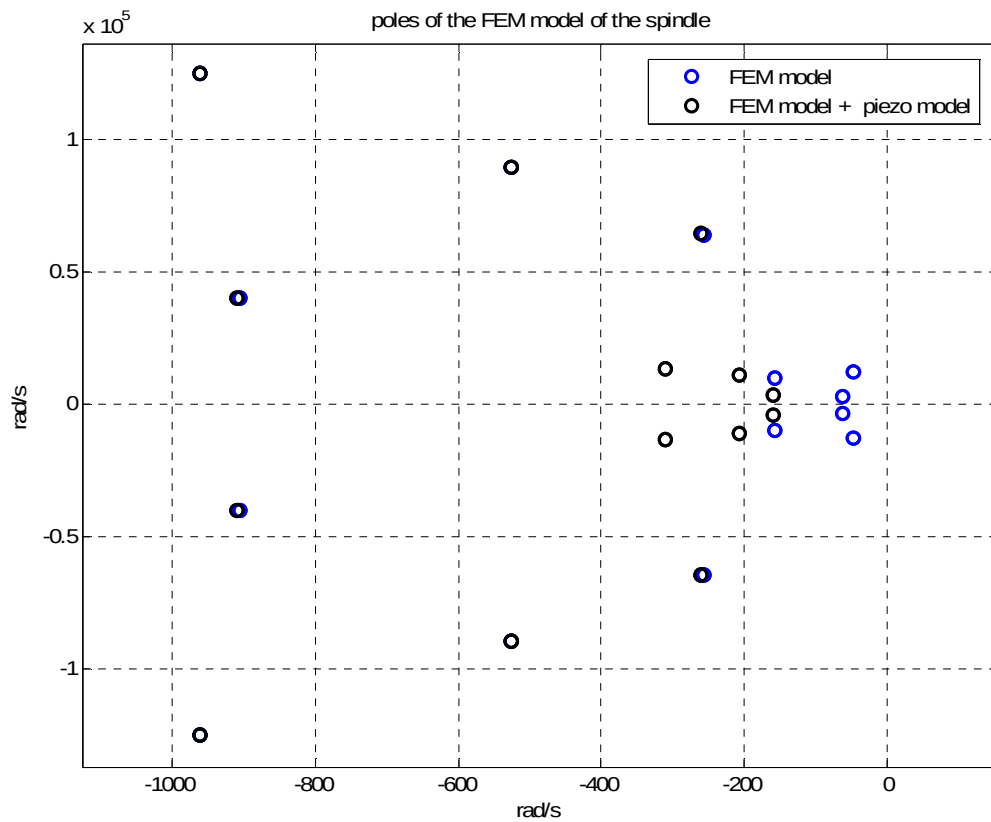


Fig. 5.34: Poles of the smart spindle system - FEM model + piezo model

This effect can be obviously well noticed especially looking at the first eigenmode that is the strongly influenced by the stiffness of the front support. This global stiffness change causes a consequent increase of the damping ratios because damping terms, that are proportional to the stiffness, have been introduced.

The global damping ratios have been tuned in order to obtain 2-5% that are typical damping ratio values for this kind of systems.

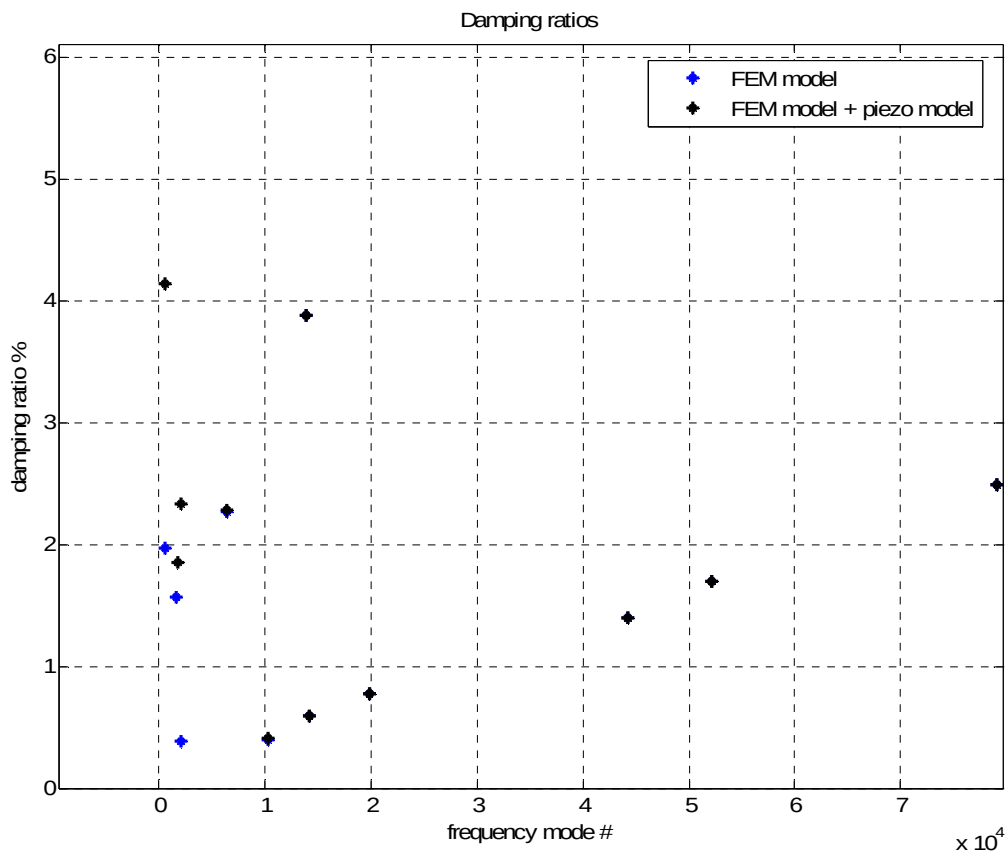


Fig. 5.35: Damping ratios smart spindle system

In Tab. 5.4 are reported both the frequency values and damping ratios of the first four modes.

Fig. 5.36 shows the poles of the active system, underlining the frequency of each mode and furthermore points out the pole that is linked to the behaviour of the piezo actuator model. It's important to notice that it isn't the pole of the piezo but at this frequency the most "oscillating quantities" are related to the piezo component.

mode	Frequency [HZ]	Damping ratios %
Mode #1	617	4.1
Mode #2	1779	1.8
Mode #3	2118	2.3
Mode #4	6362	2.3

Tab. 5.4: Frequencies and damping ratios of the active system

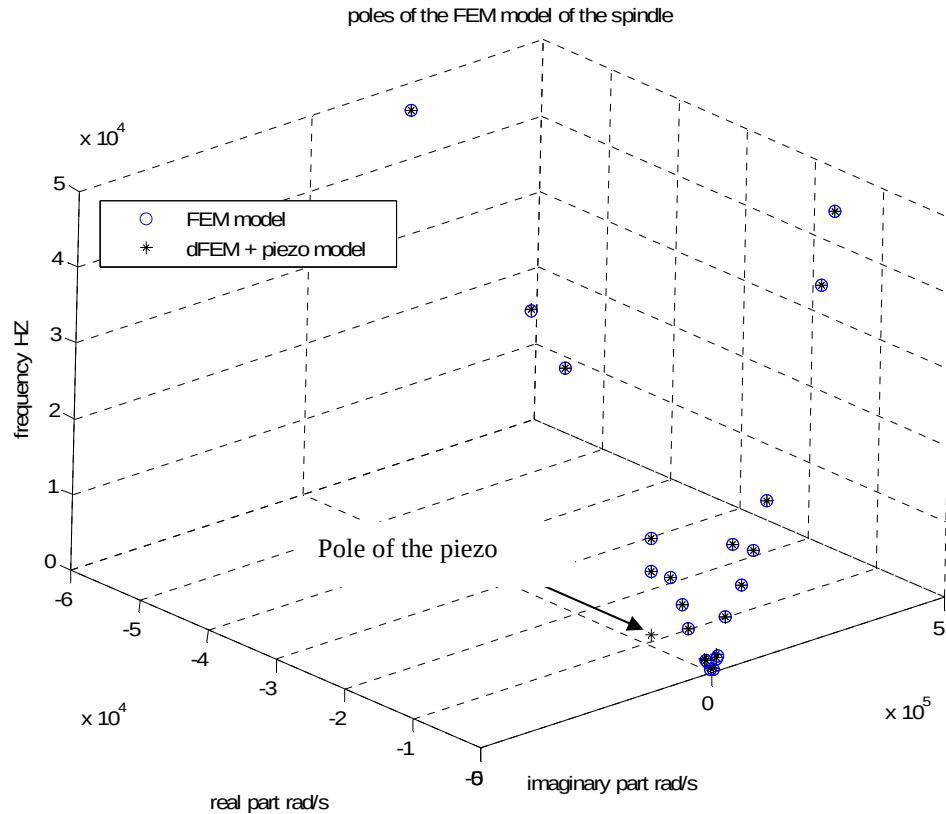


Fig. 5.36: Poles of the smart system – real-imaginary-frequency

In Fig. 5.37 and Fig. 5.38 are reported respectively the simulated dynamic compliance at the tool tip and the dynamic compliance probe displacement/tool tip force. It can be observed that the behaviour of the system, especially it is evaluated at the tool tip, is governed basically by the first dominant eigenmode @617Hz. The reason of this behaviour can be comprehended looking at the modal shapes of Fig. 5.14, the tool tip is close to be a nodal point for the second mode(@1779Hz) and the third one (@2118Hz).

This mode is very important for the cutting performance limitation; we know that the minimum of the real part directly affects the cutting stability limit.

The dynamic behaviour of the plant that is the system we are going to control is presented in the Frequency Response Function of Fig. 5.39.

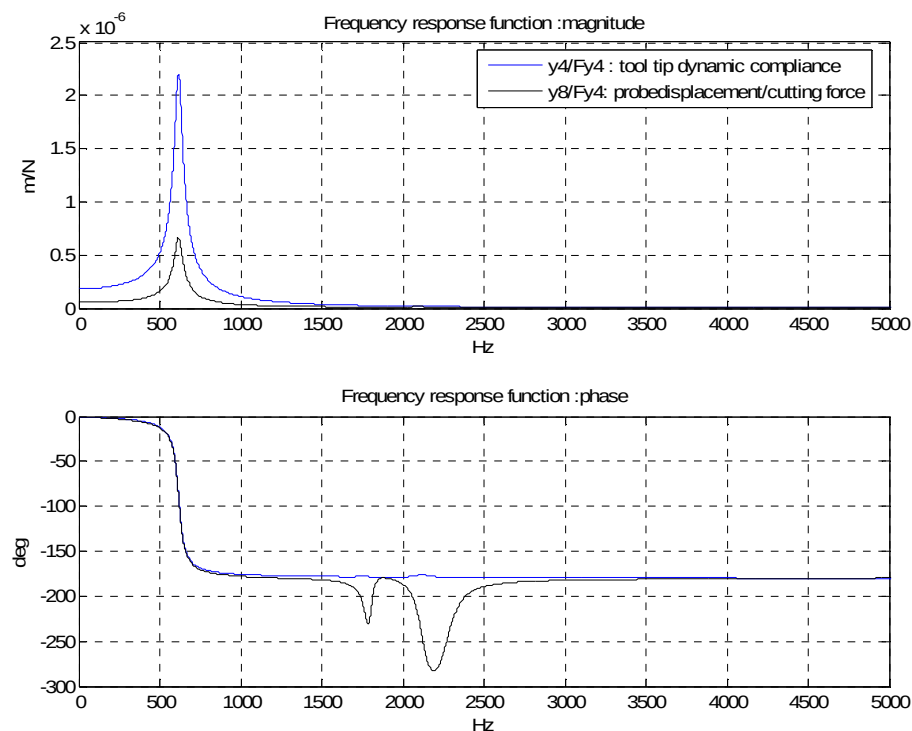


Fig. 5.37: Active spindle dynamics compliances

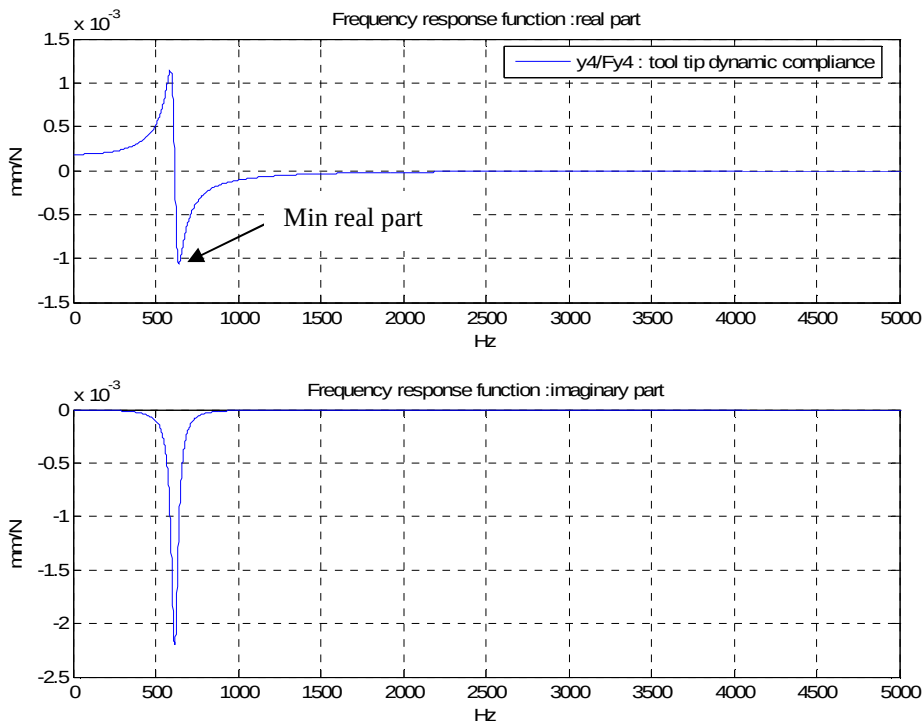


Fig. 5.38: Tool tip dynamic compliance – real vs imaginary part

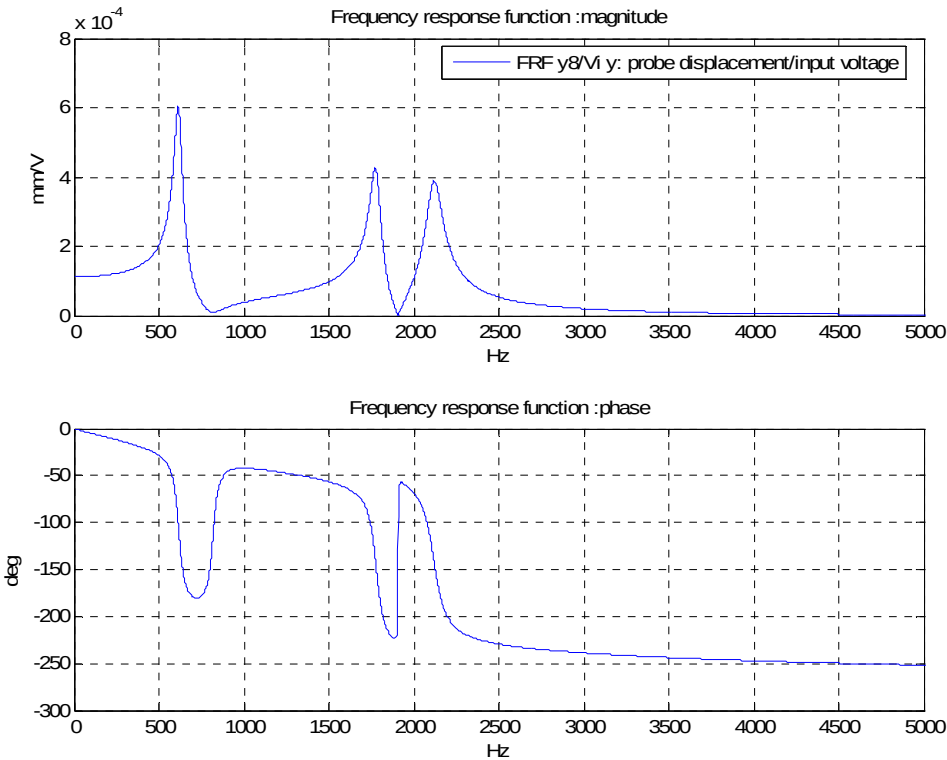


Fig. 5.39: PLANT dynamics - probe displacement/input voltage FRF

5.7 Cutting process modelling

Having modelled the active spindle it is necessary to add the model of the cutting process as illustrated in Fig. 5.40. The cutting process dynamically interacts with the spindle system: the resultant cutting forces from the process act on the tool tip but at the same time the consequent displacement of the tool tip affects the instantaneous chip thickness and so the cutting forces themselves.

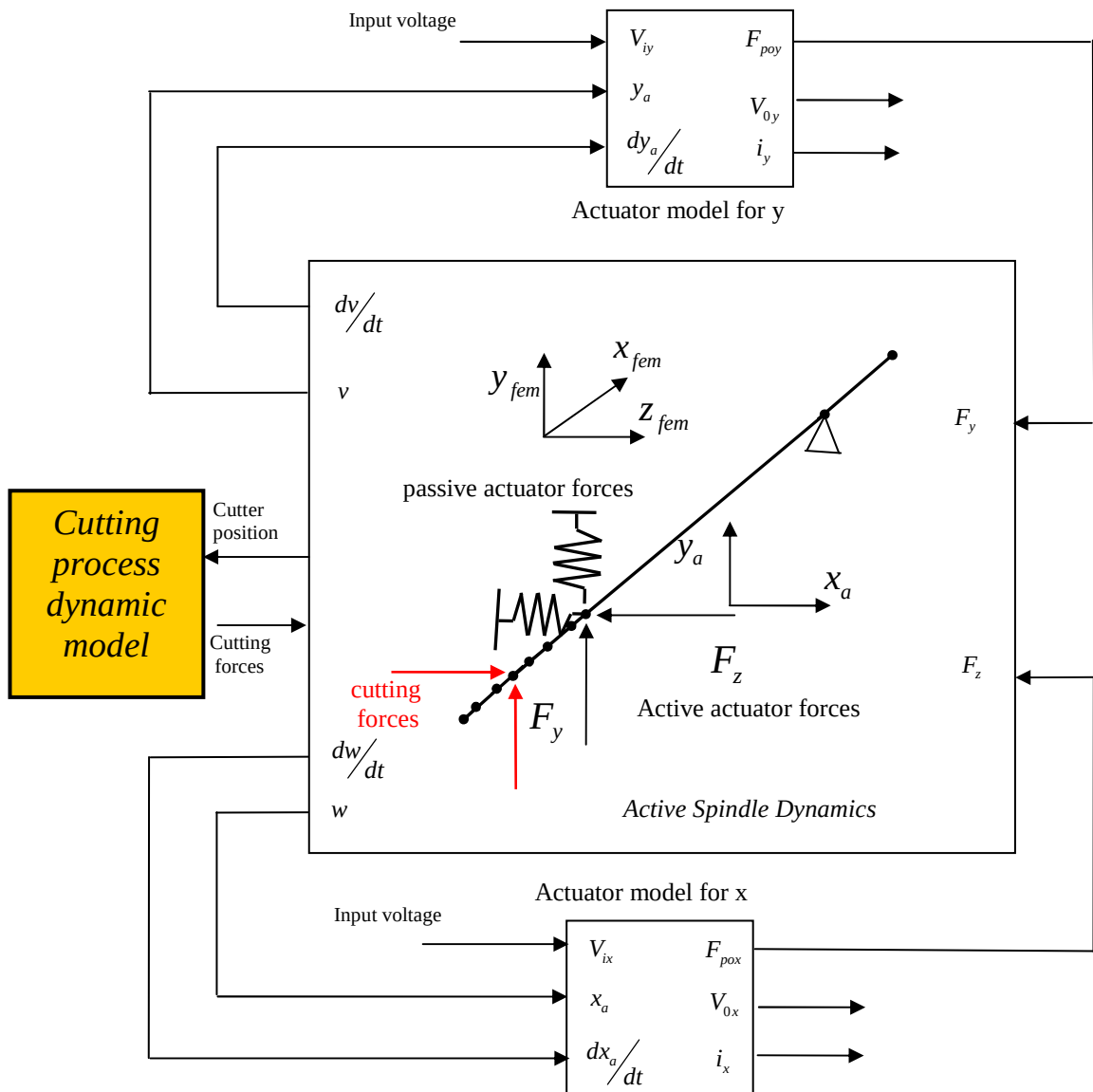


Fig. 5.40: Overall model -smart spindle and cutting process

The cutting process model used considers moreover the regenerative effects: the instantaneous chip thickness related to each cutter depends not only on the instantaneous position of the cutter that is working but on the position assumed formerly by the previous one.

The cutting process model has been described in chapter 4.3.

Using this described model it is possible to analyze the cutting process stability and the expected enhancement that the control action can bring.

An aluminium milling operation has been selected, the cutting parameters and the linked cutting coefficients are reported in Tab. 5.5.

Cutting Parameter	Value
Number of teeth	8
Tool diameter [mm]	40
Axial depth of cut [mm]	0.1
Radial immersion [%]	100
Spindle speed[rpm]	4000
Feed [mm/tooth revolution]	0.2
Kre[N/mm]	30.8
Kte[N/mm]	27.11
Kae[N/mm]	1.4
Krc[N/mm ²]	168.8
Ktc[N/mm ²]	796.1
Kac[N/mm ²]	222

Tab. 5.5: Cutting parameters – aluminium milling

5.8 Control Design Strategy

The “Optimal Control” is the control technique we have chosen for this application, its basics are well explained in many references like [117], [120] and [115]. It is basically a strategy that attempts to place the poles of the system in order to obtain the desired response; the selection of the control gains is made easier than those used in the “pole placement method” by a minimization process of a cost function “J”. This functional takes into account, setting opportune weights, both the quality of the system response and the control effort. In other way the control gains can be determined defining the desired compromise between the two requisites.

It is necessary to recall that “optimal control” is a “modal based” technique that is it uses a model of the “plant” (system to be controlled).

This “optimal control technique” generally substitutes the “pole placement” one. Pole placement needs to select a $(n \text{ (number of states)} \times m \text{ (number of input)})$ gains matrix. Generally there are infinite possibilities to obtain the desired performance and moreover, often the designer doesn’t directly know the location where to place the poles.

Having defined a dynamic system in the state space formulation, Eq. 5.23, the control action can be expressed as $u = -K \cdot x$ (x is the vector containing the system states) for that reason it is necessary to predict the states of the system.

$$\begin{aligned} \dot{x} &= A \cdot x + B \cdot u + \Gamma \cdot w \\ y &= C \cdot x + D \cdot u + H \cdot w + v \end{aligned}$$

Where H and Γ are the matrix that relates the system noise w respectively to the states x and to the output of the system y . The v is the noise on the measurements. If the optimal control, generally called “LQR Linear Quadratic Regulator”, considers stochastic noise the optimal strategy is called “LQG Linear Quadratic Gaussian”.

Like formerly mentioned the control gains are obtain from the minimization procedure of the functional J that is defined as a sum of two quadratic terms: the first one considers how it is important to obtain a good system response ($x^T Q x$) and the second one ($u^T R u$) takes into account that the control action is always limited in real applications.

$$J = \frac{1}{2} \int_0^{T=\infty} (x^T Q x + u^T R u) dt$$

Where Q and R are the associated weight matrixes.
The optimal gain matrix can be evaluated using the Eq. 5.25.

$$K = -R^{-1} \cdot B^T \cdot P$$

Where P is obtained solving the Riccati differential equation, Eq. 5.26 that is obtain from the optimization problem definition.

$$\dot{P} + P \cdot A + A^T \cdot P - P \cdot B \cdot R^{-1} \cdot B^T \cdot P + Q = 0$$

The Riccati equation is solved numerically.

5.8.1 Observer Design

As before mentioned the LQR-LQG control strategy needs to know all the states of the system. In the many cases it is impossible to know all the system states but some measurements can be frequently available. For example in the analyzed spindle system the measurements of the shaft displacement are provided by the two eddy current sensors. In all the cases like this that is when

one or more system quantity measurements are available, the key is to estimate the states using only the measurements and the knowledge of the system behaviour.

The states prediction issue can be approached using a state observer that is called Kalman Filter when the optimal solution is being sought.

The observer is based on the scheme reported in Fig. 5.41. A replication of the system has been added to the original system, it's supposed that A , B , C are known. Furthermore it is supposed system without feed through so the matrix $D = 0$.

The replication of the system differs only for the unknown initial state $\hat{x}(0)$: the uncertainty on the states produces an error between the $\hat{y}$ and the real measurement and this error is also affected by the noise on the measurement. So it could be reasonable to correct the dynamic equations of the system with a term (L) that weights the difference between the output of the system and linked estimation.

$$\begin{aligned} \dot{\hat{x}} &= A \cdot \hat{x} + B \cdot u + L \cdot (y - \hat{y}) \\ \hat{y} &= C \cdot \hat{x} \end{aligned}$$

Eq. 5.27:

$$\dot{\hat{x}} = (A - L \cdot C) \cdot \hat{x} + L \cdot C \cdot x + L \cdot v + B \cdot u$$

Eq. 5.28:

The dynamic equation of the error is therefore:

$$e = x - \hat{x} \Rightarrow \dot{e} = \dot{x} - \dot{\hat{x}} = (A - LC) \cdot e - L \cdot v$$

Eq. 5.29:

The dynamic of the dynamic depends on the matrix $(A-LC)$. If the poles of this system were arbitrarily selected the dynamic of the error would be asymptotically stable and it would be selected how fast the error tends toward zero.

This is a typical problem of “pole placement”. The problem can be solved if the couple (A, C) is observable.

Like formerly mentioned for the optimal control, the problem can be solved minimizing a functional: the variance of the error σ_{ee}^2 . In this case the observer is called Kalman Filter.

The error variance can be computed using the Lyapunov equation:

Eq. 5.30:

$$(A-LC) \cdot \sigma_{ee}^2 + \sigma_{ee}^2 \cdot (A^T - C^T \cdot L^T) + [I \quad L] \cdot W \cdot [I \quad L]^T = 0$$

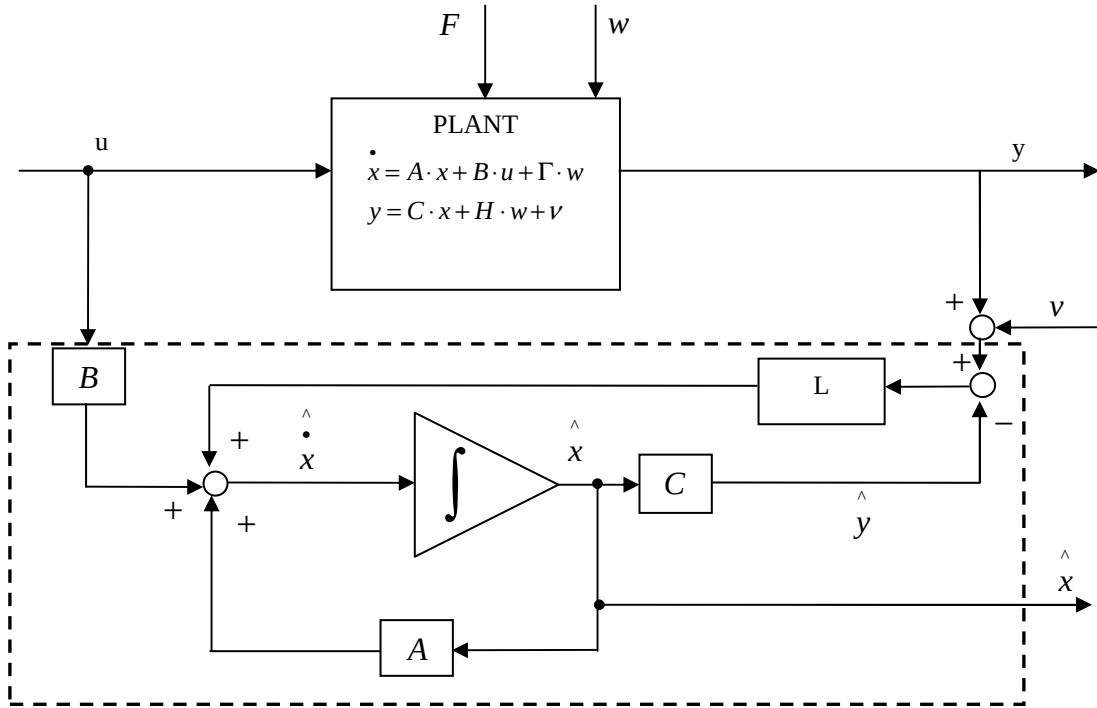


Fig. 5.41: Kalman Filter

Where the $W = \begin{bmatrix} W_{ww} & W_{vw} \\ W_{wv} & W_{rr} \end{bmatrix}$ is the covariance matrix of the white noise.

Generally $W_{ww} = Q$, $W_{vv} = R$ and $W_{rw} = W_{wr} = N$ (i.e. Matlab© code);

The optimal observer is the observer that minimizes the error variance, but the variance is a matrix so the minimum of the trace represents the proper formulation for the optimal observer.

Eq. 5.31: $\min_L (Tr(\sigma_{ee}^2))$

The optimal gain of the observer is:

Eq. 5.32: $L = \sigma_{ee}^2 C^T W_{vv}^{-1}$

Eq. 5.32 is formally analogous to the Eq. 5.25, in fact if L is substituted in Eq. 5.30 the following can be obtained:

$$\text{Eq. 5.33:} \quad A \cdot \sigma_{ee}^{-2} + \sigma_{ee}^{-2} \cdot A^T + W_{ww} - \sigma_{ee}^{-2} \cdot C^T \cdot W_{vv}^{-1} \cdot \sigma_{ee}^{-2} = 0$$

Fig. 5.33 is the algebraic Riccati equation where $\sigma_{ee}^{-2} = P$. In the algebraic

version of the Riccati equation $\dot{P} = 0$ that happens when $T \rightarrow \infty$ in Eq. 5.24.

It is important to note that there is a key principle called the “separation principle”, that permits to separately design the regulator and the states observer. Sometimes this can cause some stability problems. A technique called (LTR Loop Transfer Recovery) could be used to avoid these problems; the methodology takes into account that the open loop transfer function for the controller is completely different than the one obtained considering regulator and observer. These effects influence in a dual way the observer design.

Basically the behaviour of the Kalman Filter can be summarize considering the measurement uncertainty-states uncertainty ratio.

If this ratio is set to a high value the Kalman Filter gains L are low and the filter bases the states prediction basically on the model.

Conversely the filter relies basically on the measurements.

5.8.2 Reduced order model

In order to design the observer a reduced model of the “plant” has been obtained.

For the spindle case the full order active spindle model (78th order) has been considered as system plant and a reduced order model that considers only three modes of the system (6th order model) for the observer. In order to obtain an easy controller and considering that we are interesting in a limited frequency range, a reduced order model is advised.

In many applications taken from the specific literature, reduced models were used (i.e. Simoes [92]).

In the analyzed case reduced order model has been obtained from the full order state space one using the Matlab© function *balancmr*.

Fig. 5.42 pointed out the poles of the reduced system compared to those of the full order one.

Furthermore, Fig. 5.43 and Fig. 5.44 show the tool tip dynamic compliance (magnitude-phase and real-imaginary part).

The dynamic behaviour of the system evaluated at the tool tip mostly influences the cutting process stability and the asymptotically limit.

The reduced order seems to accurately approximate the behaviour of the plant to be controlled, this can be observed in Fig. 5.46, Fig. 5.47 and Fig. 5.43.

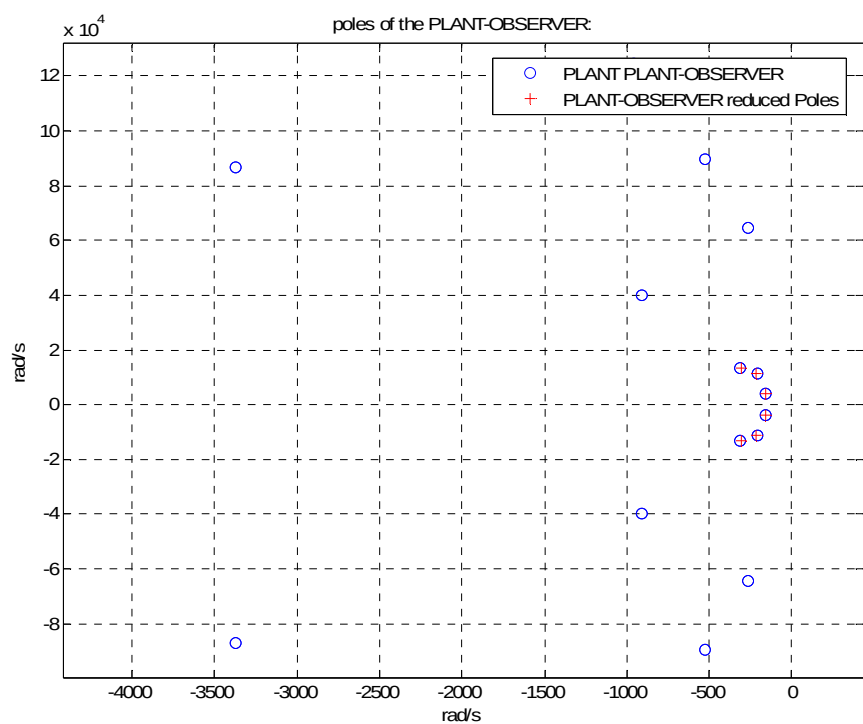


Fig. 5.42: Full order model vs reduced order model - poles

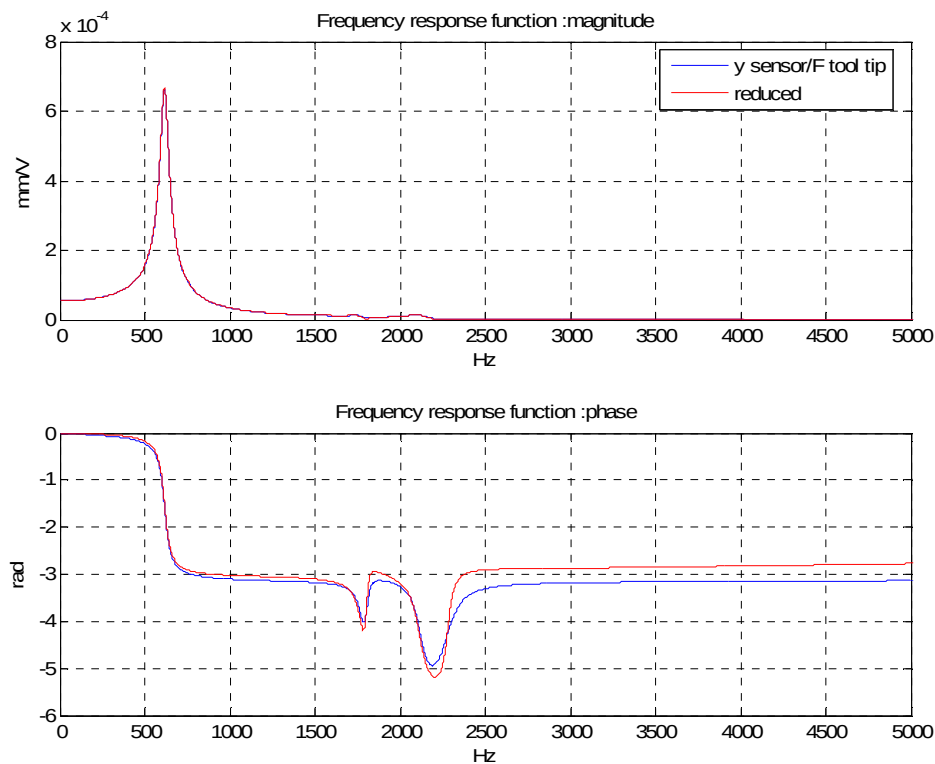


Fig. 5.43: FRF (magnitude-phase) y_{probe}/F_{cutt} (full order Vs reduced)

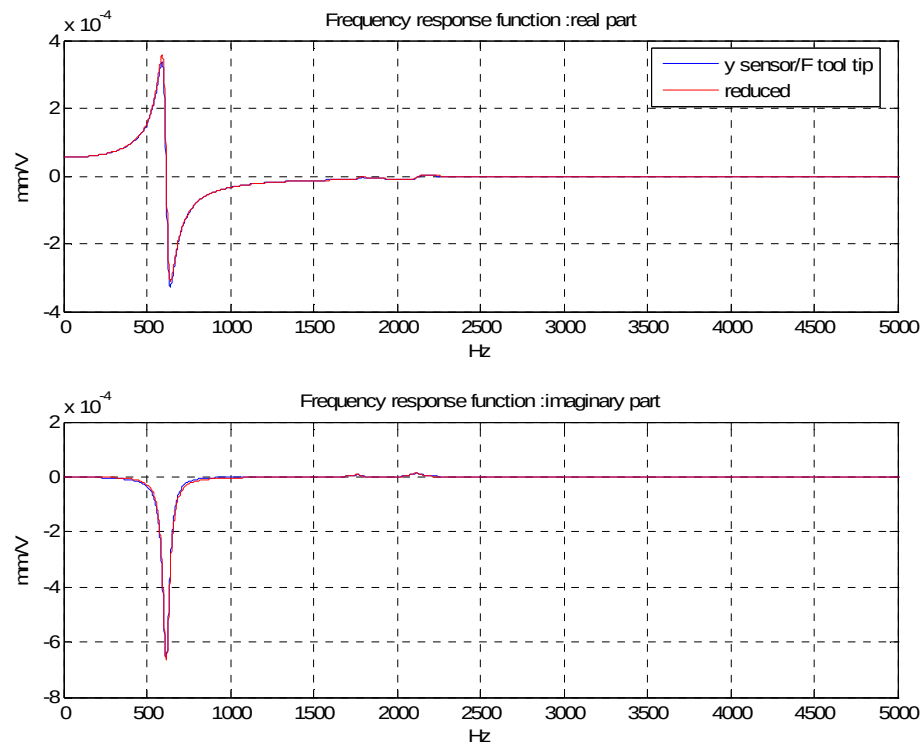


Fig. 5.44: FRF (real-imaginary part) y_{probe}/F_{cutt} (full order Vs reduced)

Obviously the reduced model doesn't approximate very well the behaviour of the plant especially close to the static behaviour of the system due to the residuals of the neglected modes. Conversely the approximation seems very good close to the resonance frequencies.

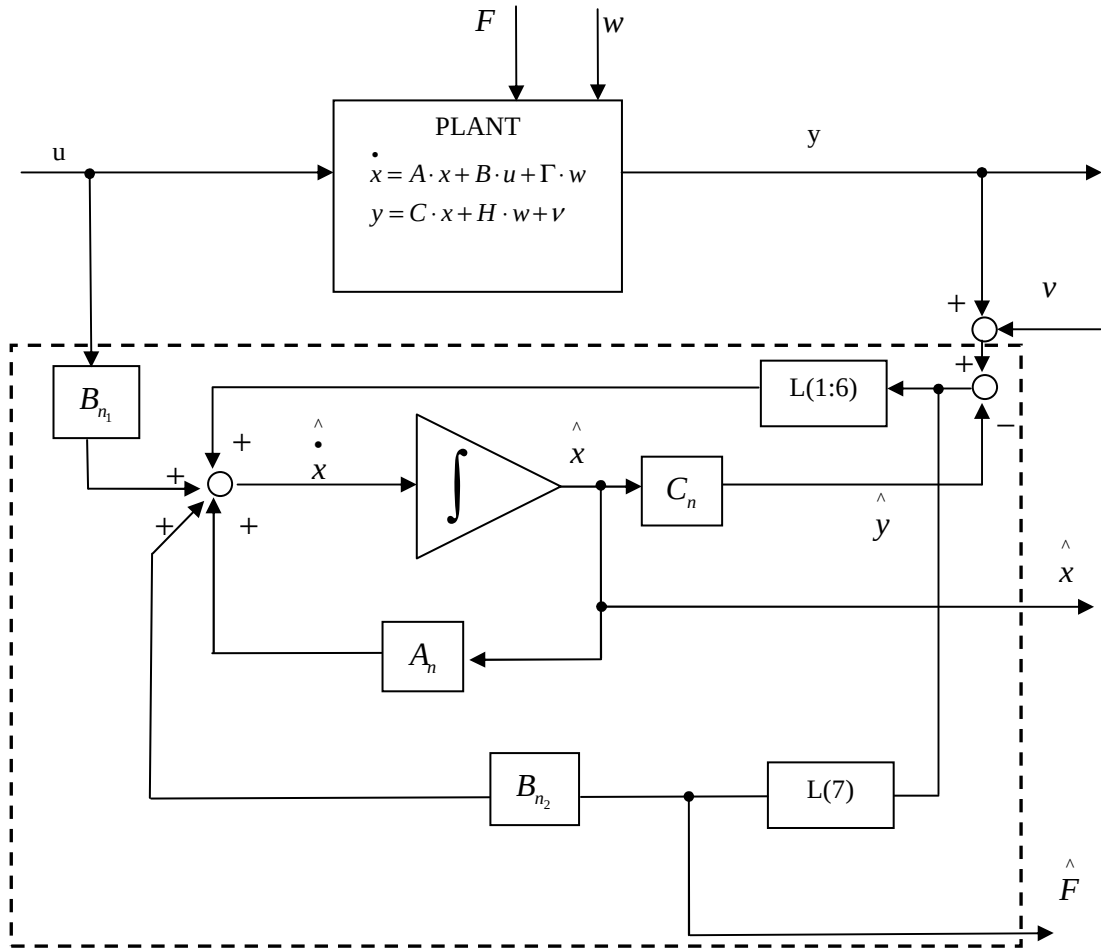


Fig. 5.45: Cutting force observer

Before designing the Kalman filter it is necessary to check the observability hypothesis.

We can do that using the Matlab© function: *obsv*

Considering the matrixes A_n, B_n, C_n :

The observability matrix K_{obs} can be computed:

$$\text{Eq. 5.34:} \quad K_{obs} = \begin{bmatrix} C_n & A_n C_n & A_n^2 C_n & \dots & A_n^n C_n \end{bmatrix}$$

If the K_{obs} matrix is full rank the system is observable.

We have developed an observer that can predict both the system states and the cutting forces. The cutting forces represent the unknown input of the system plant.

From the reduced order model the state space representation has been obtained.

$$\begin{aligned} \dot{x} &= A_n \cdot x_n + B_{n_2} \cdot F + B_{n_1} \cdot u \\ y &= C_n \cdot x_n \end{aligned} \quad \text{Eq. 5.35:}$$

Where F is the cutting force and u is the voltage input, B_{n_1} and B_{n_2} consider respectively the effect of the input voltage and the cutting force on the states of the system.

The model was expanded; a state variable linked to the cutting force was added to the system.

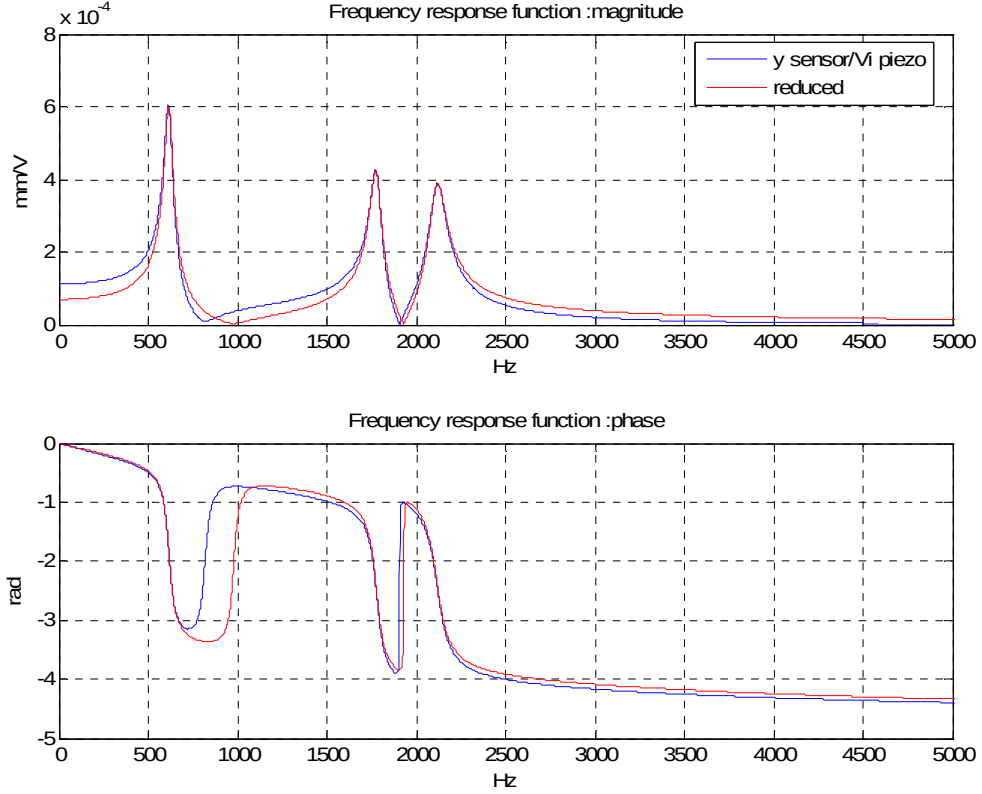


Fig. 5.46: FRF (magnitude-phase) Y_{probe}/V_{ipiezo} (full order Vs reduced)

$$\begin{aligned} \dot{x}_e &= A_e \cdot x_e + B_e \cdot u + \Gamma \cdot w \\ y &= C_e \cdot x_e + H \cdot w + v \end{aligned} \quad \text{Eq. 5.36:}$$

In this case a null matrix H has been considered.

Where $\Gamma \cdot w$ is the noise on the $\dot{x}_e$ and v is the measurement disturbances. Moreover, $x_e = [x_n \ F]$ is the new state vector. In this form the cutting forces

can be estimated like an additional state through the Kalman filter designed for the expanded model.

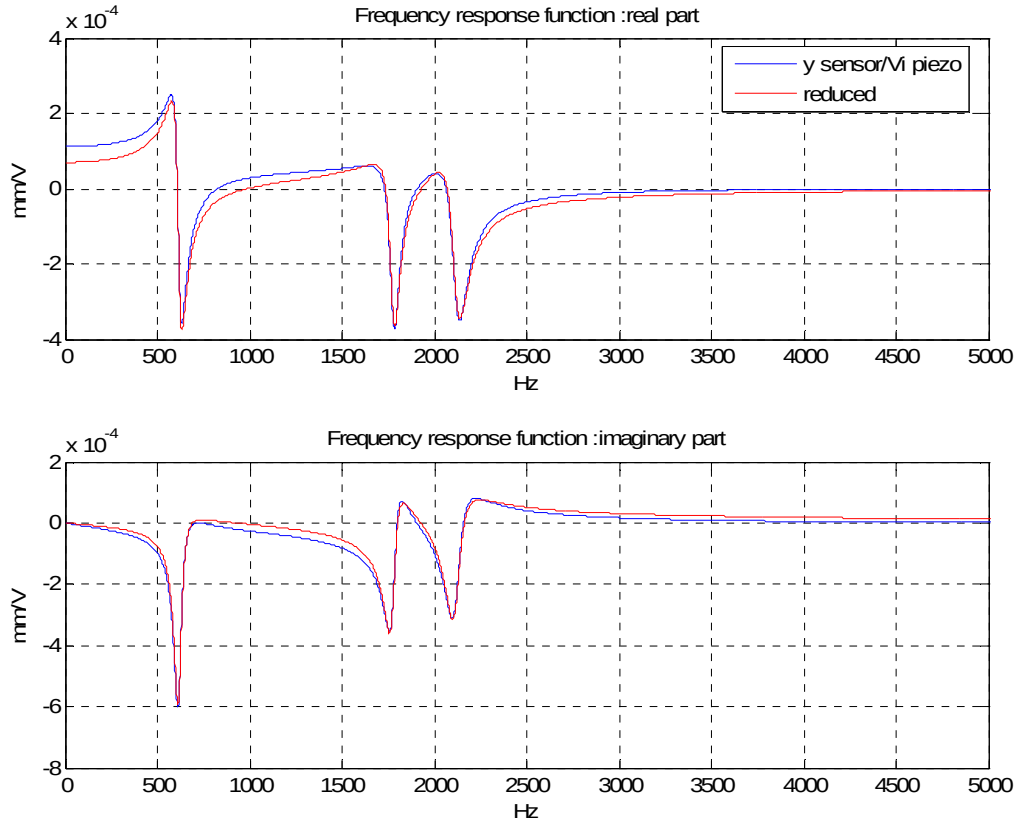


Fig. 5.47: FRF (real-imaginary part) Y_{probe}/V_{ipiezo} (full order Vs reduced)

The graphically realization of the cutting force observer is given in Fig. 5.45. We decided to consider two sources of system disturbance, basically on the cutting force and on the input voltage thus $w = [w_1 \ w_2]$ and the Γ is a $\Gamma(7 \times 2)$ matrix.

Eq. 5.37:

$$B_e = \begin{bmatrix} B_{n_1}(1,1) \\ B_{n_1}(2,1) \\ B_{n_1}(3,1) \\ B_{n_1}(4,1) \\ B_{n_1}(5,1) \\ B_{n_1}(6,1) \\ 0 \end{bmatrix}, \quad C_e = \begin{bmatrix} C_n(1,1) \\ C_n(2,1) \\ C_n(3,1) \\ C_n(4,1) \\ C_n(5,1) \\ C_n(6,1) \\ 0 \end{bmatrix}^T$$

And the structure of the system noise:

$$\text{Eq. 5.38:} \quad \Gamma = \begin{bmatrix} B_{n_1}(1,1) & 0 \\ B_{n_1}(2,1) & 0 \\ B_{n_1}(3,1) & 0 \\ B_{n_1}(4,1) & 0 \\ B_{n_1}(5,1) & 0 \\ B_{n_1}(6,1) & 0 \\ 0 & 1 \end{bmatrix}$$

So the A_e can be expressed as:

$$\text{Eq. 5.39:} \quad A_e = \begin{bmatrix} A_e(1,1) & A_e(1,2) & \dots & \dots & \dots & A_e(1,6) & B_{n_2}(1,1) \\ A_e(2,1) & A_e(2,2) & \dots & \dots & \dots & \dots & B_{n_2}(2,1) \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ A_e(6,1) & \dots & \dots & \dots & \dots & A_e(6,6) & B_{n_2}(6,1) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

And y is the probe displacement measurement

So the correction originated from the measurement

$$\text{Eq. 5.40:} \quad \begin{aligned} \dot{\hat{x}}_e &= A_e \cdot \hat{x}_e + L \cdot (y - C_e \cdot \hat{x}_e) + B_e \cdot u \\ y_0 &= C_0 \cdot \hat{x}_e \end{aligned}$$

And L is the gain matrix of the Kalman filter.

The following relationship can be obtained:

$$\text{Eq. 5.41:} \quad \hat{F} = \left\{ \frac{C_0 \cdot \text{adj}[sI - (A_e - LC_e)]}{\det(sI - (A_e - L \cdot C_e))} \cdot L \right\} \cdot \delta_F$$

There is also an equivalent digital representation.

The Kalman filter gain matrix is obtained through the solution of the Riccati equation:

$$\begin{aligned} \dot{P} &= A_e P + P A_e^T + \Gamma Q \Gamma^T - P C_e^T R^{-1} C_e P \\ L &= P C_e^T R^{-1} \end{aligned}$$

Eq. 5.42:

Where:

Γ is the system noise matrix.

Another assumption that has been introduced is that the system and the measurements noise are uncorrelated.

$$\begin{aligned} Q &= E[w \ w^T] \\ R &= E[v \ v^T] \\ E[w \ v^T] &= 0 \end{aligned}$$

Eq. 5.43:

R is obtained from the RMS of noise of the sensor computed from a steady state spindle experimental test.

Initially the first value used for the noise is $R = 1 \cdot 10^{-11} mm^2$. A “trial and error” tuning phase allows to obtain a proper value of the Q matrix that was tuned to obtain the compensation.

The Matlab© function used to compute the L gain is “*kalman*”.

$$Q = \begin{bmatrix} 100 & 0 \\ 0 & 1 \cdot 10^{11} \end{bmatrix}$$

Eq. 5.44:

If one wants the regulator poles to dominate the closed-loop response, the observer poles should be faster than the regulator poles. This will ensure that the estimation error decays faster than the desired dynamics and the reconstructed states follow the real ones (at least without noise and modelling error). As a rule of thumb, the observer poles should be 2 to 6 times faster than the regulator poles. With noise measurements, one way wish to decrease the bandwidth of the observer by having the observer poles closer to those of regulator. This produces some filtering of the measuring noise. In this case however the observer poles have a significant influence on the close loop response, Preumont [99].

In Fig. 5.48 some preliminary simulation tests are reported, a good predicting capability of the cutting force observer can be underlined.

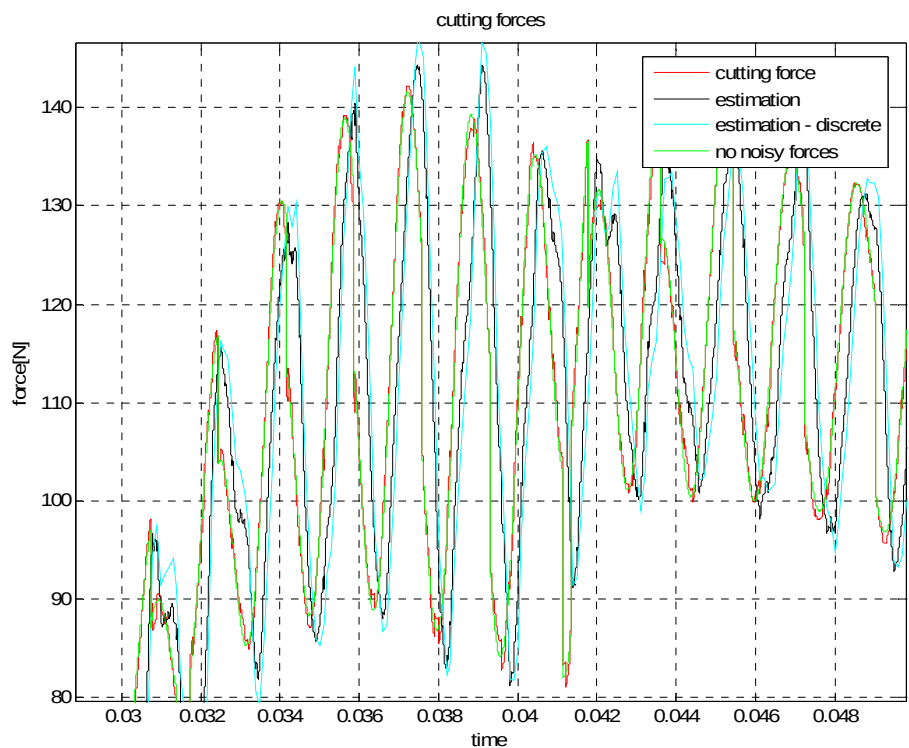


Fig. 5.48: Cutting force prediction capability

5.8.3 LQG Regulator Design

Before designing the Linear Quadratic Regulator (LQR-LQG) the controllability hypothesis has been checked.

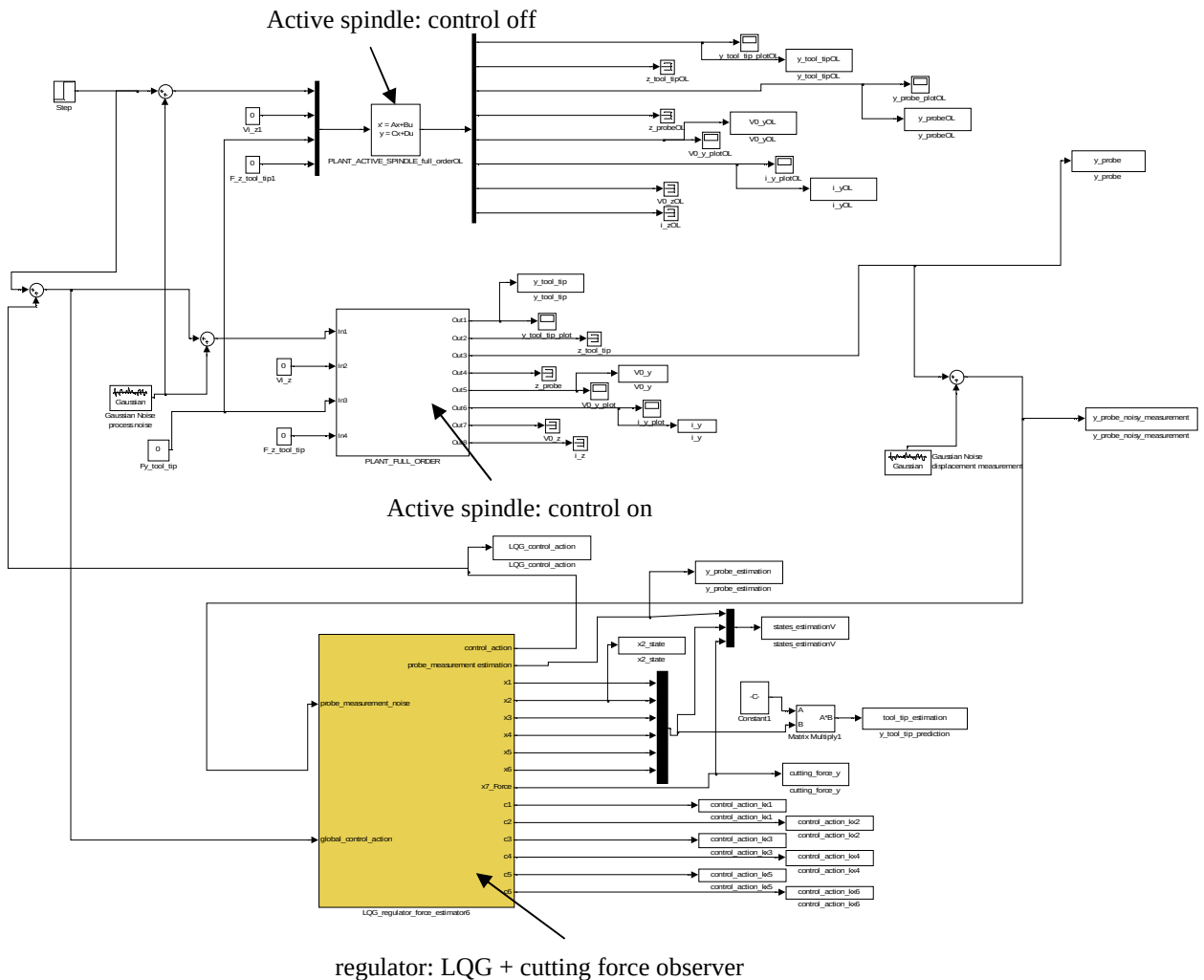


Fig. 5.49: Active spindle model Simulink© – response force step

In Fig. 5.49 the model used to tune the Q-R values of the regulator is depicted. This model allows comparing the transient response to an input step force on the tool tip or an input voltage step.

Having fixed the R_{LQG} value because there is only an output, if only one direction is considered, different values of Q_{LQG} were tested. Obviously a value of input noise has been set in the model.

The Q_{LQG} matrix has to be diagonal and there are different techniques to select the weights, for example the Bryson's rule but generally the tuning phase is a

trial-and-error iterative design procedure like we did for the Kalman Filter tuning.

There is also the possibility to minimize the following functional in which there is a term that weights the output instead of the states.

$$\text{Eq. 5.45} \quad J = \frac{1}{2} \int_0^{T=\infty} (y^T Q y + u^T R u) dt$$

The used Matlab© function is *lqry*.

In this case the Q_{LQG} matrix collapses to a single value for each direction.

The choice to develop separately the two regulators for the corresponding axis is due to the possibility to tune differently the linked gains. This could be useful especially when the controllers will be implemented on the prototype, in fact the dynamic behaviour of the real plant in the two directions slightly differs due to the structure that support the spindle.

In Tab. 5.6 are reported the final selected weights and in Tab. 5.7 the corresponding gains vector of the controller.

# parameter	value
R_{LQG}	$R_{LQG} = 1$
Q_{LQG}	$Q_{LQG} = 8 \cdot 10^{06}$
Noise on the measurement	Gaussian - $\sigma^2 = 1 \cdot 10^{-10} \text{ mm}^2$, $f_c = 10000 \text{ Hz}$
Noise on the input	Gaussian - $\sigma^2 = 1 \text{ V}^2$, $f_c = 10000 \text{ Hz}$

Tab. 5.6: Q - R weights

K_i	K_1	K_2	K_3	K_4	K_5	K_6
value	-595.35	457.7	254.1	-490.1	-155.42	574.15

Tab. 5.7: State gains

The Fig. 5.50 shows the effectiveness of the designed feedback control to damp the system.

The control action value seems to be reasonable for the piezo considered, this is pointed out in Fig. 5.51.

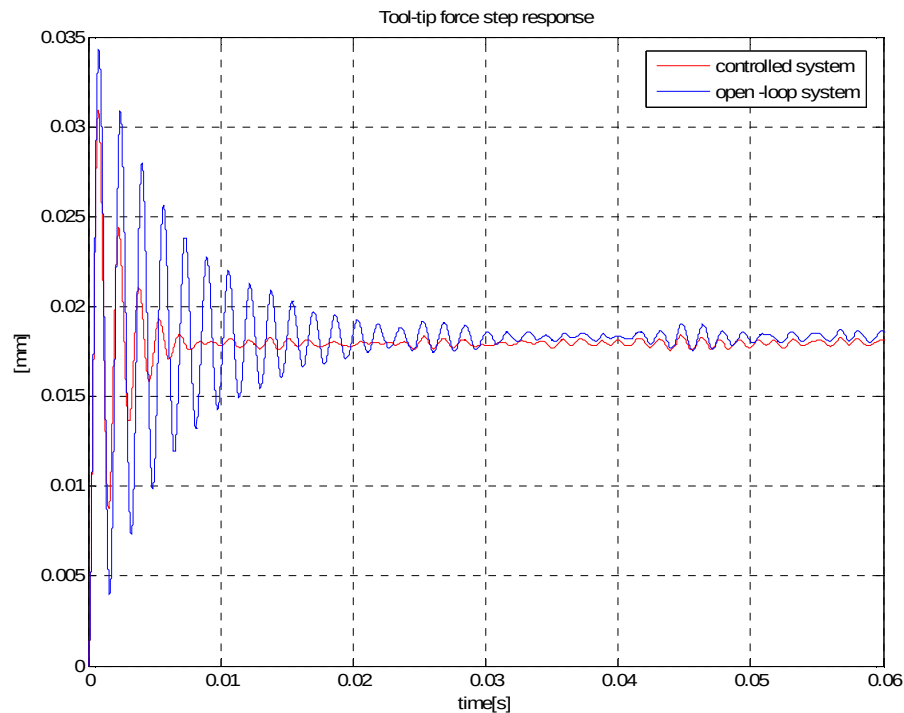


Fig. 5.50: Tool tip response to step force on the tool tip

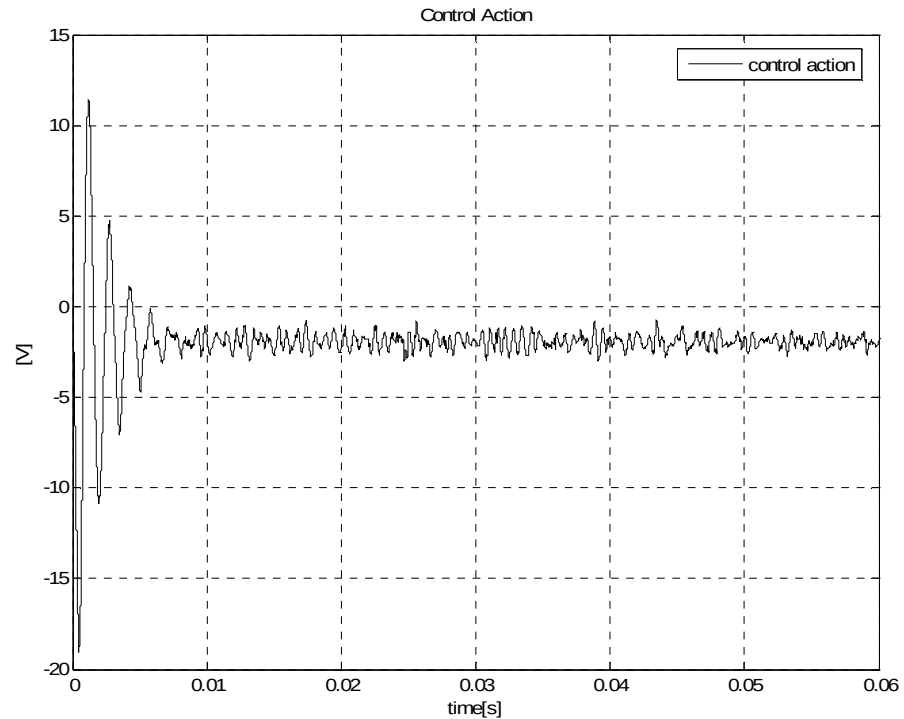


Fig. 5.51: Control action

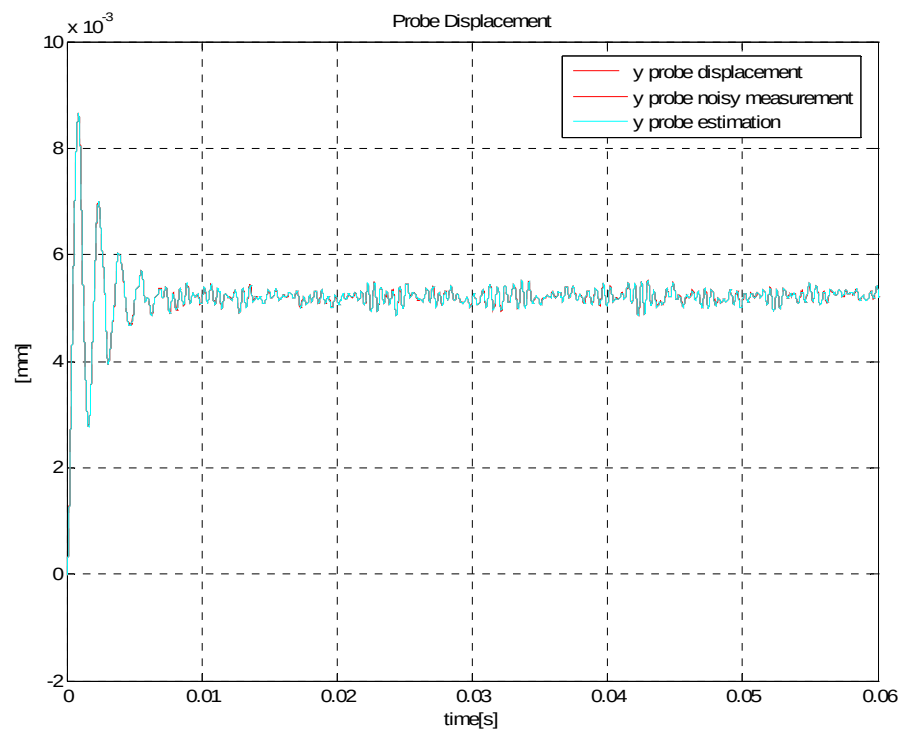


Fig. 5.52: Probe displacement – measurement and estimation

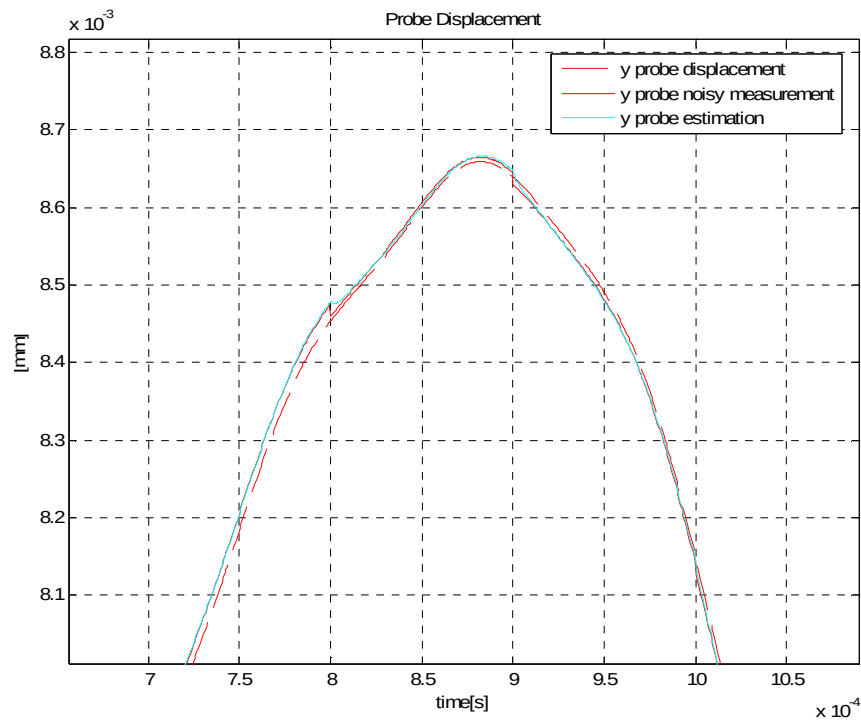


Fig. 5.53: Zoom

Fig. 5.52 and Fig. 5.53 show the good performance of the observer. Obviously the observer, if the gains are quite high, follows accurately the measurements. If the measurements are very noisy or the noise on the system is very high and the Kalman Filter gains are too high, the noise is propagated into the states estimation. In this case it is necessary to reduce the gains of the observer in order to increase the filtering action.

A balance between the speed of the observer and the capability of the observer to filter the noise frequencies that are in the measurement should be found. This balance obviously depends strongly on the measurement noise and on the noise on the system. In order to implement the control strategy on the real time application the regulator (LQG+ observer) needs to be retuned in order to consider the real noise ratio and possible inaccuracies of the model.

The Fig. 5.54, shows the capability of the observer to predict the tool tip deflection and in Fig. 5.55 is reported the predicted tool tip force. From this pictures we can conclude that the Kalman filter seem to be very fast.

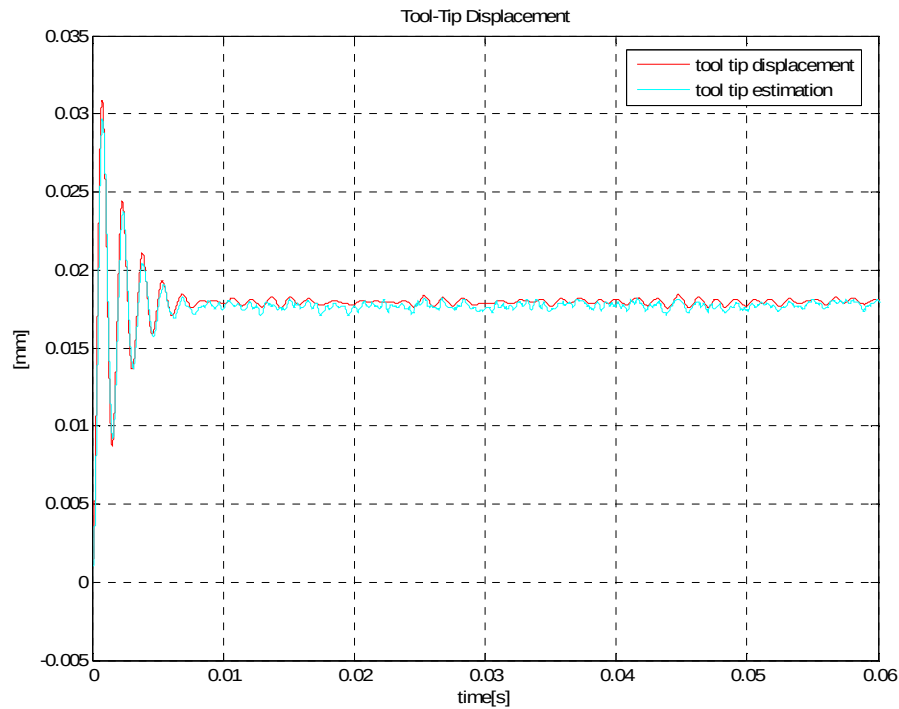


Fig. 5.54: Tool tip displacement

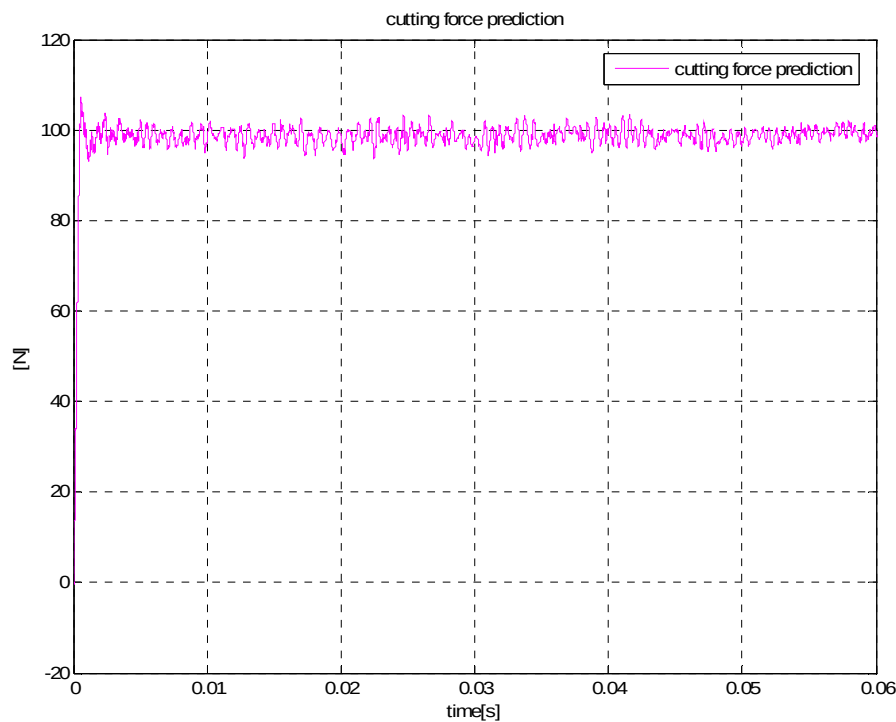


Fig. 5.55: Cutting force prediction (force step on tool tip=100N)

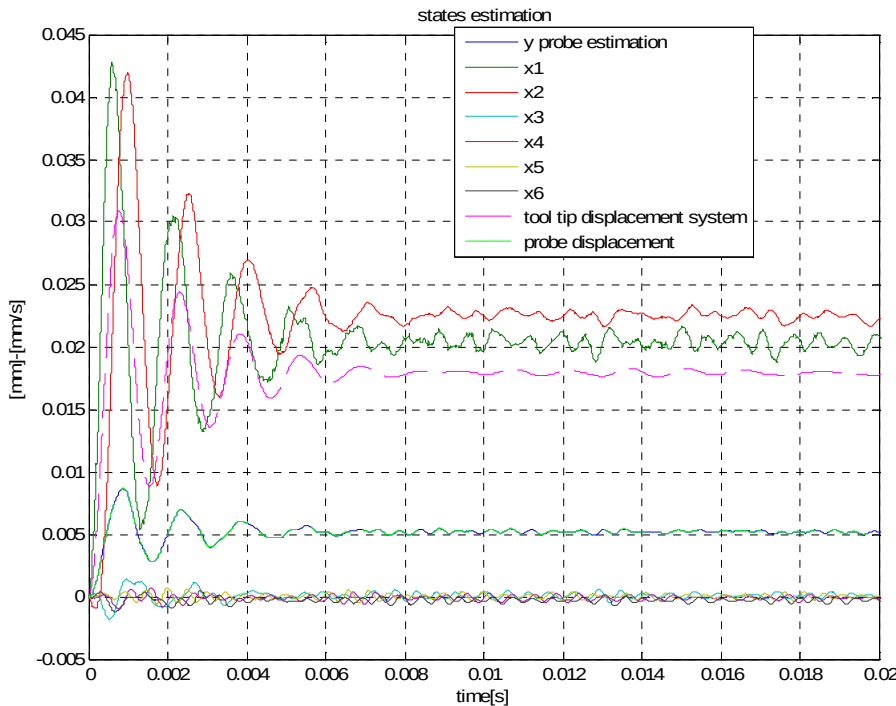


Fig. 5.56: States estimation from observer

In Fig. 5.56 are depicted some estimated quantities such as the states of the system, the probe measurement and the tool tip deflection.

It can be observed that the states of the system don't represent directly a physical quantity of the spindle (i.e. the displacement at the tool-tip) in fact the C matrix of the state-space representation permits to understand how each single state contributes to the system physical output. Looking at Fig. 5.57: the states x_1 and x_2 represents the main contributes to the tool tip displacement. Probably x_4 , x_5 and x_6 are linked to the derivative of the states.

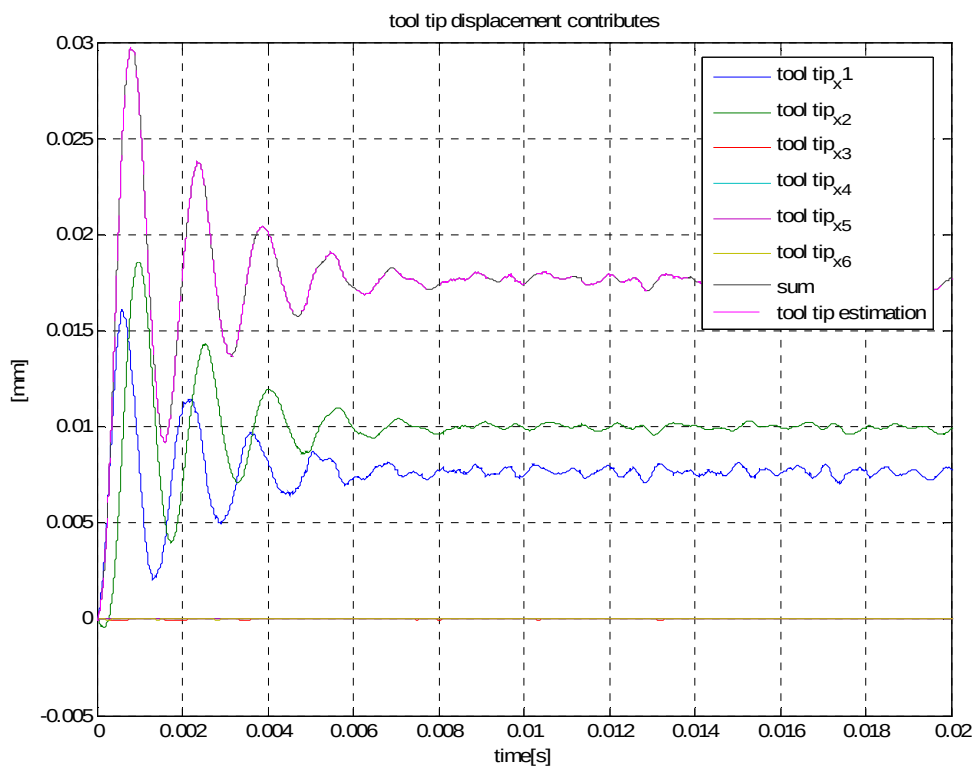


Fig. 5.57: Tool tip displacement states contributes (weighted by C matrix)

Fig. 5.58 points out the control action comparing it to the tool tip displacement. and the next one shows which state strongly contributes to the global control action

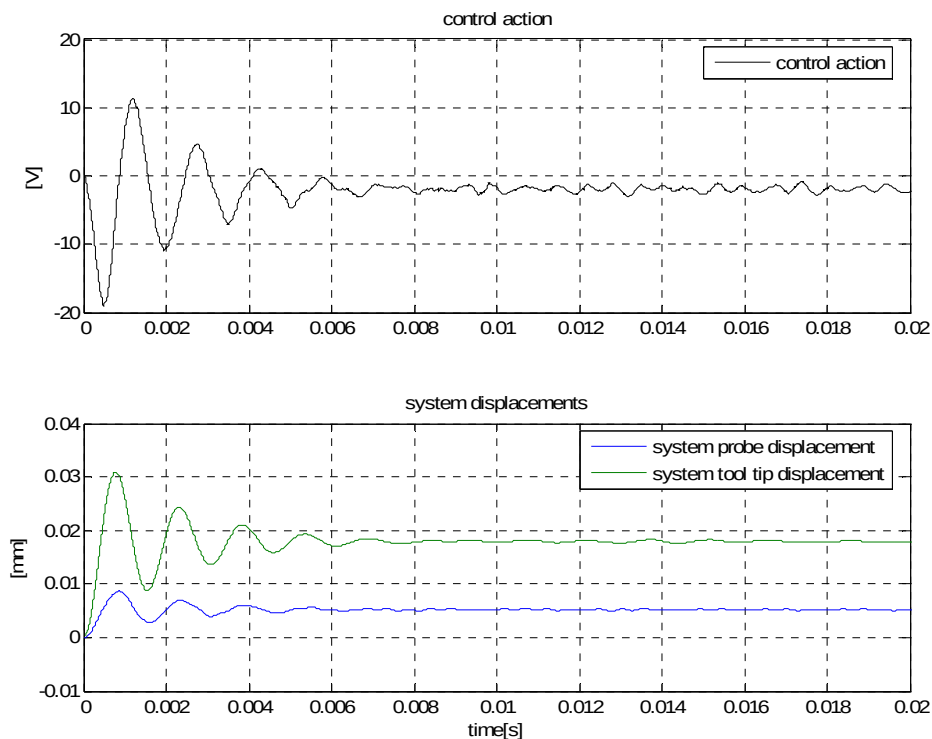


Fig. 5.58: Control action – system displacement

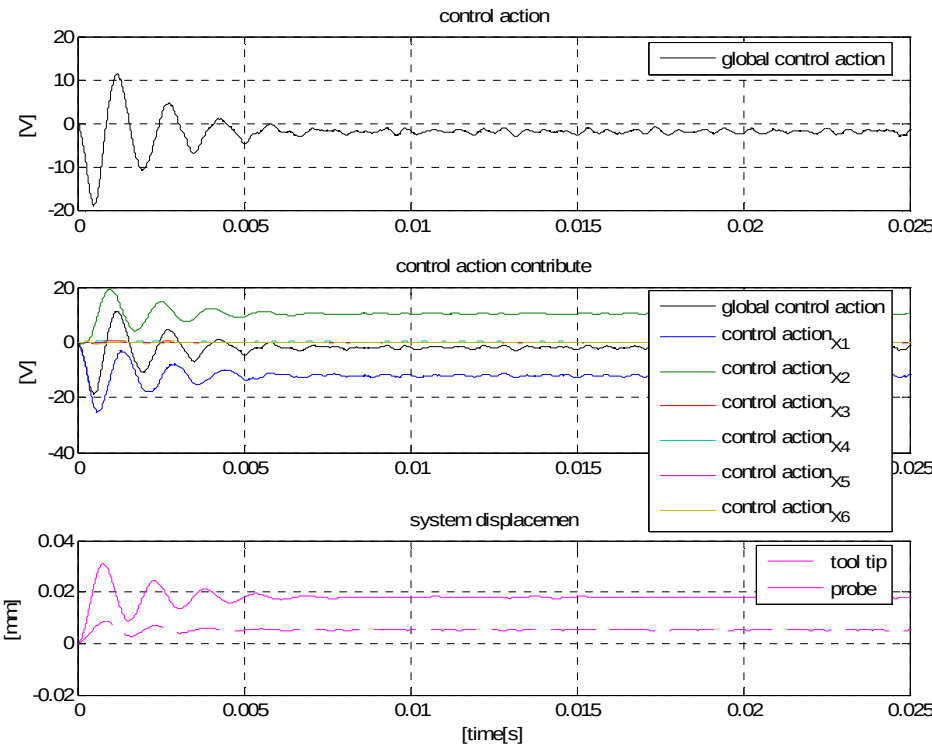


Fig. 5.59: Control action contributes

The global control action is basically due to the first and second state. Different tunings have been considered and they are resumed in Tab. 5.8.

Case #	R_{LQG}	Q_{LQG}
1 (reference case)	$R_{LQG} = 1$	$Q_{LQG} = 8 \cdot 10^6$
2 (decreasing gain)	$R_{LQG} = 1$	$Q_{LQG} = 5 \cdot 10^6$
3 (increasing gain)	$R_{LQG} = 1$	$Q_{LQG} = 13 \cdot 10^6$
4 (increasing gain+)	$R_{LQG} = 1$	$Q_{LQG} = 20 \cdot 10^6$
5 (increasing gain++)	$R_{LQG} = 1$	$Q_{LQG} = 30 \cdot 10^6$

Tab. 5.8: Different tunings

The effects of the different analyzed tunings is presented in Fig. 5.60 and how the control action changes.

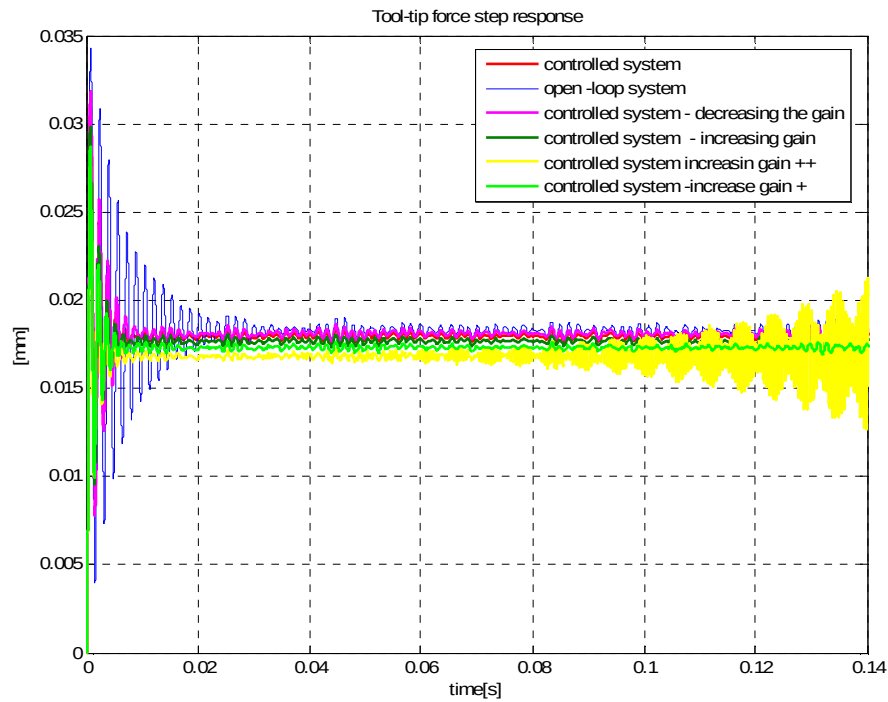


Fig. 5.60: Tool tip response - different weights

If the gains of the regulator are too high the system response is bring to instability.

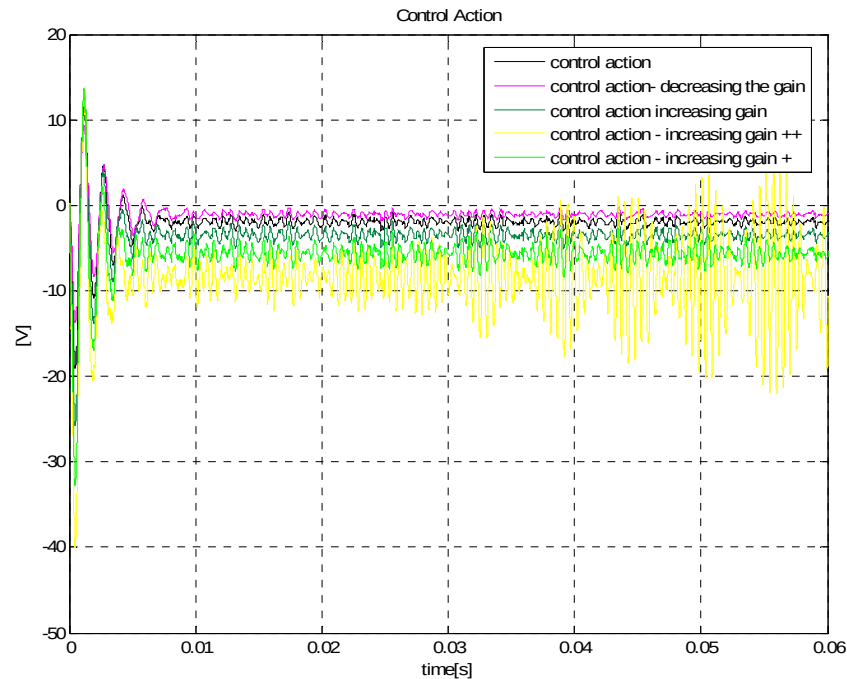


Fig. 5.61: Control action – different tunings effects

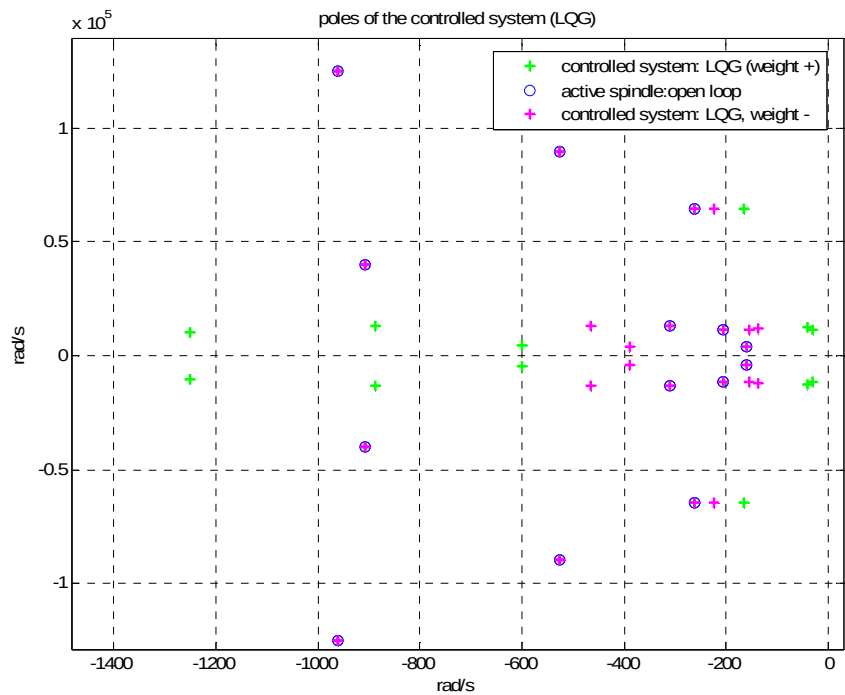


Fig. 5.62: Poles of the system – open loop Vs closed loop

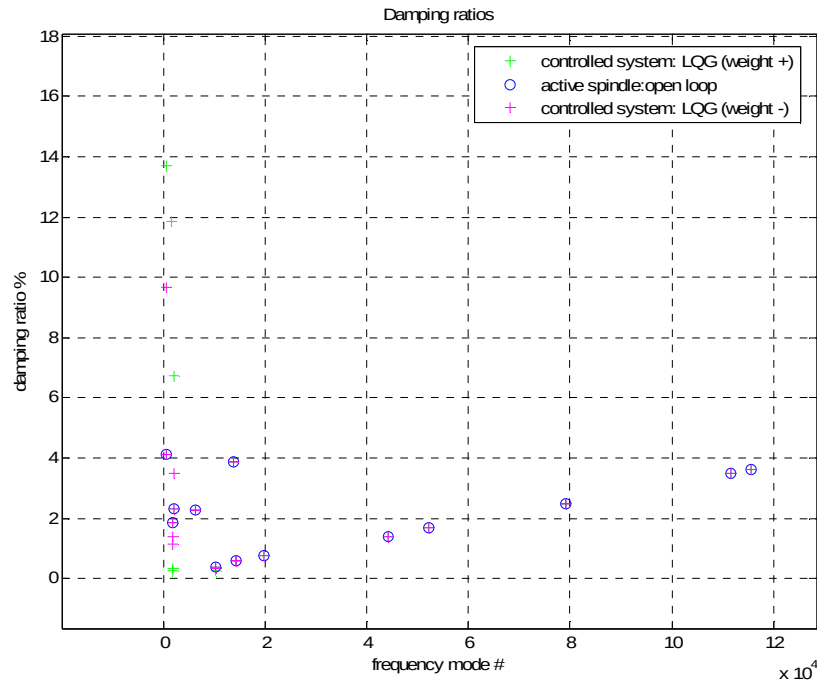


Fig. 5.63: Damping ratios - open loop Vs closed loop

As formerly described, a similar result can be obtained using also the states weight whit the Matlab© function *lqr*.

In the previous figures the effects of the gains of the regulator on the poles of the spindle system have been analyzed.

Basically we can note an increment in the damping ratios linked to the first and third mode Fig. 5.63, but a decrease of the damping ratio of the second eigenmode, this is an usually effect due to the presence of uncontrollable modes, Jeffrey [91]. Probably in this case the main reason is the limited bandwidth of the drive.

Only a slightly increase of the natural frequencies has been pointed out, Fig. 5.64.

The conclusion formerly presented can be observed also analyzing the Frequency Response Functions depicted in Fig. 5.66 e Fig. 5.67.

From the cutting stability point of view Fig. 5.67 is very meaningful. A decrement of the absolute value of the minimum of the real part of the tool tip dynamic compliance can be appreciated; this effect is due to the damping effects introduced into the plant by the regulator.

In the following section the behaviour of the controller during the cutting operation will be analyzed.

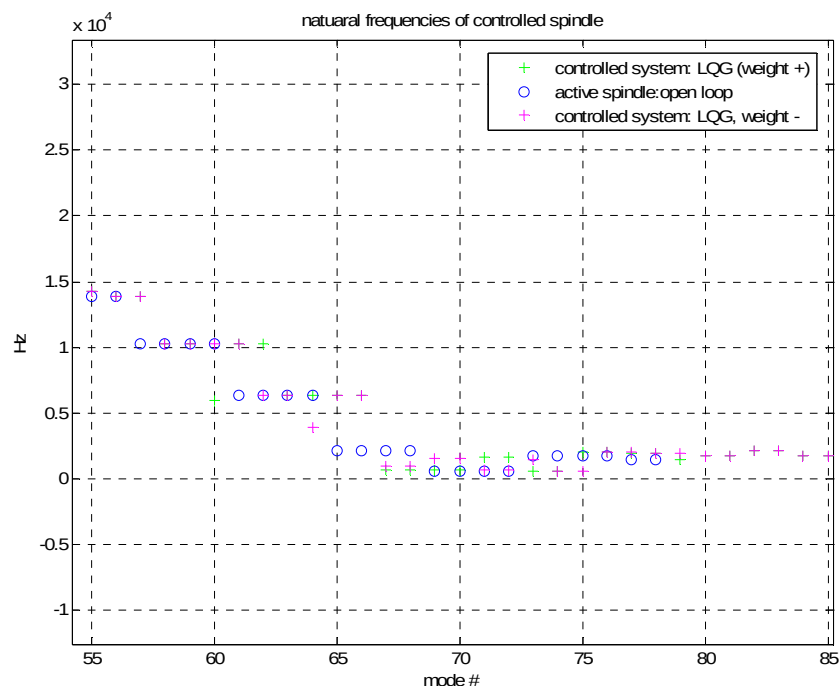


Fig. 5.64: Natural frequencies of the system – open loop vs closed loop

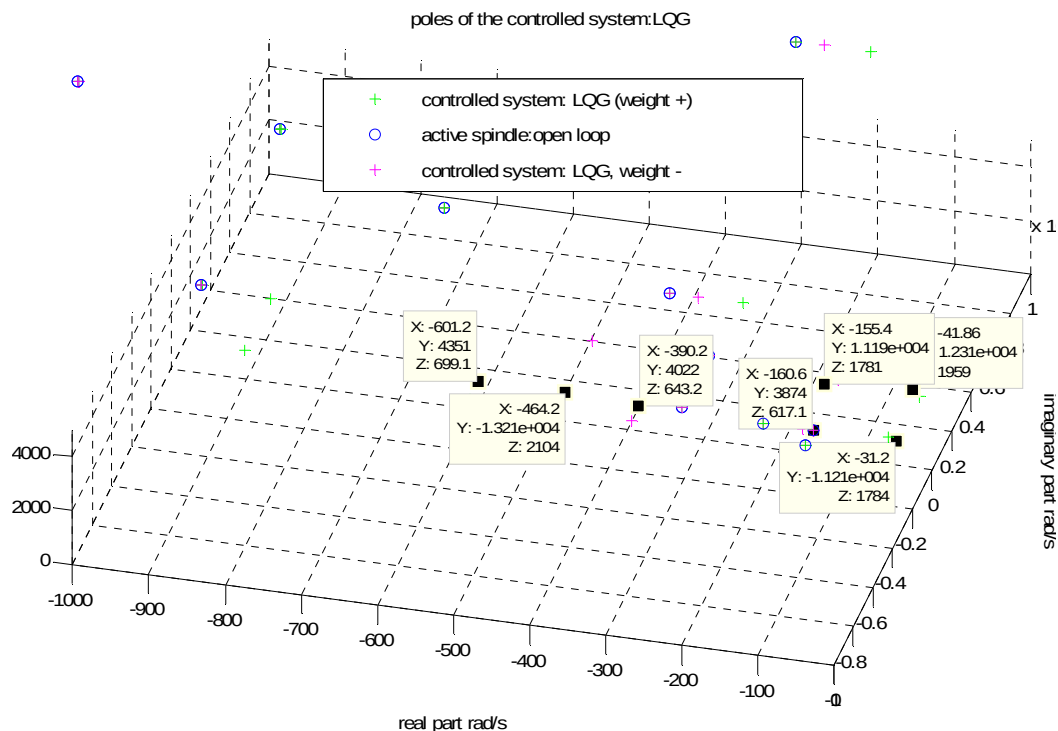


Fig. 5.65: Zoom – poles of the system - labels

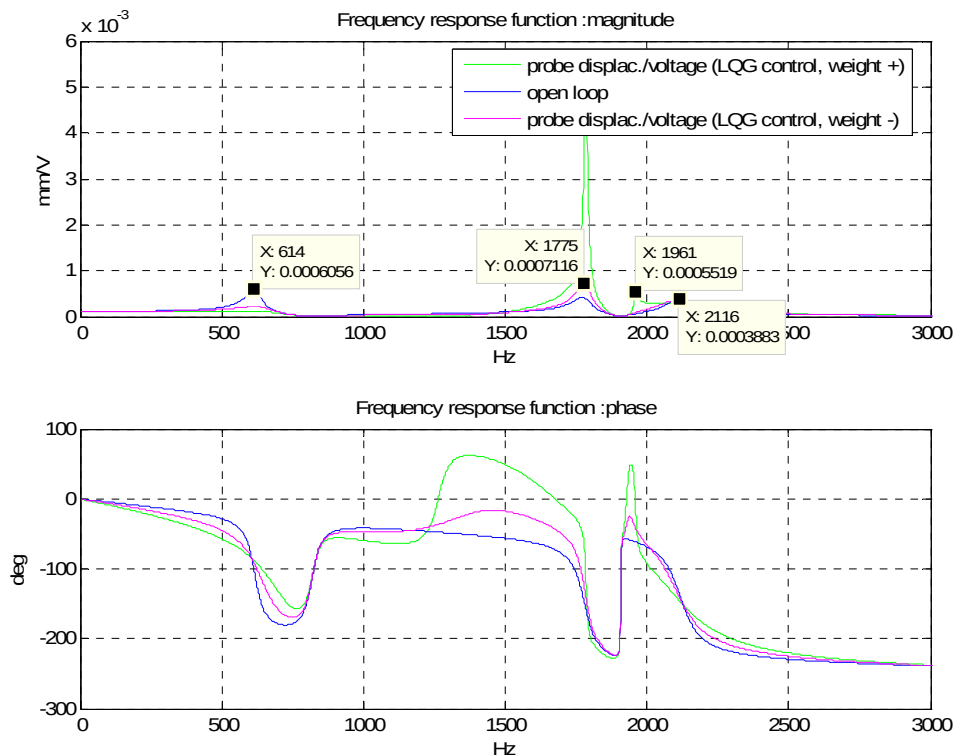


Fig. 5.66: Plant dynamics, Frequency Response Function – Effect of the regulator gains

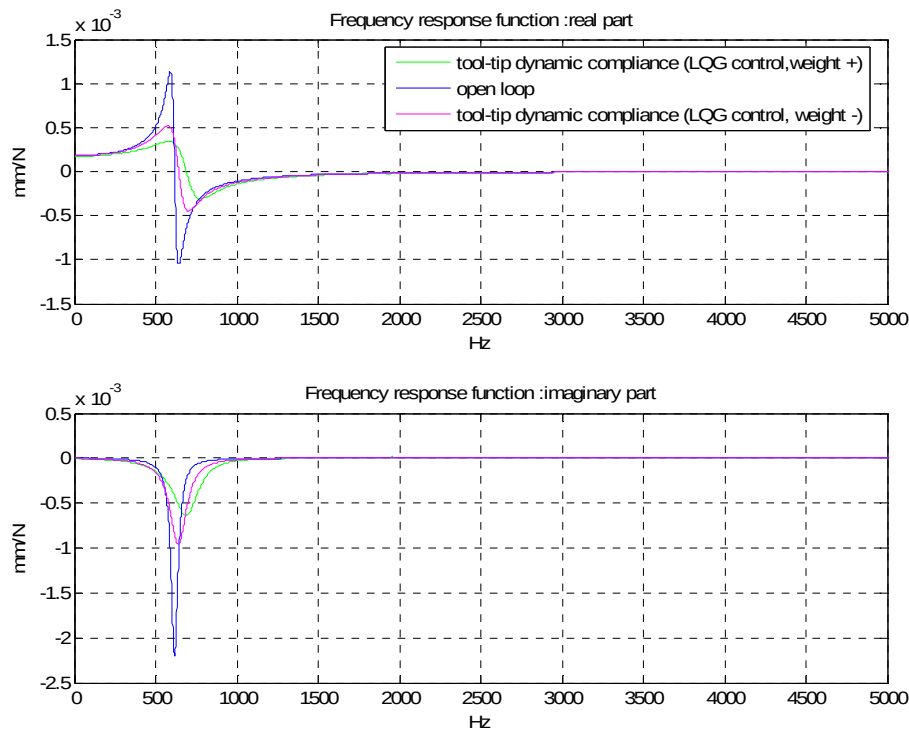


Fig. 5.67: Tool tip FRF, real-imaginary part - effects of control gains

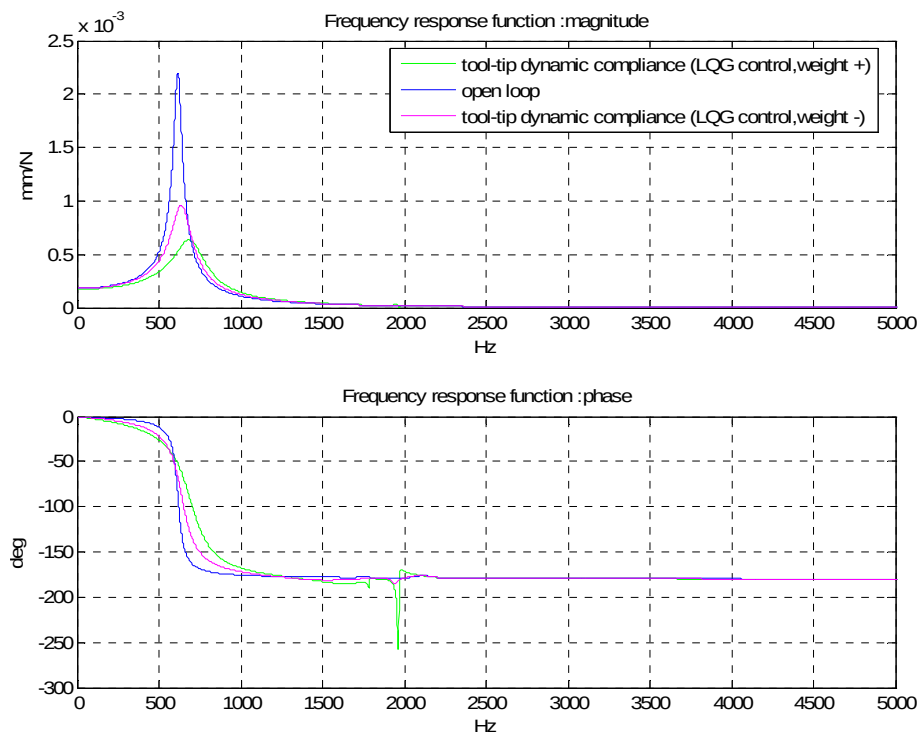


Fig. 5.68: Tool tip FRF, magnitude-phase - effects of control gains

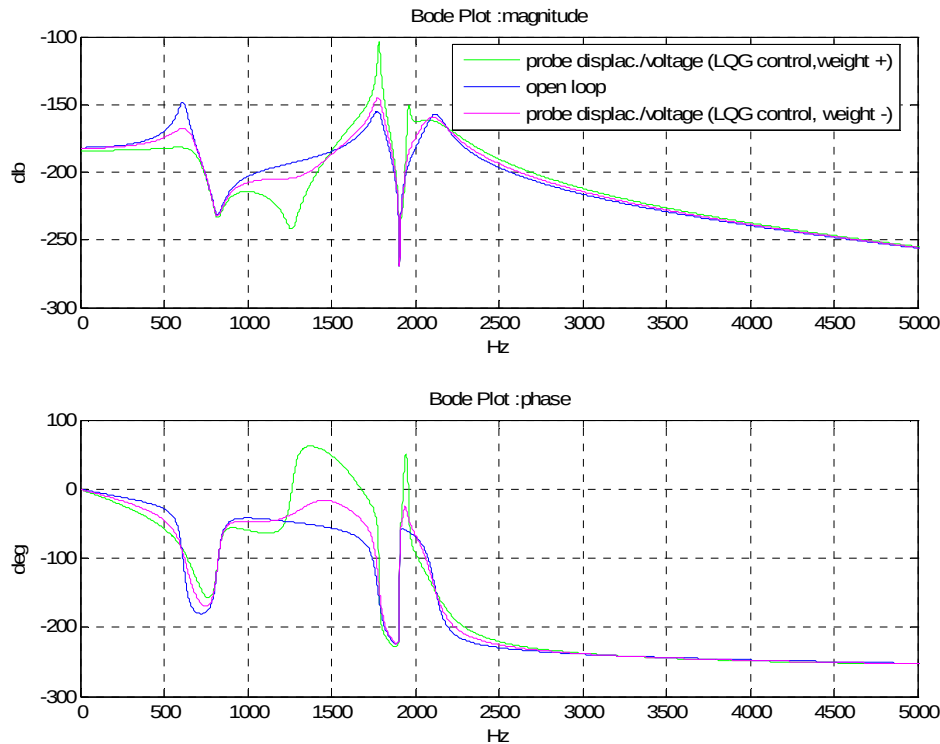


Fig. 5.69: Bode diagram – controlled system

5.9 Stability lobes prediction

In this section the behaviour of the controller during the cutting operation has been evaluated and moreover the enhancement on the cutting stability limit has been verified using both the traditional frequency approach and simulations in time domain.

5.9.1 Cutting Process Enhancement (MRR)

The Lobes Diagrams depicted in Fig. 5.70 are obtained using the classical frequency approach, Altintas et. al. [10], they are linked to the aluminium cutting operation defined in Tab. 5.5.

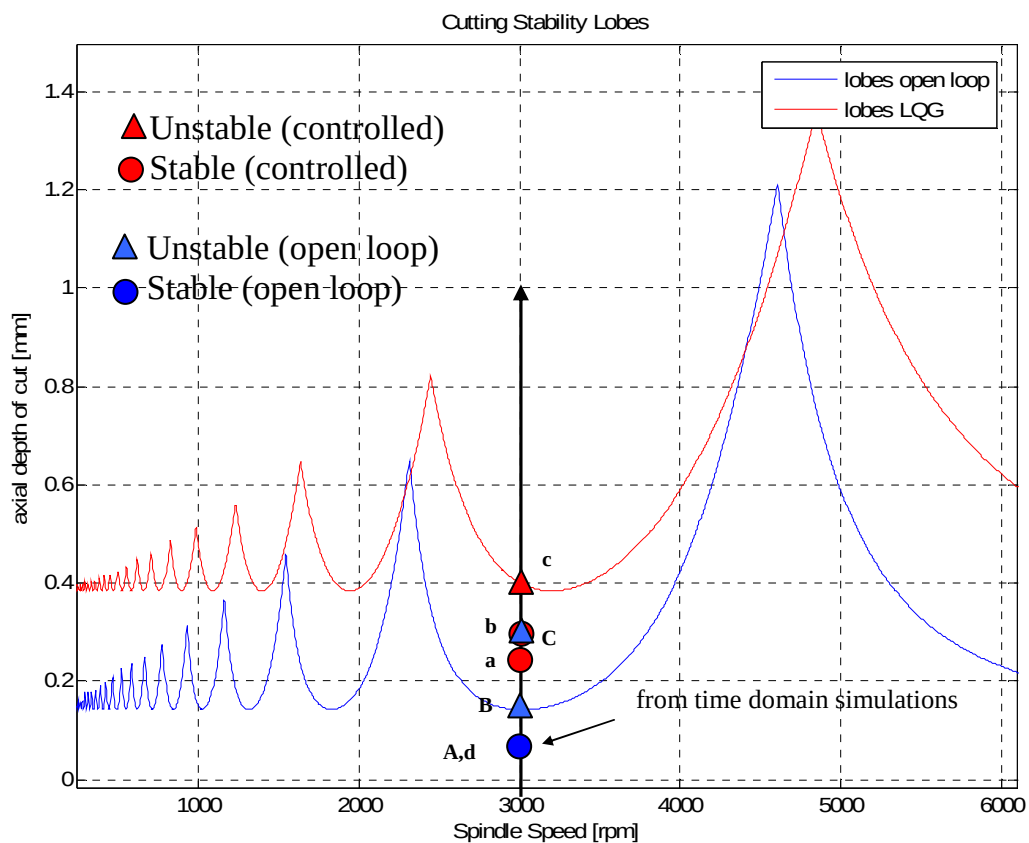


Fig. 5.70: Lobes diagram

An important enhancement on the asymptotical stability limit can be observed: the minimum axial depth of cut has been almost doubled.

On the lobes diagram are located the simulated milling operations that have been performed in order to analyze both the controller behaviour and the spindle system performances during the cutting.

The Fig. 5.71 shows the lobes diagram with the corresponding chatter frequencies for both cases: open loop system and controlled one.

It is interesting to observe that the control action significantly modifies the chatter frequencies. This phenomenon was observed also in an active damping application developed by Ganguli in [97].

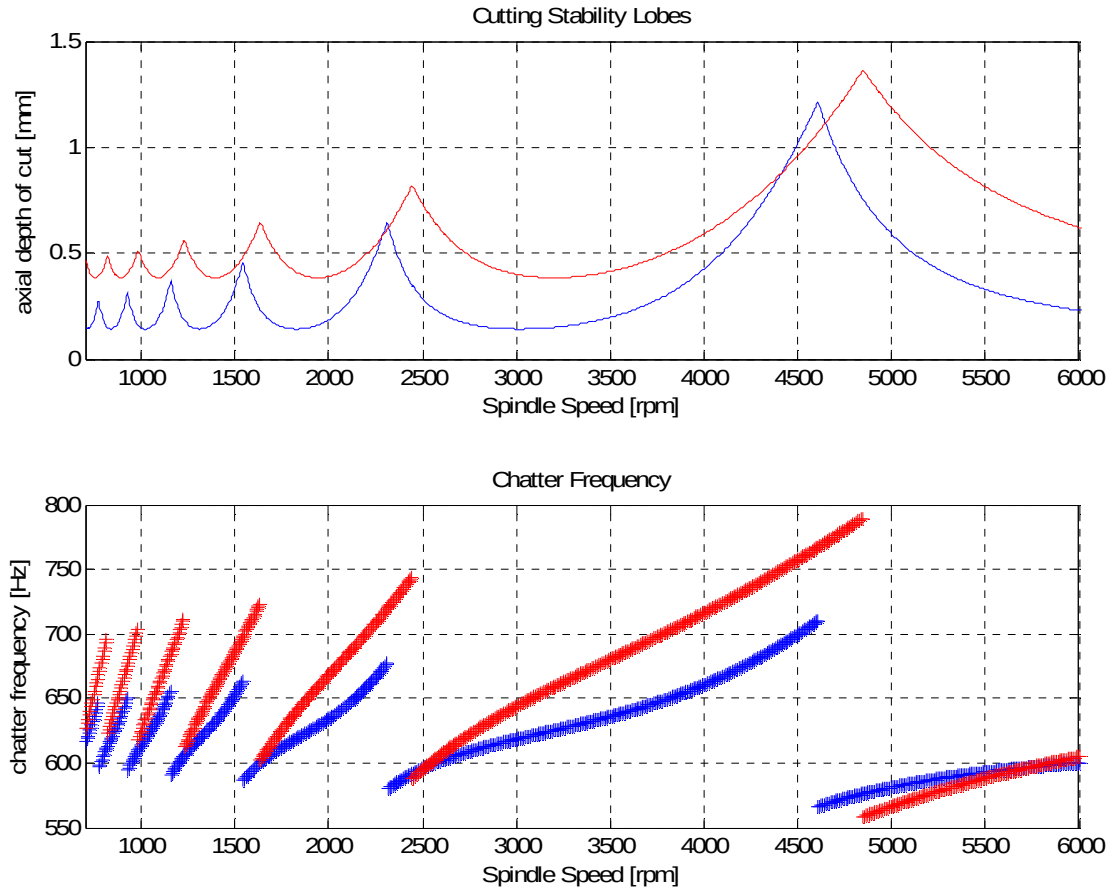


Fig. 5.71: Lobes diagram and chatter frequencies

5.9.2 Time Domain Simulations

As formerly introduced, some simulations have been performed and the presentation of the results follows. First we can assert that the time domain simulations confirm the results predicted by the frequency approach: an important enhancement on the Material Removal Rate has been obtained.

The analyzed milling operations are resumed in Tab. 5.9.

<i>Milling operation</i>	<i>System</i>	<i>Axial depth of cut [mm]</i>	<i>Spindle Speed [rpm]</i>	<i>Stable-Unstable</i>
A	Open Loop	0.1	3000	stable
B	Open Loop	0.14	3000	unstable
C	Open Loop	0.3	3000	unstable
a	Controlled	0.25	3000	stable
b	Controlled	0.3	3000	stable
c	Controlled	0.4	3000	unstable
d	Controlled	0.1	3000	stable

Tab. 5.9: Simulated milling operations

5.9.3 Effects of the LQG regulator

Considering the milling operations C and b.

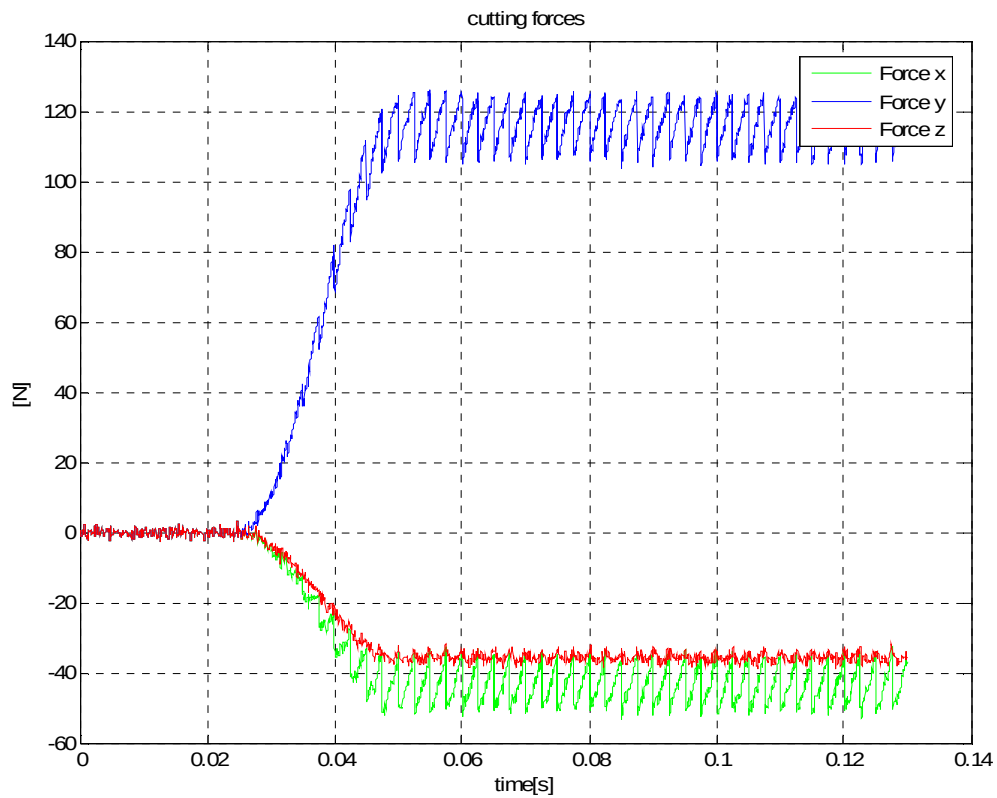


Fig. 5.72: Cutting forces – milling b

The diagrams reported in Fig. 5.72 and Fig. 5.73 show the cutting forces along the main directions respectively during the milling C and b.

The system in open-loop configuration can't avoid the cutting process instability so the cutting forces grow up to unacceptable values and consequently the displacement of the tool tip and the roughness of the machined surfaces. The transient in which the cutting forces increment occurs can be observed in Fig. 5.73.

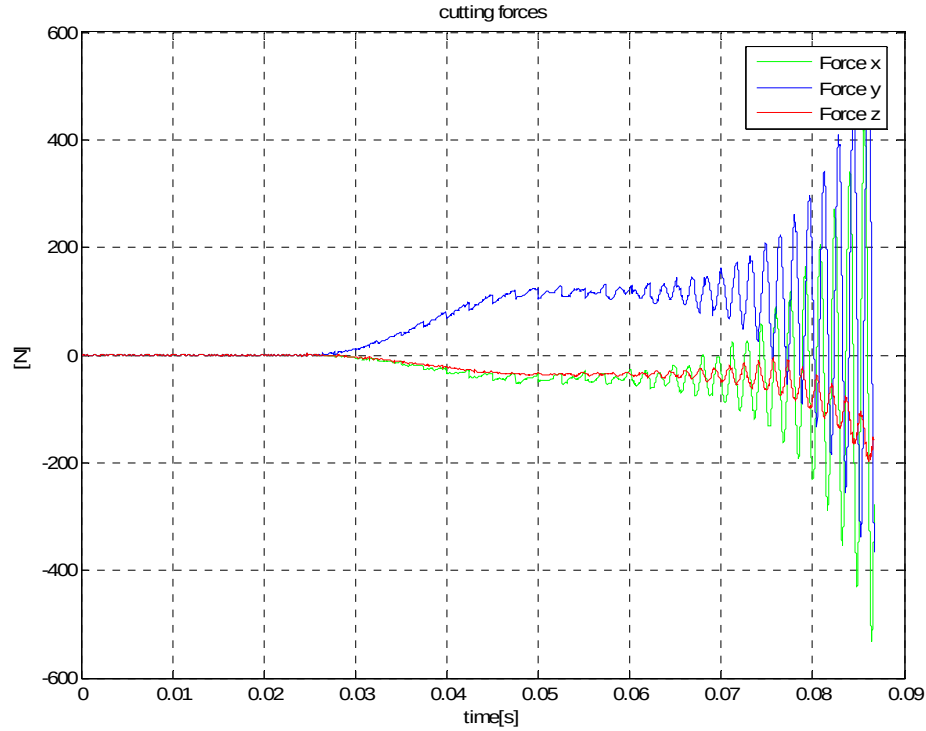


Fig. 5.73: Cutting forces – milling C

Both the tool tip increment and the eddy current probe measurement can be appreciated in Fig. 5.74 and Fig. 5.75.

Moreover, in Fig. 5.76 and Fig. 5.77 the control actions in both directions are reported.

Comparing the tool tip displacement and the control action (Fig. 5.77) we can appreciate how the controller acts on the piezoelectric actuators in order to damp the vibration and to stabilize the cutting.

The effectiveness of the observer based on the Kalman Filter can be appreciated in Fig. 5.78 and in Fig. 5.79: respectively the prediction of the cutting forces and the displacement at the tool tip seems quite good.

These information can be used to design an additional control contribute that counteracts the effect of the cutting forces on the system.

In fact the optimal control strategy is generally designed without considering the effects of the external disturbances.

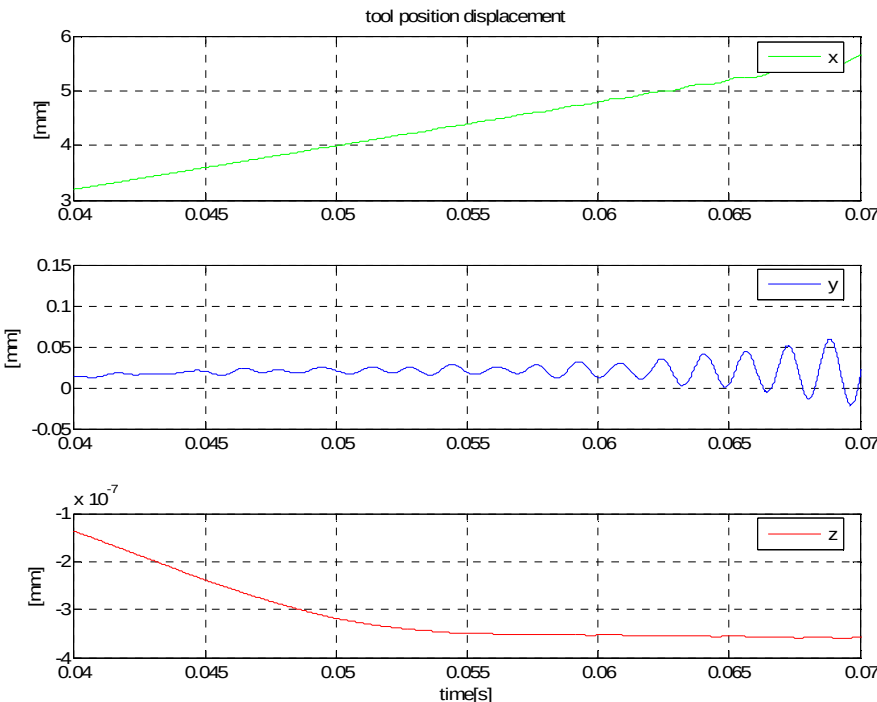


Fig. 5.74: Tool tip displacement – milling C

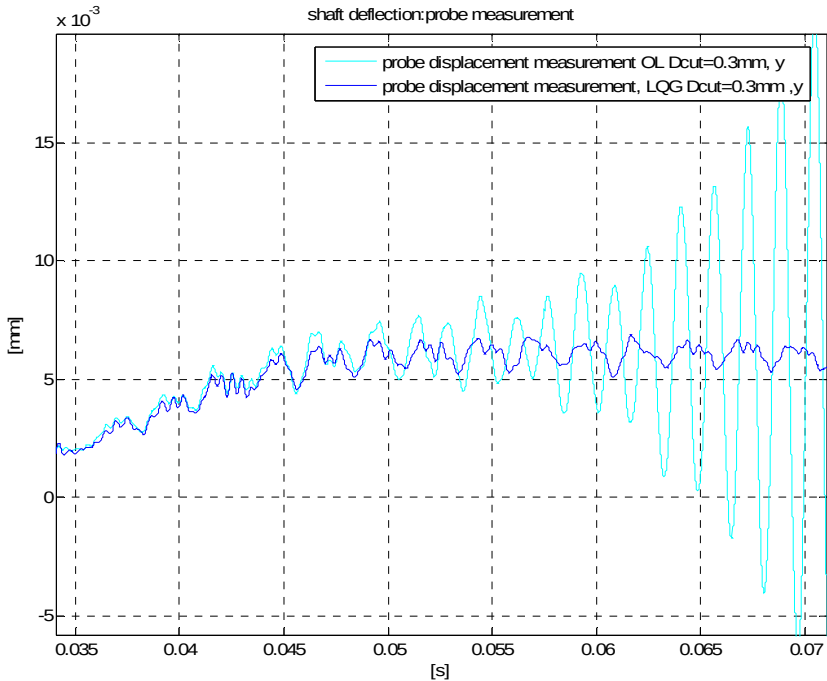


Fig. 5.75: Probe measurement comparison - open Loop vs LQG - milling b and C

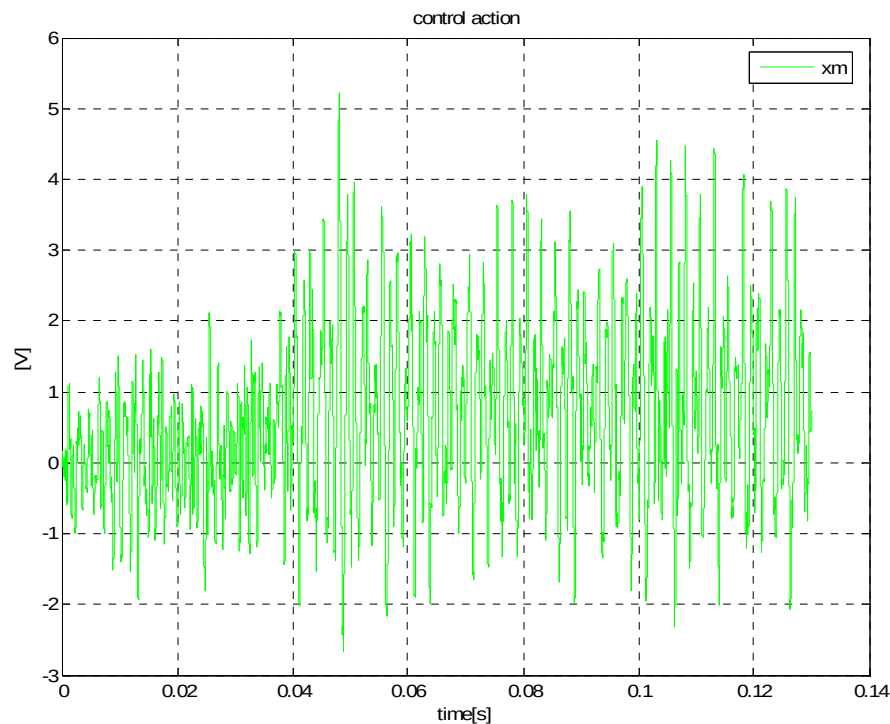


Fig. 5.76: Control action x axis, milling b

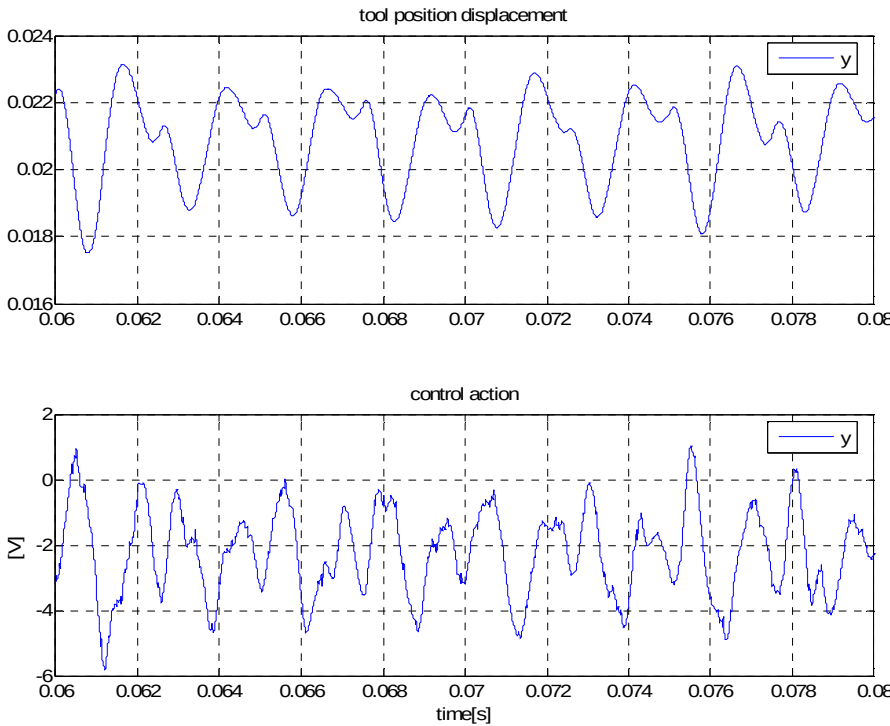


Fig. 5.77: Tool tip displacement and control action - y direction - milling b

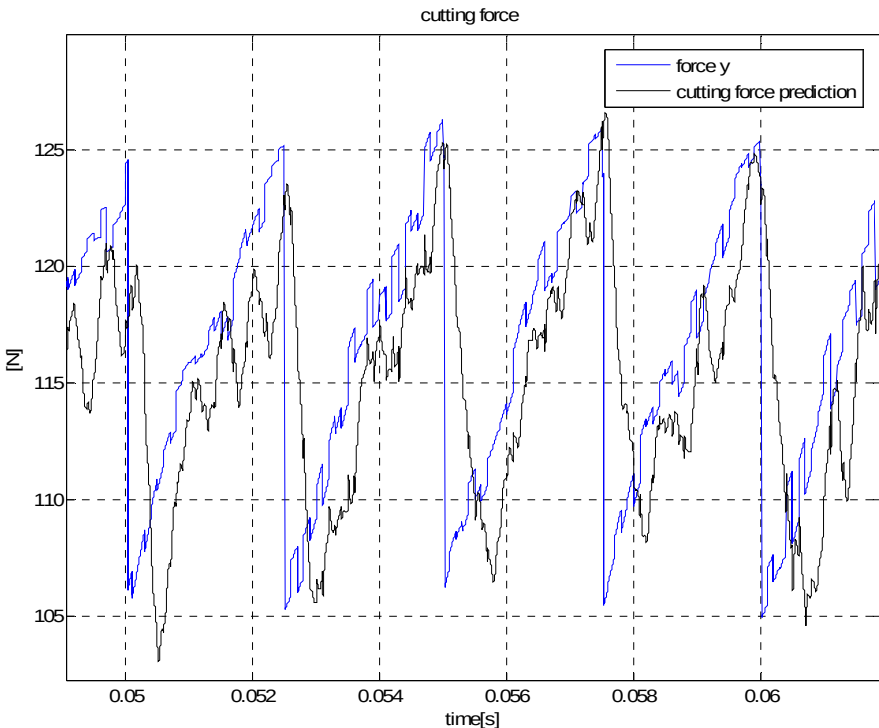


Fig. 5.78: Cutting force prediction, milling *b*

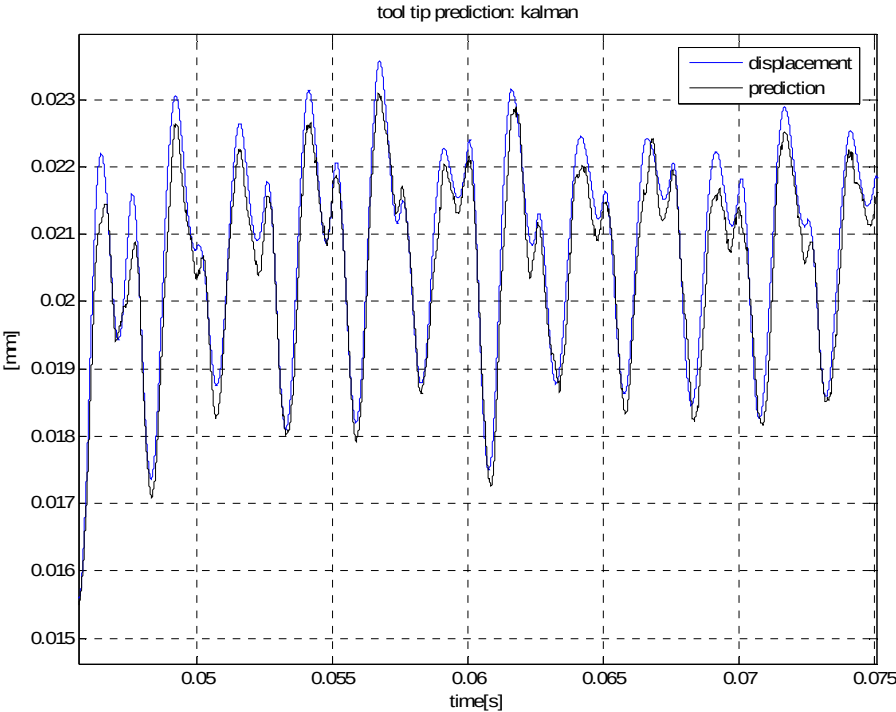


Fig. 5.79: Tool tip displacement prediction, milling *b*

Furthermore, the spindle system in both configurations during milling “A” and “d” in which the axial depth of cut is equal to 0.1mm was analyzed.

A reduction of the amplitude of the tool tip displacement can be observed so this enhancement is transferred to the quality of the machined surfaces.

Generally the LQR regulator have efficient damping proprieties for frequencies close to the resonances; in this case the tooth passing frequency is 400Hz that is lower than the first frequency resonance so we can observe that the vibration reduction of the tooth passing frequency component is not very high.

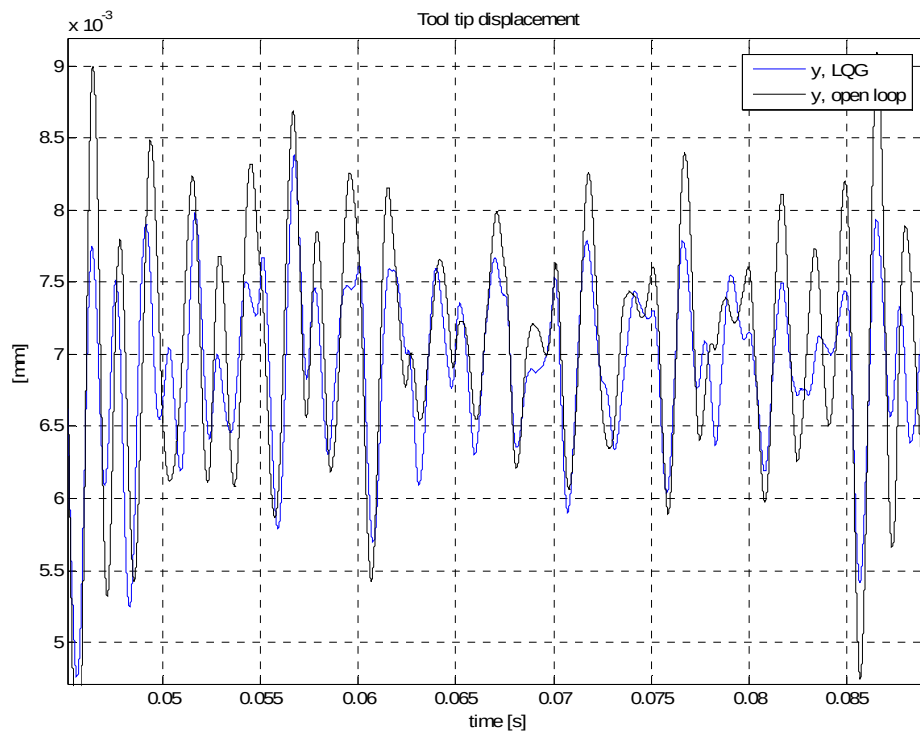


Fig. 5.80: Tool tip displacement – milling A

The results obtained simulating milling c (axial depth of cut=0.4mm) follow.

In these cutting conditions the instability occurs.

Probably is necessary to put a saturation block to limit the control action that tends to grow when the instability occurs; this is needed in order to protect the piezo-actuator from high voltage command signals.

In Fig. 5.81 and Fig. 5.82 are respectively depicted the tool tip displacements that show the instability occurrence and the control action before the chatter vibration is completely manifested.

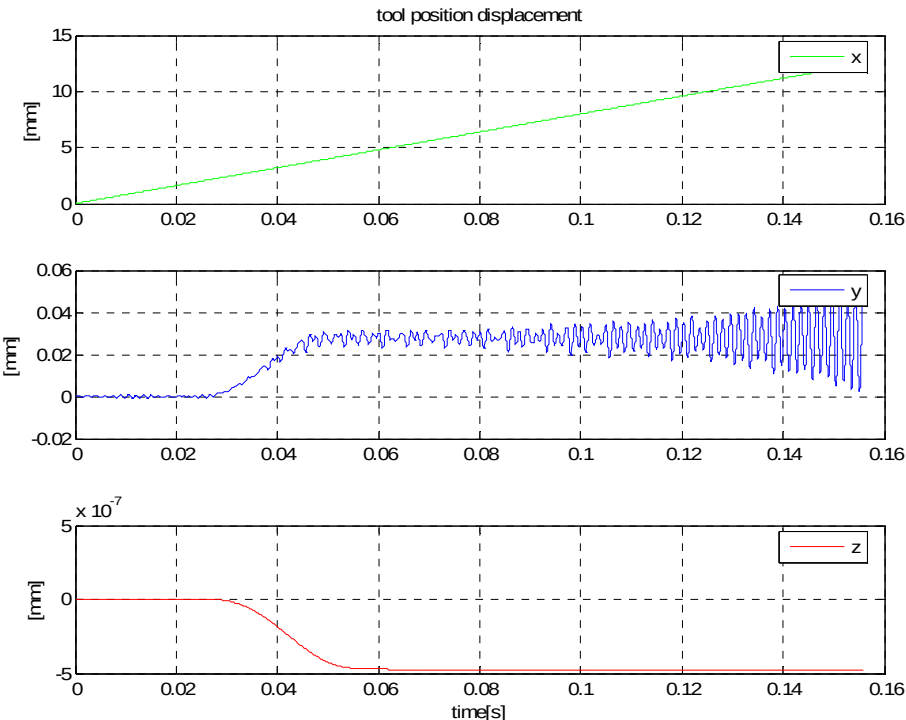


Fig. 5.81: Tool tip displacement - milling c

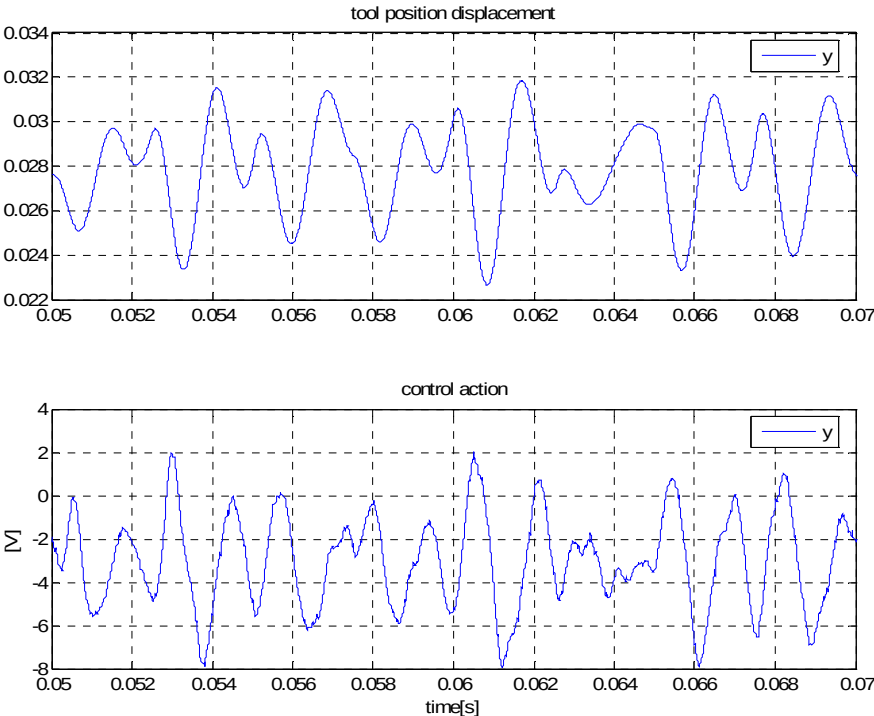


Fig. 5.82: Control Action and tool tip displacement - milling c

5.10 Additional control action

Ho in [94] presented a control approach based on the combination of the LQG (optimal control) approach and the input estimation one. This combined control strategy allows getting better control performances.

This control strategy is based on the estimation of the unknown input forces that act on the system; in that case the dynamic system is a generic multi degrees of freedom lumped structure. The unknown external forces that are considered as disturbances on the system were predicted using an additional Kalman filter and they were used to compute a supplementary control action. The added control action was obtained using the inverse dynamics of the system.

In this section we want to test the described mixed control strategy on the spindle system application in order to obtain further enhancements.

We have encountered some problems during the plant dynamic inversion; these are mainly caused by the presence in the system transfer function of some positive real part zeros.

We can roughly banalize that the inversion issue is like a poles-zeros exchange problem thus positive real part zeros can make the system unstable.

In this case it is necessary to use an approximated inverted transfer function.

The scheme used to solve the inversion problems has been depicted in Fig. 5.83.

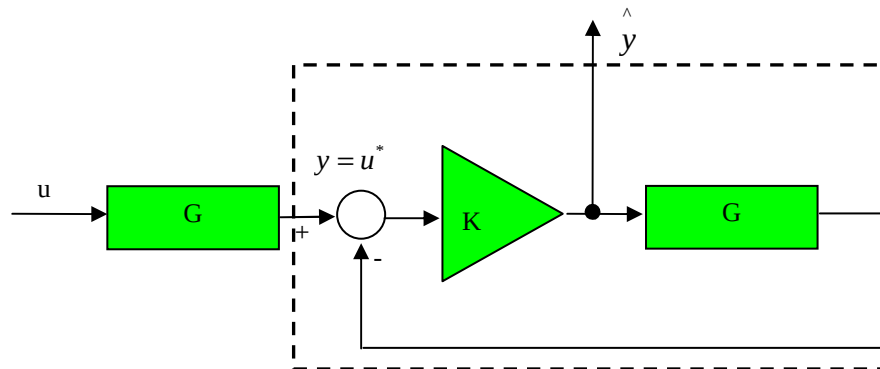


Fig. 5.83: Scheme used to invert the dynamic

We can write the following equations:

$$\begin{aligned}
 \text{Eq. 5.46} \quad G^*(s) &= \frac{\hat{y}}{u^*} = \frac{K}{1 + K \cdot G(s)} = \frac{K}{1 + K \cdot \frac{\text{zeros}}{\text{poles}}} = \\
 &= \frac{K \cdot \text{pole}}{\text{poles} + K \cdot \text{zeros}}
 \end{aligned}$$

Where G is the transfer function of the system and G^* is the inverted transfer function.

From Eq. 5.46 we can assert that the zeros of G^* are the pole of G (always, independently of the magnitude of gain K). Conversely if K is very small the poles of G^* “start” at the pole of G ; if K increases and approaches infinity, the poles of G^* approach the zeros of G .

The position of the poles of G^* can be adjusted using the knob K .

$$\text{Eq. 5.47} \quad \lim_{K \rightarrow \infty} G^*(s) = \frac{\text{poles}}{\text{zeros}}$$

and if is necessary the static gain:

$$\text{Eq. 5.48:} \quad G(0) \cdot G_p \cdot G^*(0) = 1 \rightarrow G_p = \frac{1}{G(0) \cdot G^*(0)} = \frac{1 + K \cdot G(0)}{K \cdot G(0)}$$

Again, a trade-off has to be found, because on the one hand high gains reduce the stationary gain error and the dynamic input of the propering poles, while on the other hand, they increase the bandwidth of the inverse and therefore challenge the integration algorithm.

Using this technique a supplementary control action was added to the LQG control contribute.

In order to not obtain an unstable system a relatively low gain K for the G^* has been selected.

The simulated cutting tests, corresponding to milling “c”, performed with the proposed combined control strategy has been depicted in Fig. 5.84; different gains were used.

Not brilliant enhancements in the tool tip displacement have been obtained during the simulated cutting tests.

The static tool tip deflection has been reduced by 4% and the amplitude of the component linked to the cutting forces has been reduced by the 15%.

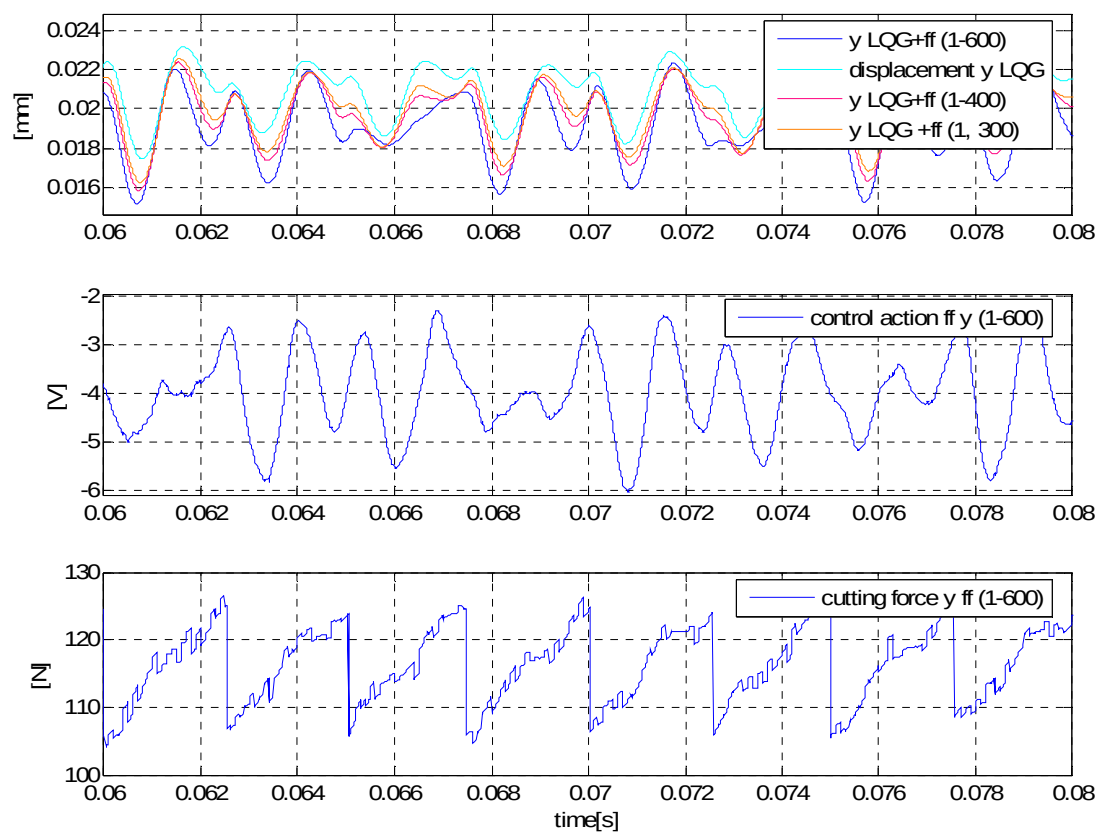


Fig. 5.84: Tool tip displacement – additional control action

5.11 Preliminary experimental tests

The designed control strategy has been implemented on the real time application directly converting the tuned controller in the continuous time version.

Generally two different approaches can be followed to implement the control strategy on a real controller: the first one, as previously described, consists on the direct continuous-discrete conversion of the tuned control action while in the second one the controller is designed directly in the discrete domain using “z transform”.

We have selected the first approach because the sampling frequency of the real time controller was set to 4000Hz, this is a reasonable approach because the sampling frequency is higher than the controller desired bandwidth.

From the first experimental test we have noticed that a retuning of the control gain is necessary. These activities will be further developed.

Moreover a first stage of experimental tests have been conducted on the test bed in order to check the reliability of the proposed model and furthermore to investigate the behaviour of the piezoelectric actuators.

Another important critical aspect that could influence the closed loop performances is the inadequateness model used to design the controller: some robustness problems can occur.

The Fig. 5.85 shows the comparison between the simulated Frequency Response Function that describes the dynamic behaviour of the plant and the experimental one.

The experimental one has been computed from different system responses to voltage impulse tests (amplitude=45V and width=0.3ms). The experimental FRF presents a coherence function close to 1 from 0Hz to 2400Hz except for the frequencies close to 800Hz where a zero of the system can be observed.

The measured plant dynamics has been reported in Fig. 5.85.

Analogous results are obtained from a sinusoidal sweep test, they are reported in Fig. 5.86.

The experimental results seem to not exhibit a good agreement compared to the simulated ones.

Obviously it is necessary to remember that behaviour of the piezo-actuator is nonlinear so the is not proper to define a FRF that is based on the linearity hypothesis.

If you look at the Fig. 5.85 the presence of additional modes due to the dynamic behaviour of the structure on which the spindle is mounted can be observed. The frequency value corresponding to the first mode of the spindle seems well modelled nevertheless the dynamic influence of the structure.

We can observe a phase lag in the Frequency response function of the plant.

This supplementary phase lag that is not present in the model can be probability associated to the discretization effect.

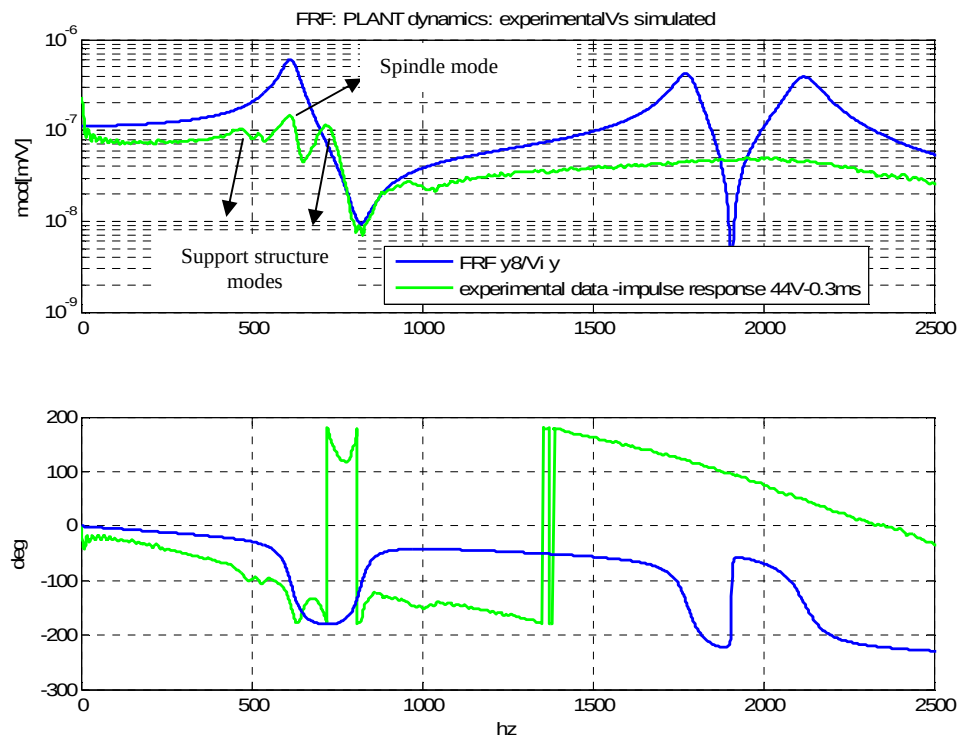


Fig. 5.85: Plant dynamics – experimental vs simulated

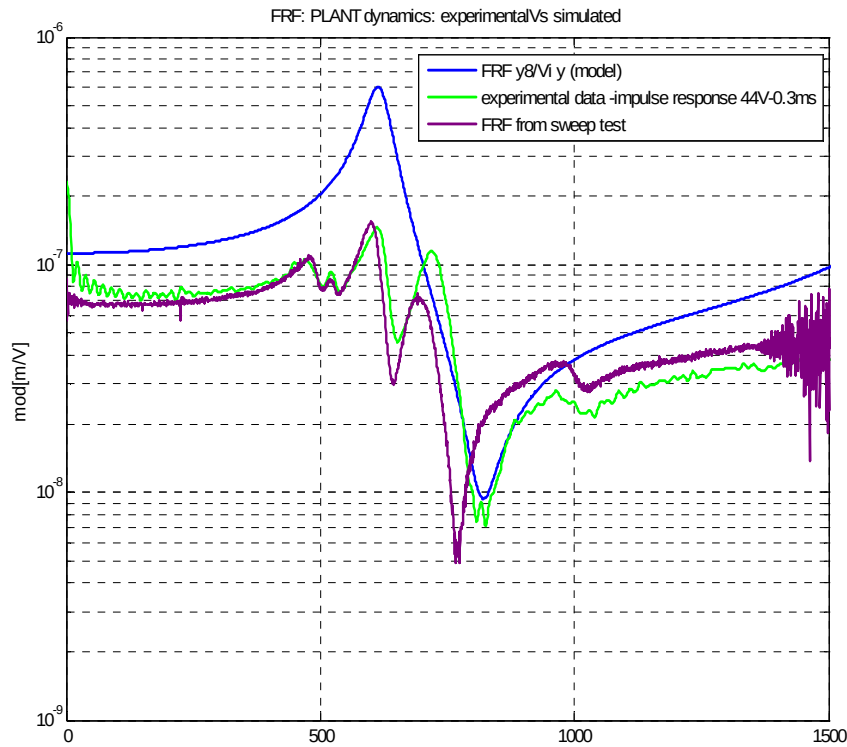


Fig. 5.86: Plant dynamics – sweep test

The differences observed in the magnitude diagram reveal an higher static gain of the model compared to the experimental test bed.

In Fig. 5.86 we can moreover appreciate the effects of the pole linked to the bandwidth of the power amplifier (@1450Hz).

Some static tests have been also performed, [105]. The force and the displacement are respectively applied and measured in different positions of the shaft. Generally these tests reveal that the physical prototype seems to be a little bit more flexible than the model, this indication contrast the one from the FRF depicted in Fig. 5.86.

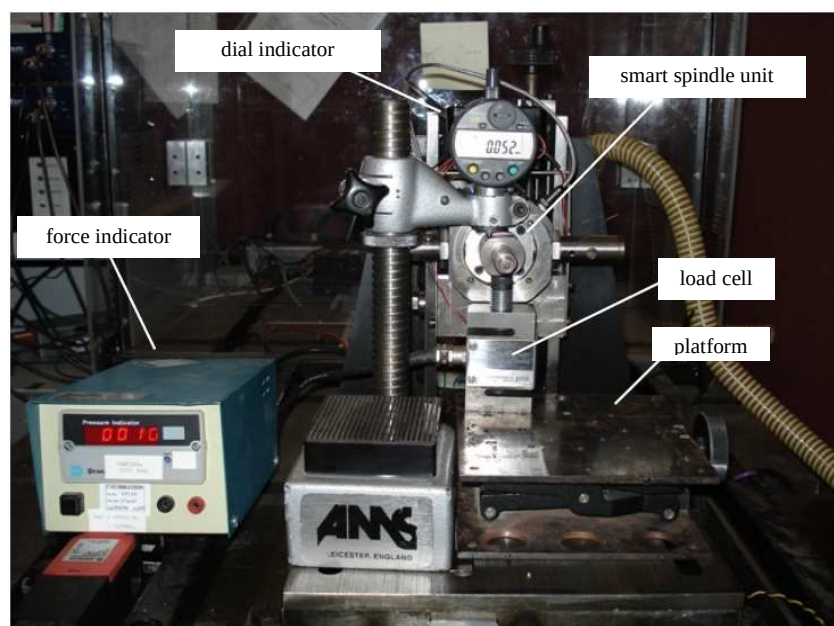


Fig. 5.87: Static tests

This result is also confirmed from some dynamic compliance measurements done on the mechanical part of the test rig, as depicted in Fig. 5.88 and Fig. 5.89.

The measured dynamic compliances are reported in Fig. 5.90.

We can observe that the model seems more rigid than the spindle prototype.

It is necessary to assert that the confidence interval obtained from the measurements is quite high due to the non simultaneous sampling of the force from the hammer and the displacement measured by the eddy current probes.

Moreover the effects of the piezo voltages have been studied: nevertheless this high confidence interval it seems that the voltage doesn't affect the dynamic behaviour of the system.

We are going to perform some other additional tests to increase the reliability of the proposed models.

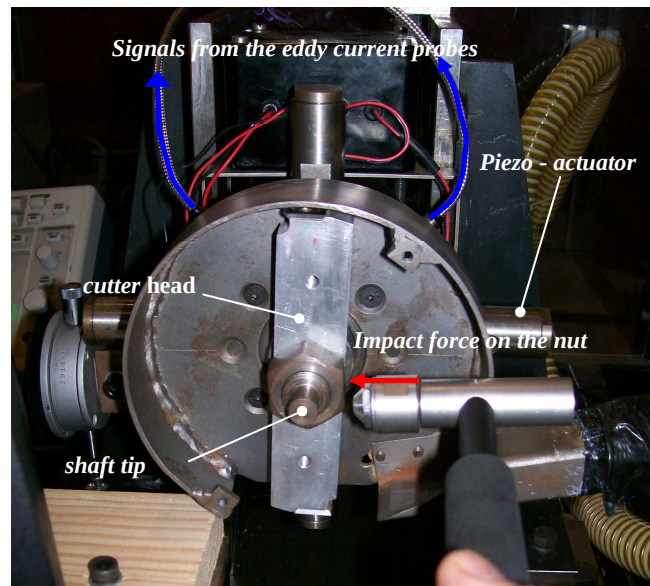


Fig. 5.88: Dynamic compliance measurement

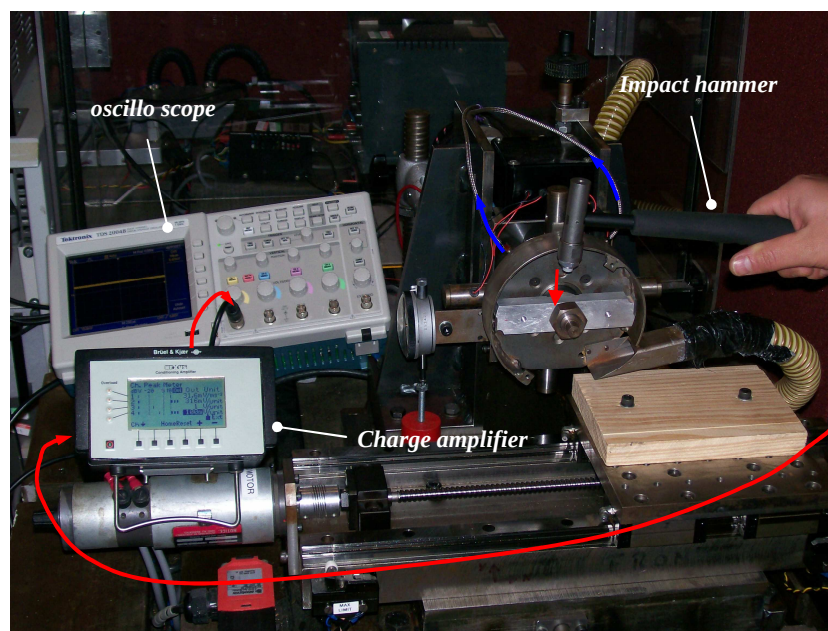


Fig. 5.89: Instrumentation used for the dynamic compliance measurement

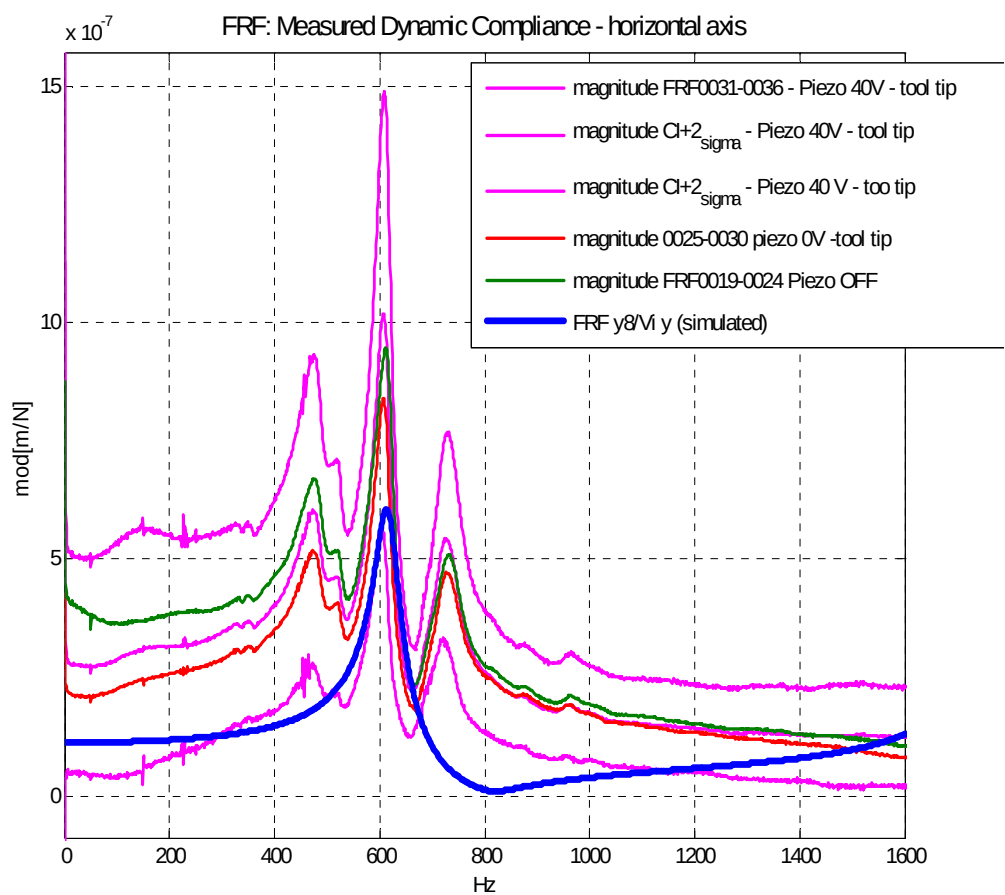


Fig. 5.90: Dynamic compliance measurement – tool tip/eddy current sensors

5.12 Discussion and future works – active vibration control

In this work the capabilities of an active system to enhance the Material Removal Rate during milling has been investigated. The technique seems very promising. The asymptotical stability limit was almost doubled; this has been verified both via time domain simulation and frequency domain approach.

After the definitive implementation and the consequent adjustment of the control strategy on the spindle prototype the damping capabilities will be experimentally verified.

Obviously the improvement of the dynamic performances of the spindle can be exploited also during wood cutting thus some cutting tests will be performed.

The expected enhancements especially on the surface quality will be opportunely investigated and moreover the dependence of the control strategy efficacy on the cutting parameters will be deeply analyzed.

As formerly explained in wood cutting applications the forces linked to the unbalance generally represent an important source of vibration, conversely in metal cutting the cutting forces play the dominant role.

In wood cutting application the regenerative chatter instability isn't a limiting factor so a more appropriate control strategy can be developed to reduce the tool tip vibration. The Iterative Learning Control (ILC) seems to be the adequate strategy to improve the performance of periodic process. We are moreover interesting to develop a combined technique based on LQG control and ITL.

The ILC isn't suitable for metal cutting because the regenerative chatter occurrence can't warrant the process periodicity. These future activities can be developed via the collaboration of the Laoughborough University.

6 Conclusion

In this thesis the research efforts have been focused mainly on the spindle system performance improvement especially for High Speed Machining applications. Different design solutions and strategies have been analyzed and developed in order to increase the Material Removal Rate and to obtain better workpiece surface quality.

In High Speed Machining the most important limiting factor is the occurrence of the cutting process instability usually called chatter instability. Frequently this phenomenon is strictly related to the spindle dynamic behaviour so a methodology to model and deeply comprehend the spindle system has been proposed. An intensive experimental activity was performed in order to update the proposed spindle models. These experimental modal tests, arranged on different spindle setups, have allowed to characterize and to evaluate the contribute of some important spindle components to the overall spindle dynamic behaviour.

The results of these activities consist of a set of modelling guidelines necessary to build reliable spindle models. These modelling methodologies and the linked updating tips can be used to easily test innovative design solutions. The design of a carbon fiber-epoxy composite material spindle shaft has been presented as a concrete example. Considering the hypotheses introduced in the proposed first rough analysis; the use of the designed composite shaft seems surprisingly not particularly promising if the MRR improvement is considered as the most important target.

Moreover, the dynamic interaction between the spindle system and the machine tool has been studied in this thesis. Two different modelling approaches to evaluate the influence of the machine structure on the dynamic behaviour of the spindle have been proposed. The first one is particularly suitable for spindle designers while the second one can be more easily adopted by the machine-tool designers. The analysis and the experimental tests performed on different industrial cases reveal the strong interaction between the two components especially considering the frequencies involved in high speed machining. It is therefore more important to consider these aspects during the spindle design to correctly evaluate the stability lobes diagram and the chatter free axial depths of cut.

In the forth chapter of this thesis the Spindle Speed Variation technique has been studied as a promising chatter suppression method. Particular attention has been focused on the deep comprehension of the complex effects of speed modulation on the regenerative phenomenon. The instantaneous chip thickness modulation that strictly depends on the SSSV parameters has found to be the key factor for the effectiveness of the technique. An energetic analysis has been proposed to properly select the main SSSV parameters. More precisely, the parameters selection is based on the minimization of the energy introduced in the machine

tool system by the cutting process. The reliability of the proposed fast approach has been checked comparing the results obtained from time consuming simulations.

The SSSV seems to be very effective in the chatter vibration reduction especially for low speed machining; in these regions of the stability chart the right selection of the modulating parameters is less critical than in high speed machining and therefore the technique shows an important robustness.

An experimental cutting test campaign would be necessary to strengthen the results obtained via time domain simulations.

Active vibration damping strategies have been analyzed as chatter suppression technique. An application to an active spindle system with four piezo actuators has been developed. A model based control strategy has been proposed in order to control the tool tip displacement and thus to improve the quality of the machined surfaces. Relevant enhancements in the cutting stability were observed from simulated results: the asymptotical stability limit was almost doubled. An additional control strategy based on the cutting force prediction has been designed in order to reduce the tool deflection occurred during the machining. A first stage of experimental tests has been performed in order to verify the damping properties provided by the control action. The necessity of a further tuning of the regulator has been observed.

A. Appendix: Machine tool experimental dynamic characterization

A.1. Dynamic compliance measurement

In this appendix it is presented the procedure used to measure the tool tip dynamic compliance. The Frequency Response Function has been computed from multiple (n_d) impact tests. It is possible to simultaneously measure the impact forces and the accelerations as described in Fig. 6.1.

It is necessary to compute the auto-spectrum of the force (X) and the cross-spectrum of the acceleration (Y) to evaluate the frequency response function by the estimator H_1 , Eq. 6.1.

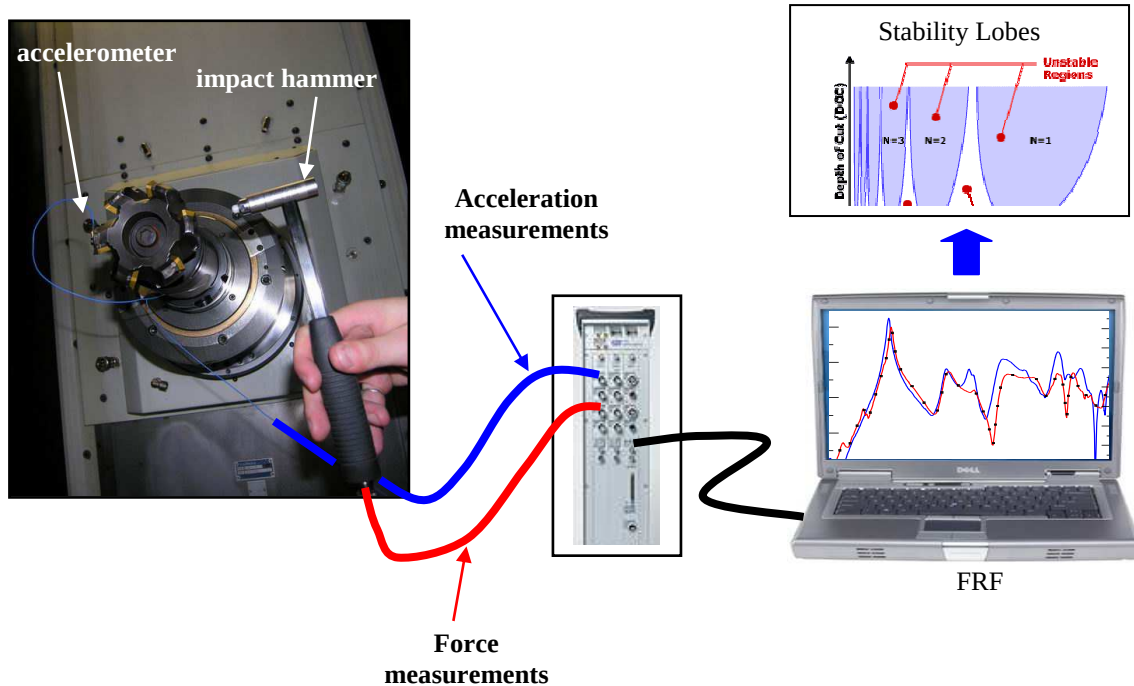


Fig. 6.1: Dynamic compliance measurements and lobes diagram computation

The coherence function can be used to check the reliability of the dynamic compliance estimation.

Moreover the modal parameters, Ewins [47] linked to the first three modes of the machine depicted in Fig. 3.14 have been computed and presented in Tab. 6.1 and Tab. 6.2.

Some cutting tests were performed in order to validate the lobes diagram.

The performed milling operations described in chapter 4 are depicted in Fig. 6.2.

Eq. 6.1:

$$\hat{G}_{xx}(f) = \frac{1}{n_d} \sum_{i=1}^{n_d} X_i^*(f) \cdot X_i(f)$$

$$\hat{G}_{xy}(f) = \frac{1}{n_d} \sum_{i=1}^{n_d} X_i^*(f) \cdot Y_i(f)$$

$$H_1(f) = \frac{\hat{G}_{xy}(f)}{\hat{G}_{xx}(f)}$$

	Mode#1	Mode#2	Mode#3	Mode#5
Modal stiffness [N/mm]	3.11E5	2.69E6	1.57E5	1.84e5
Frequency[Hz]	225	334	451	732
Damping ratio[%]	9.4	0.5	2.4	2.9

Tab. 6.1: Identified modal parameters – X axis

	Mode#1	Mode#2	Mode#3	Mode#5
Modal stiffness [N/mm]	1.81E5	1.55E6	1.84E5	1.81e5
Frequency[Hz]	290	460	747	1191
Damping ratio[%]	7.8	3	2.13	1.3

Tab. 6.2: Identified modal parameters – Y axis

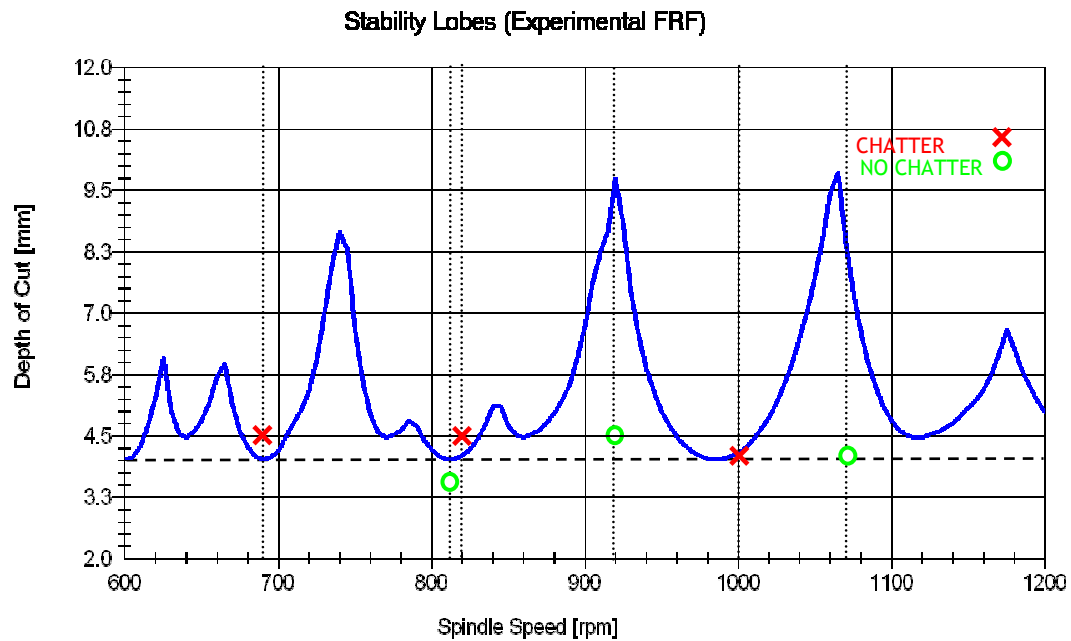


Fig. 6.2: Lobes diagram and cutting tests

B. Appendix: FEM spindle model

A.2. FEM description

Generally the rotor systems can be modelled with sufficient accuracy by one-dimensional elements, which make the actual implementation much easier. The basis of the FEM was adopted from Slavik et al. (1997) [148] and used to implement a simple FE pre-processor in MATLAB, Hynek [105]. A brief description of the FEM follows.

The rotor system is divided into elements with the use of nodes. In order to achieve maximum flexibility, 6 degrees of freedom (DOF) is considered in each node. While FE model with 4 DOF for each node is necessary to model rotor's transverse vibration, FE model with 6 DOF for each node also allows to model torsional and longitudinal vibration. Eq. 6.2 shows the vector of the node displacements that correspond to 6 DOF

$$\text{Eq. 6.2} \quad q_n = [u \quad v \quad w \quad \varphi \quad \psi \quad \theta]^T$$

where u , v , w are displacements in direction of x , y , z axes respectively and φ , ψ and θ are angular displacements around axes x , y , z respectively. The finite element model of the rotor system is created by assigning elements to the nodes. For example, rotor representing element is assigned to two nodes and disc-representing element is assigned to just one node. A number of element types, listed in Tab. 6.3, have been implemented to model typical parts of a rotor system such as rotor, disk, and bearing. Each element type has one or more of the following element matrixes

- M_c , element mass matrix
- K_c , element stiffness matrix
- B , element damping matrix
- G_c , element gyroscopic matrix

The element matrixes are used to assemble global matrixes for the whole rotor system to form a matrix equation of motion for the rotor system. A brief description of the basic element types follows.

The main element type is the element for modelling the rotating part of the rotor system (ROT). It is depicted in Fig. 6.3. The element has two nodes A (for $x=0$) and B (for $x=l$). The displacements of the two nodes form a local element displacement vector q_l^e which is expressed as follows.

Eq. 6.3:

$$q_l^e = \begin{bmatrix} q_{nA}^T & q_{nB}^T \end{bmatrix}$$

$$q_l^e = \begin{bmatrix} u_0 & v_0 & w_0 & \varphi_0 & \psi_0 & \theta_0 & u_l & v_l & w_l & \varphi_l & \psi_l & \theta_l \end{bmatrix}^T$$

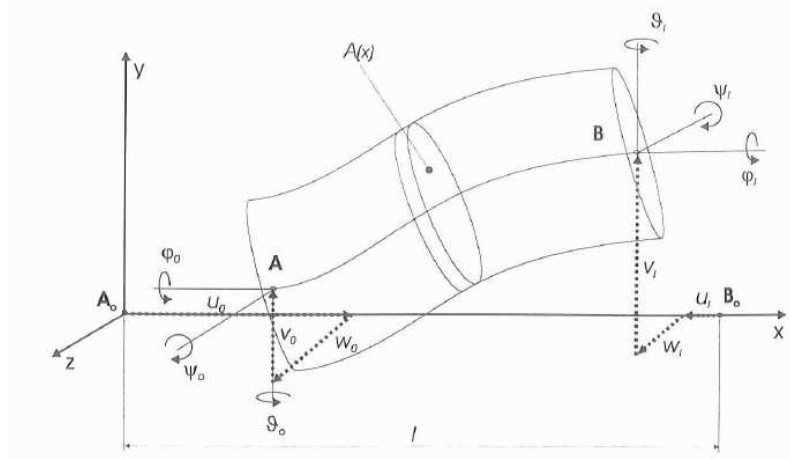


Fig. 6.3: Rotor element

The lateral deformation (i.e. v , w , ψ , θ) are approximated by a third order polynomial along the element length in order to ensure continuity and smoothness of the deformed rotor centre line. The longitudinal and torsional deformations (i.e. u , φ) are approximated by a linear polynomial along the length of the element.

Disc-like features, such as flywheels, cutter heads etc, are modelled as a rigid disc attached to the rotor. The element for modelling disc-like features (DISK) is shown in Fig. 6.4. It has only one node A and the element local displacement vector is as follows.

Eq. 6.4:

$$q_l^e = q_{nA}^T$$

$$q_l^e = \begin{bmatrix} u_0 & v_0 & w_0 & \varphi_0 & \psi_0 & \theta_0 \end{bmatrix}^T$$

Bearings and similar features that provide a point support for the rotor are modelled by the element (SUPPORT) depicted in Figure 7-3. The symbols in Fig. 6.5 with one index represent translatory stiffness and viscous damping in direction of corresponding axis (e.g. k_x , b_x , are stiffness and damping in direction of axis x). The symbols with two indexes represent stiffness and viscous damping around corresponding axis (e.g. k_{xx} , b_{xx} , stiffness and damping around axis x).

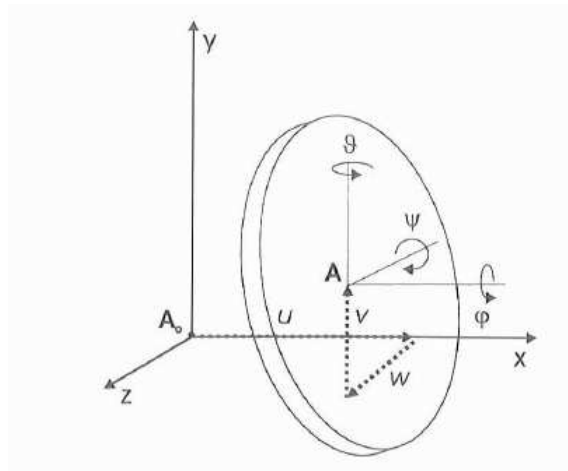


Fig. 6.4: Disk element

The local element displacement vector is identical to Eq. 6.4. There is also a variation of the element “SUPPORT” named “LINK”, which is intended to model a flexible link between two nodes. The “LINK” element has essentially two nodes and its local element displacement vector is identical to Eq. 6.3.

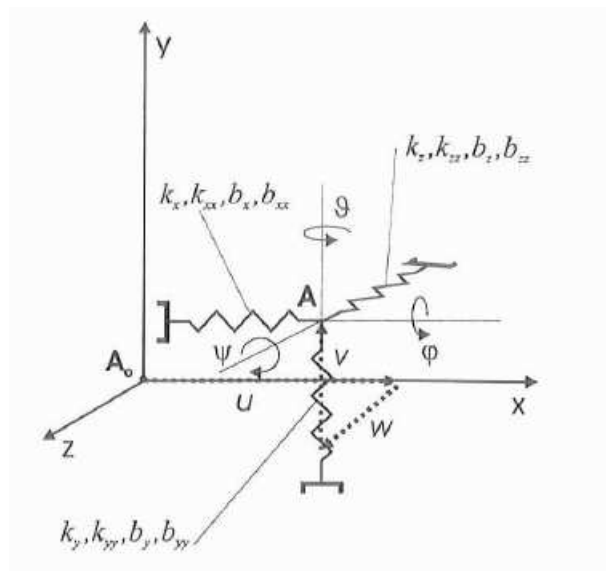


Fig. 6.5: Support element

Element name	Number of nodes	Number of DOF	Element matrix				Function
			$\mathbf{M}_e$	$\mathbf{K}_e$	$\mathbf{B}_e$	$\mathbf{G}_e$	
ROT	2	12	yes	yes	no	yes	Modelling of a rotating part of a rotor system
STATOR	2	12	yes	yes	no	no	Modelling of a stationary part of a rotor system
DISK	1	6	yes	no	no	yes	Modelling of symmetrical discs attached to rotor (i.e. flywheel, cutterhead)
S_DISK	1	6	yes	no	no	no	Modelling of symmetrical discs attached to a stationary part of a rotor system
SUPPORT	1	6	no	yes	yes	no	Modelling of a flexible rotor/stator support
LINK	2	12	no	yes	yes	no	Modelling of a flexible link between rotor and stator (similar to SUPPORT)

Tab. 6.3: Elements for modelling rotor system

The matrix equation that describes the FEM model has been reported in Eq. 6.5

$$\text{Eq. 6.5:} \quad \mathbf{M} \cdot \ddot{\mathbf{q}} + (\mathbf{B} + \omega \cdot \mathbf{G}) \cdot \dot{\mathbf{q}} + \mathbf{K} \cdot \mathbf{q} = \mathbf{f}$$

where $\mathbf{M}$ is the global mass matrix, $\mathbf{B}$ is the global damping matrix, $\mathbf{G}$ is the global gyroscopic matrix, $\mathbf{K}$ is the global stiffness matrix, $\mathbf{f}$ is the vector of external forces, $\mathbf{q}$ is the global displacement vector and ω is rotor speed.

The model of a rotor system consists of a number of nodes and elements that are assigned to the nodes. The node displacements from each node are assembled into the global displacement vector $\mathbf{q}$ as follows

$$\text{Eq. 6.6:} \quad \mathbf{q} = [\dots u_i \ v_i \ w_i \ \varphi_i \ \psi_i \ \theta_i \ \dots]^T$$

where index i denotes node displacements of i^{th} node. The size of the global displacement vector n is equal to the total number of DOF. The global displacement vector forms a global configuration space to which all the local element matrixes have to be transformed prior to assembling them into the global system matrixes. The transformation is performed with the use a transformation matrix $\mathbf{T}$, which depends on the element interconnection and needs to be determined for each element separately.

The transformation basically involves expanding the element's local displacement vector (i.e. equation Eq. 6.3 and Eq. 6.4 to the size of the global displacement vector q and rearranging the element node displacements so that their position in the expanded vector corresponds to their position in the global displacement vector Eq. 6.6. The size of the transformation matrix T is $n \times m$ where n is the total number of DOF of the rotor system model and m is the element number of DOF. Furthermore, the transformation matrix has non zero elements equal to 1 positioned so that the element's node displacements are placed on position that they have in the global displacement vector q .

The actual form of the transformation matrix depends on the nodes numbering.

The transformation of the i^{th} element displacement vector to the global configuration space is expressed as follows

$$\text{Eq. 6.7:} \quad q_{gi}^e = T_i \cdot q_{li}^e$$

The global matrixes of the FE model are then assembled by summing all the transformed local element matrixes as follows.

$$\text{Eq. 6.8:} \quad X = \sum_{i=1}^k T_i^T \cdot X_{ei} \cdot T_i = M, B, G, K$$

The FEM described in this section is implemented in MATLAB. The main part of the implementation is a simple FE pre-processor. The pre-processor assembles the global model matrixes from an element list, provided in an input text file (example of the input text file can be found in FEM input file). The pre-processor provides following functions

- Arbitrary constant boundary conditions can be introduced in each DOF. A coupling can be defined between two DOF, which are then treated as if they were identical. It is an analogy to connecting two DOF with rigid link. For example if torsional displacement in two element nodes is coupled, the element is torsionally rigid but it can still rotate as a rigid body.
- An additional proportional damping in the following form can be introduced. This is useful for introducing internal damping found in any structure.

$$\text{Eq. 6.9:} \quad B_p = \alpha \cdot M + \beta \cdot K$$

- The pre-processor can assemble an output matrix T_{out} which can be used to select only particular DOF from the global displacement vector as follows

Eq. 6.10: $q_{sel} = T_{out} \cdot q$

This is useful when FE model is converted into a state space model for assembling system output matrix.

The pre-processor can assemble an input matrix T_{in} , which can be used to create the vector of external forces f_{in} equation Eq. 6.5 as follows

Eq. 6.11: $f = T_{in} \cdot f_{in}$

This is useful when FE model is converted into a state space model for assembling system input matrix.

The parameters of the elements can be defined as a MATLAB expression, which are evaluated during the matrix assembly. This is used for building parameterised models, which can be then used for optimisation or model tuning. The parameters of the FE model (e.g. spindle diameter) can be controlled from a MATLAB script.

The main advantage of the FEM implementation in MATLAB is that it provides an open system that can be easily adjusted to particular requirements or integrated into more complicated model. Moreover, the computational and visualisation tasks can be transferred to the standard MATLAB functions, which simplified the actual implementation

A.3. FEM input file – active spindle

FEM Input file: mechanical model of the spindle

% wood spindle

% Elements definition

<MESH>

% EN	Name	Nodes	Par	l	d	d0	dens	E	nu	k
E 1	rot N 1	2	P	0.010	0.010	0	7800	2.1e11	0.3	1
E 2	rot N 2	3	P	0.011	0.016	0	7800	2.1e11	0.3	1
E 3	rot N 3	4	P	0.013	0.016	0	7800	2.1e11	0.3	1
E 4	rot N 4	5	P	0.006	0.016	0	7800	2.1e11	0.3	1
E 5	rot N 5	6	P	0.025	0.032	0	7800	2.1e11	0.3	1
E 6	rot N 6	7	P	0.007	0.022	0	7800	2.1e11	0.3	1
E 7	rot N 7	8	P	0.005	0.017	0	7800	2.1e11	0.3	1
E 8	rot N 8	9	P	0.087	0.0155	0	7800	2.1e11	0.3	1
E 9	rot N 9	10	P	0.054	0.008	0	7800	2.1e11	0.3	1

```
%E enum support N n1 P kx ky kz kxx kyy kzz bx by bz bxx byy bzz
```

```
E 10 support N 8 P 0 14e6 14e6 0 0 0 0 10 10 0 0.3 0.3
```

```
E 11 support N 9 P 0 0 0 0 7e3 7e3 0 0 0 0 0 0
```

```
% EN Name Nodes Par m IO I a e fi ni k
```

```
E 12 disk N 8 P 0.15 0.054e-3 0.028e-3 0 0 0 0 1
```

```
E 13 disk N 3 P 0.40e-1 0.166e-5 0.00000113 0 0 0 0 1
```

```
E 14 disk N 4 P 0.11 0.110e-3 5.7e-5 0 0 0 0 1
```

```
%E 11 disk N 3 P 0.15 0.054e-3 0.028e-3 0 0 0 0 1
```

```
</MESH>
```

```
% Component definition
```

```
<COMP>
```

```
all | 1 2 3 4 5 6 7 8 9 10
```

```
</COMP>
```

```
% Boundary condition definition
```

```
<BC>
```

```
N all | 1 0 0 1 0 0 | 0 0
```

```
N 9 | 0 1 1 0 0 0 | 0 0
```

```
</BC>
```

```
% Proportional damping
```

```
<PD> 0 1e-7 </PD>
```

```
%<PD> 1e7 1e-7 </PD>
```

```
% Output DOF definition
```

```
<OUT>
```

```
N 1 | 0 1 1 0 0 0
```

```
N 2 | 0 1 1 0 0 0
```

```
N 3 | 0 1 1 0 0 0
```

```
N 4 | 0 1 1 0 0 0
```

```
N 5 | 0 1 1 0 0 0
```

```
%N 6 | 0 1 1 0 0 0
```

```
%N 7 | 0 1 1 0 0 0  
N 8 | 0 1 1 0 0 0
```

```
</OUT>
```

```
% Input forces definition
```

```
<IN>
```

```
N 1 | 0 1 1 0 0 0  
N 2 | 0 1 1 0 0 0  
N 3 | 0 1 1 0 0 0  
N 4 | 0 1 1 0 0 0  
N 5 | 0 1 1 0 0 0  
%N 6 | 0 1 1 0 0 0  
%N 7 | 0 1 1 0 0 0  
N 8 | 0 1 1 0 0 0
```

```
</IN>
```

```
% XYZ nodes coordinates (used only for modes plotting)
```

```
<N>
```

```
N 1 | 0 0 0  
N 2 | 0.010 0 0  
N 3 | 0.021 0 0  
N 4 | 0.034 0 0  
N 5 | 0.040 0 0  
N 6 | 0.065 0 0  
N 7 | 0.072 0 0  
S 8 | 0.077 0 0  
S 9 | 0.164 0 0  
N 10 | 0.218 0 0
```

```
</N>
```

7 Bibliography

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